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SIGNALS OF HUMAN OPERATOR STATE

by A. N. Luk'yanov and M. V. Frolov

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A. N. Luk'yanov and M. V. Frolov

ANNOTATION

This book is dedicated to results of quantitative processing of measurements of electrical activity of the brain, intonation coloring of speech and a number of vegetative functions characteristic for the state of active attention and emotional stress in man. These data are necessary for objective testing of the state of man performing control functions, and for determination of his reliability under difficult (emergency) situations. In addition to the information produced in model psychophysiological experiments, the book contains a description of methods of processing signals recorded.

The book is designed for physiologists, psychologists, doctors, engineers, working in the area of the psychophysiology of labor and sports. Eighty illustrations, five tables, 105 bibliographic references.

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From the Authors

This book presents an analysis of the results of investigations of speech and electrophysiological signals as applicable to states of attention and emotional stress of operators. Also, methods are presented for processing, and a number of the results produced can be used to evaluate other human states. Where possible, we have attempted to present formulas and recommendations convenient for direct practical utilization. The book does not analyze problems related to evaluation of the state of an operator on the basis of a set of signals. The reader can acquaint himself with some information from this area in other works: R. S. Dadashev, Ye. N. Murashov, "The Possibility of Solving the Problem of Diagnosis of the Functional State of Man Using Computers," in the book *Chelovek i Avtomat* [Man and Automaton] (Nauka Press, 1965); D. Louli, A. Maxwell, *Faktornyy Analiz Kak Statisticheskiy Metod* [Factorial Analysis As a Statistical Method] (Mir Press, 1967); G. S. Sebestian, *Protsessy Prinyatiya Resheniy pri Raspoznavanii Obrazov* [Processes of Decision-Making in Pattern Recognition] (Kiev, Tekhnika Press, 1965). /3*

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* Numbers in the Margin indicate pagination in the foreign text.

Introduction

The Problem of Dynamic Observation of Operator State

Modern technology, so broadly extending man's control over his /5 environment, at the same time has placed new and increased demands on his labor. The reliability of the "human link" has become one of the decisive factors in the effective utilization of various control systems. The struggle for failure-free functioning of this link is conducted in three main directions. First of all, along the line of organizing the work of the operator so as to correspond to the maximum to the psychophysiological capabilities of man, his capacities for receiving and processing incoming signals into actuating commands. The complex range of problems of this type is one of the principal subjects of study for engineering psychology. The second condition of effective participation of man in "man and machine" systems is the degree of his training for the activity before him, improvement of training methods, the search for optimal volumes of training exercises and drills. Finally, considerable attention is given to professional selection of persons most suited for this type of activity. The individual specifics of the nervous system, the rapidity and accuracy of reactions, emotional stability, self-control during critical situations can be determined to a certain extent before the operator takes his place at the control panel.

Still, practice has shown that none of these methods eliminates the necessity of dynamic observation of the state of the operator during the process of his work, particularly if we are concerned with activity where even a brief decrease in reliability of the "human link" can lead to dramatic consequences. Human states can be divided into two large groups. One includes those states which practically exclude the operator from the control process, form a clear break in the previously closed circuit /6 (sleep, extreme fatigue, loss of consciousness). When states of the second type (strong emotional stress, distraction of attention) arise, the operator continues to participate in the control process, although his activities become less effective, the threat of missing significant signals and the appearance of false, nonmotivated commands arises. It is this second type of state which is the subject of the investigation whose results are the subject of this present report.

Before going over to the problem of the possibility of objective diagnosis of emotional stress and attention in humans, we must give at least a brief characterization of the nature of these two states. We assume that the physiological basis of emotions is activity of a special nervous apparatus, compensating in the process of adaptive behavior for a deficit of information necessary for organization of actions to satisfy the existing demands (Simonov, 1965a, 1966a, b).

The principal points of the "informational concept of emotions" can be formulated in the following five statements.

1. Emotions in the natural sense of this term (fear, joy, alarm) are different in principle from the so-called emotional tone of sensations (pain, contact satisfaction, etc.), so that the emotional tone carries within itself some significance concerning the subject, while emotions arise as a result of prognostic evaluation of the situation.

2. Emotions are not reducible to human demands or to actions designed to satisfy them.

3. There is a quantitative dependence of the degree of emotional stress on the demands and the difference between the information prognostically necessary and available at the given moment in the subject. This dependence can be represented in the form of the rule $E = -D(In-Is)$ where E is the emotion, -D is the demand, In is the information prognostically necessary for organization of actions to satisfy the given demand, and Is is the information actually available to the subject. We emphasize, that here and in the following the term "information" is used considering its pragmatic meaning, which can be defined as the change in probability of achieving a purpose due to the receipt of a message (Kharkevich, 1960). The correctness of the rule formulated above can be experimentally proven whenever the degree of emotional stress, magnitude of demand and deficit (or excess) of pragmatic information can be quantitatively evaluated.

4. Necessary and possible actions are compared not only in the process of performance of the latter (Anokhin, 1964), but before any action in the purely informational plane on the basis of "probabilistic prediction" (Feygenberg, 1963). Since the prediction may not correspond to /7
the information objectively sufficient for achievement of a purpose, the emotional reaction necessarily takes on a subjective character.

5. Positive emotions arise as a result of an excess of the available information (information received) over that predicted by the subject, i.e. when Is is greater than In. The temporary excess of pragmatic information appearing in a situation and the positive emotions stimulate the operator to set a new goal, not looking out over the "information desert." Thus, the compensatory significance of nervous mechanisms of the emotions in the process of the entire adaptive behavior of higher living organisms is seen once more.

The applicability of "information concepts" to evaluation of emotional stress arising in the process of labor activity has been demonstrated in studies both of comparatively elementary motor skills (Simonov, 1966b), and of complex actions of pilots in emergency situations (Yankelevich, 1965). Modifying our rule, B. M. Yankelevich introduced a number of clarifying coefficients. One of these coefficients determines the degree of danger through the probability of an accident, i.e. through the ratio of the number of accidents known to the pilot to the total number of flights of this type.

Many experiments using animals and precise psychological investigations have indicated the dependence of the success of performance of a task on the degree of emotional stress (Jerks-Dodson rule). Investigations have shown that for each type of activity there is a certain optimum of emotional stress, during which the reactions are most highly perfected and effective. Decreasing this emotional tone as a result of low demands or complete information on the subject (monotonous, stereotyped actions) leads to dreaming, loss of attentiveness, missing of significant signals and delayed reactions. On the other hand, extreme emotional stress disorganizes activities, complicates tendencies to untimely reactions, to reactions to extraneous and insignificant signals, to primitive actions such as blind search by the trial and error method. This is the reason dynamic observation of the degree of emotional stress of an operator can aid in predicting possible worsening of his working ability before it is reflected in the control itself.

The second state of interest to us -- attention -- can be defined as the selective (narrowed) readiness of the brain for certain reactions to strictly defined signals. This state is characterized by: a) inhibition (blocking) of signal channels entering the brain with the exception of the channel through which the most significant signals are expected or arrive; b) an increase in the sensitivity of the analyzer for the significant signals; c) an increase in the readiness of the actuator apparatus for reactions to this signal, which is manifested as a shortening of the latent periods. /8

The integral of all these changes (degree of inhibition of secondary channels, degree of increase of sensitivity of visual, auditory or other analyzer, degree of decrease in reaction time) can be looked upon as a measure of the degree of attention, as a measure of its intensity, stress, resistance to distracting factors. In other words, if the operator does not miss significant signals (is not distracted), but reacts to them rapidly and correctly, we can say that he has a high degree of attention. However, if he is distracted, reacts slowly, makes errors, we can say that his level of attention is low, if the degree of training of the operator for the given activity is equivalent in both cases.

Doubtless, attention depends on the significance of the signals, their value, which we will define as the change in the probability of achieving the goal (satisfaction of demands), resulting from the receipt of the given message. This dependence of attention on the demand and value of signals expected, common for attention and emotions, drawing these two states closer together, frequently masks the difference between them. However, emotion is always a function of the pragmatic uncertainty; emotion is stronger, the less the information the operator has as to how he should satisfy the demands being presented to him. In contrast to emotion, attention is more concentrated, more narrow, the fuller the information as to where, when and how the signals should appear, how and when the operator should react to it.

Like any stereotype, the state of attention has its deficiencies: the tendency to automatization, sluggishness, inertness, minimization of energy expenditures, etc. Therefore, the ideal working state for a human operator should include a combination of attention with a certain level of emotional excitation, i.e. with elements of pragmatic uncertainty introduced to the situation (Menitskiy, 1966).

In contrast to the concentrated attention, emotion is characterized by readiness to react to a broad range of presumed significant signals (in its extreme manifestation -- to any signals: "The burnt child dreads the fire"). Emotion converts attention to vigilance. The weak side of vigilance is the danger of false reaction to an indifferent (insignificant) signal. The weak side of automatized, narrow attention is the danger of missing a significant signal (the operator may become distracted or dreamy while waiting). Concentrated attention causes rapid fatigue. Excitation of narrowly tuned nervous structures is soon replaced by their inhibition, which leads either to predominance of the activity of other nerve elements (distraction of attention) or to general mental fatigue (drowsiness). Emotional excitation, on the one hand, causes the attention to become /9
broader (vigilance), and on the other hand increases the general tone of the brain.

Thus, attention causes the brain to decrease the number of effective signals (signals received and processed, causing a reaction), while emotion causes the number of these signals to increase. The optimum state involves a combination of these two opposite tendencies, the interaction of which is shown schematically on Figure 1.

Changes in the discriminating capacity of the brain have been the subject of systematic experiments conducted by L. D. Chaynova (1965). She showed that as habits become fixed, reaction rates increase to a certain extent due to minimization of the analysis of the input signal. This results in a danger of a decrease in the differentiating capacity, the appearance of false alarms. Precise analysis of stimuli requires a definite characteristic of the state of attention, involving special training.

The degree of emotional stress and attention can be judged primarily on the basis of the results of actions. Clearly, this judgment, although quite suitable in the process of training or selective testing, is not highly applicable to situations where operator errors can lead to very serious results. A second method of evaluating the state of the operator consists of sending test signals in parallel, in measuring actions in the control of the regulated object. We have in mind all sorts of tests which periodically appear against the background of the primary operator activity and require various response reactions. This sort of /10
activity testing makes it possible to detect decreases in the reliability

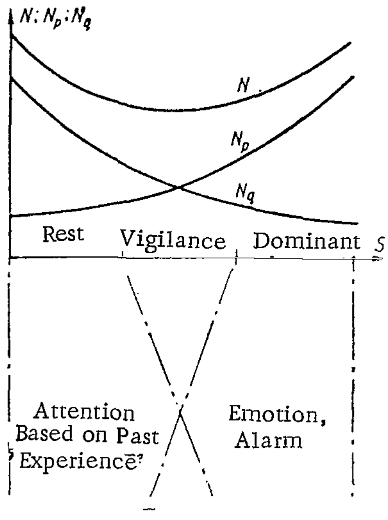

Figure 1. Nature of the Dependence of Errors on Human Operator State. S, Operator state; N_q, Quantity of significant signals missed; N_p, Number of false reactions to insignificant signals; N, Total number of errors; $(N = N_p + N_q)$

of the operator and provide timely reassignment of his function to a substitute (back-up device) or determination of the required changes in the operating regime (volume and rate of information flow, time given for decision-making, etc.) to correspond to the state of the operator. However, essential defects in this method are obvious.

First of all, the test signals frequently distract the operator from its main activity, disturbing him and creating an additional load. Secondly, the knowledge that of the signals arriving only a certain percentage are actually important for the control function creates a complex psychological situation in which the responsibility for action is decreased (after all, some of the reactions are only test, "game" reactions). Finally, the responses to the test signals, being arbitrary, undergo the influence of the intentional efforts of the operator, sometimes masking his true state. At the cost of mobilization of all his efforts, the operator will be able, up to a certain moment, to react effectively not only to the "main," but also to the "test" signals, although his functional state has already gone beyond the limits of optimal working ability. If the test signal does not react, the termination of compensation may be so sudden that it cannot be foreseen and prevented.

All of this stimulates us to turn our attention to testing using involuntary changes in physiological operator functions, objectively recorded during the process of his operator activity. Various symptoms of emotional stress and attention described in the literature are presented in Table 1. We can see that the set of these symptoms is not as broad as the engineering psychologist would like, particularly considering that many of the indicators (for example, changes in the chemical composition of the blood or urine) are practically inaccessible for observation during the process of operator activity. However, still more difficulties arise to hinder technical realization of the automatic testing system.

In the investigations of psychophysiologists, in works on professional selection, evaluation of the electrophysiological and vegetative changes is generally performed by a specially trained person (doctor, psychologist) on the basis of his experience, observations and intuition. The

TABLE 1. OBJECTIVELY RECORDED SIGNALS DURING EMOTIONAL REACTIONS AND THE STATE OF ATTENTION IN MAN

Physiological Indicator	Emotional Reactions		Quiet Attention
	Negative	Positive	
Electrical activity of the brain			
Electroencephalogram with surface leads	Depression (in many cases -- exultation) of α rhythm, reinforcement of θ rhythm within 4-7 Hz range and high-frequency β-activity;	Reinforcement of θ rhythm	Tendency to depression of α rhythm
Electroencephalogram with leads from deep structures	Tapered oscillations in hippocampus and amygdaloid nucleus		
Level of constant potential with leads from deep structures	Oscillations in level stronger, the stronger the emotional reaction of the subject	Change less stable the more negative the emotions.	
Responses caused			Increase in amplitude, change in composition and manifestation of components
Electrical phenomena in skin			
Fluctuations of potential	Manifest with unexpected inputs		
Change in resistance to external current	Decrease in resistance		
Heart rhythm	Increase, less frequently decrease in pulse rate	Increase in rate	Tendency to decrease and stabilization
Overall blood pressure	Increase	Increase, lagging behind negative emotions	
Respiration	*Change in frequency, depth, form and relationship between duration of inhalation and exhalation*		
Main reflexes			
Pupil diameter	Primarily expanded		
Rapid eye movement	Decrease in movement		
Blinking	Increase in blinking		

TABLE 1. (Continued)

Physiological Indicator	Emotional Reactions: Negative	Emotional Reactions: Positive	Quiet Attention
Reaction of musculature			
Stress of skeletal muscles	Increases	Increases moderately, sometimes decreases	Increases moderately
Tremor	Reinforced		
Smooth skin muscles	Contraction occurs with certain strong effective reactions		
Intonation of Speech	May be subjected to objective instrumental analysis		
Content of biologically active materials in the blood	Fear -- increase in adrenalin and decrease in noradrenalin Rage -- increase in noradrenalin decrease in adrenalin		
	Alarm -- increase in both materials		
	Grief -- decrease in both materials		
Content of biologically active materials in urine	Change in content of adrenalin, noradrenalin and 17-ketosteroids		

available literature contains practically no quantitative, unambiguous data on symptoms of emotions and active attention which could be used to develop an automatic diagnosis system. Comprehensive investigation and strict evaluation of signals of the human operator state is currently a central problem in the search for means of truly objective testing of this state.

The work of A. N. Luk'yanov and M.V. Frolov represents one of the first attempts to fill in this gap. Lased on a diagram of the combination of "man and machine" new in principle, where the influence of man on the machine is considered with dynamic consideration of the reverse influence of machine on operator (Simonov, Temnikov, 1965; Ivanov, Simonov, 1965; Luk'yanov, Frolov, 1966), they present a quantitative evaluation of a number of objectively recorded changes which characterize the state of rest, active attention and emotional stress in man. The initial data were produced by the authors in experiments modeling certain aspects of operator's activities. Changes in the electrical activity of the brain, skin, pulse frequency and respiratory movements were recorded and processed. One interesting and promising signal was found to be the intonation characteristic of the speech. Doubtless, many problems touched upon in these investigations require further development. Nevertheless, the results produced can be used as a good basis for practical realization of a system of dynamic observation of the state of the human operator. /13

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CHAPTER 1

THE HUMAN OPERATOR IN THE CONTROL SYSTEM

ABSTRACT. This chapter presents a broad overview of the problem of reliability of the human operator in the control system. It is pointed out that human operator reliability is maximal only at a certain level of operator stress, stress levels' either too low or too high resulting in a decrease of operator reliability and performance. Various methods of analyzing man-machine systems, looking upon the operator as the controlling or the controlled element, are briefly analyzed.

Throughout the history of the development of technology, the position /14
of man in relation to machines has been changing. For many years, machines were primarily used to transform human energy, lightening physical labor; man acted as the source of energy. The primary functions of man in relation to technology were his energy functions. With the development of technical progress, the position of man in the "man and machine" system has changed. The accumulation of knowledge on the functions of the human operator have allowed us to create machines in which the activity of the operator has been partially or completely replaced by semiautomatic and automatic devices. The role of man has become different, reduced to observation and control of the work of the automated devices, and the performance of preventative maintenance. With the construction of electronic computers, logical automata and other cybernetic devices, designed to mechanize the solution of a number of problems earlier performed by man, thereby becoming an extension and reinforcement of his brain (Kopnin, 1964), the possibilities of man increased significantly. However, even in these systems the role of man remains as before: control, observation and in case of failure -- active interference. Man performs another role in systems designed for operation with incomplete initial information, under conditions in which the hard programs are not able to provide for the solution of the problems required. In systems with incomplete initial information, sometimes nondeterministic systems, man is one of the principal links of the system, since he is capable of making decisions in situations which could not be predicted, and in situations in which available prepared solutions cannot provide satisfactory results for the operations.

§1.1. Necessity of Testing of Operator State

In *nondeterministic systems, the functions of the human operator* as a link in the system can be reduced to perception of and evaluation of incoming information with subsequent decision-making and output of commands for actuation. The quality of performance of these demands by the human operator, which determine the functional operation of the link, also /15
defines the quality of functioning and, in the final analysis, the reliability of the entire system. The reliability here is taken to mean the capability of the system under certain operating conditions to retain its characteristics within limits providing acceptable performance of the tasks for which the system is designed. One possible means leading to an increase in reliability is optimal matching of elements of machine design to the "parameters" of the human operator, effective distribution of functions between man and machine, optimal organization of the operator's activity and, finally, education, training and selection of operators for work in automatic control systems.

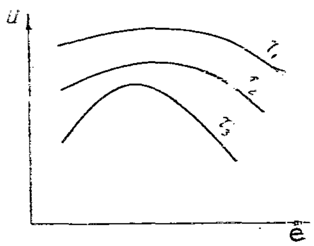


Figure 2. Effectiveness of Work (u) As a Function of Emotional Stress (e) for Work of Varying Difficulty (τ) ($\tau_1 < \tau_2 < \tau_3$)

The participation of man in the "man and machine" complex unavoidably introduces the factor of emotionality. Emotions have a varied and powerful influence on the vital activity of the organism: they mobilize the energetic resources, increase the functional capacities of the muscles, heighten the sensitivity of receptors, increase the tone of the higher segments of the brain, "open" the storehouse of subconscious memory to which we attribute the *finding of intuitive solutions*. At the same time, emotions help us to concentrate our attention, suppressing secondary activities and the demands on which they are based (Simonov, 1966c).

In addition to the positive influence of emotions on the activity of the organism, the negative effects of high levels of emotional stress are well known. These effects, expressed as a reduction in the functional state of the organism, appear most clearly with negative emotions in stress situations. This factor is well illustrated by the Jerks-Dodson principle (Figure 2).

In correspondence with the Jerks-Dodson principle (Jones, 1962), the effectiveness of work (u) is a function of emotional stress (e): $u = f(e)$ when $\tau = \text{const}$, where τ is the difficulty of the work being performed. It is interesting to note that for a certain type of work ($\tau = \text{const}$), the effectiveness of performance reaches a maximum $u = u_{max}$ only at a certain level of emotional stress $e = e_{opt}$. A change in the emotional stress in either direction from e_{opt} results in a decrease in the effectiveness of /16 the work. It is not difficult to see from this that measures taken earlier to facilitate an increase in the reliability of the operator are somewhat static, since they do not relate to the current state of the operator (his attention, level of ability, emotional stability, etc.) which is a characteristic of the "man and machine" system which changes dynamically during the process of work itself.

A second means is available for increasing the reliability of the "human operator" link, namely: testing the state of the operator during the process of work, primarily testing his attention and emotional stress as the determining states in the activity of operators in "man and machine" systems.

Testing the state of the operator using tests has a number of serious defects. Special test signals, the responses to which could characterize the state of the operator, are a form of outside "intervention" in his activity, distract his attention and are hardly suitable for unforeseen, emergency situations. However, this is only one aspect of the matter. Another, equally important aspect is that the arbitrary motor reactions attract the attention of higher compensatory mechanisms, and disruption of motor reactions may appear too late; it may immediately precede the critical decrease in reliability involving loss of consciousness and complete failure of the control system. In this sense, changes in the bioelectric activity of the brain and vegetative functions have a number of advantages: their recording does not complicate the activity of the operator, they are developed involuntarily and can be used as a timely signal of any decrease in operator reliability.

§1.2. Electrophysiological Testing

In our analysis of the problem of testing the operator's state[1] we will base ourselves on the following position. Each state of the human organism, representing a set of interrelated systems (central nervous system, cardiovascular system, etc.) can be set in correspondence to the values of the physiological parameters characterizing the activity of these systems. If we know the mean values of physiological parameters for each state, we can use comparison of these values to the instantaneous states to determine which of the M possible states corresponds to the instantaneous state.

For a number of reasons, direct observation of changes in physiological parameters is difficult; therefore, they are investigated using /17

[1] Electrophysiological testing means testing using electrical processes which are reflections of the bioelectrical activity of the systems of the organism, and are the results of conversion of changes in nonelectrical quantities to electrical quantities using special transducers.

indirect methods. At the present time, one of the principal methods of studying biological processes, for example in investigations of certain aspects of the activity of the central nervous system of animals and man (Kozhevnikov, Meshcherskiy, 1963) is the method of studying the electrical activity of various organs. Interest in this method can be explained by two factors: on the one hand, the accessibility of recording changes in biopotentials which accompany changes in a number of physiological parameters of the primary systems of the organism (brain, heart, muscles, etc.) at various points on the body, and on the other hand, the fact that electrical signals are the most convenient form of representation of changes in physiological parameters, allowing effective storage, performance of mathematical operations, speed of operation, etc. For this same reason, transducers which convert nonelectrical quantities to electrical quantities are used to record changes in a number of physiological parameters which are not accompanied by electrical activity, particularly respiration in the pneumogram (PG).

Thus, information on the state of the operator may be represented in the form of a set of values of parameters (components) of electrical signals characterizing the physiological parameters of the entire organism.

In geometric interpretation, these values can be looked upon as the coordinates of a multivariate vector in space, the coordinate axes of which correspond to the components of the electrical signal being analyzed. If Q^j electrical signals are selected for testing the j-th system of the organism, each signal being characterized by several components, the description of the j-th system in the i-th state of the organism A^{ij} can be represented by the matrix

$$A^{ij} = \begin{Vmatrix} x_{11}^{ij} & x_{12}^{ij} & \dots x_{1h}^{ij} & \dots x_{1H_1^j}^{ij} \\ x_{21}^{ij} & x_{22}^{ij} & \dots x_{2h}^{ij} & \dots x_{2H_2^j}^{ij} \\ \dots & \dots & \dots & \dots \\ x_{q1}^{ij} & x_{q2}^{ij} & \dots x_{qh}^{ij} & \dots x_{qH_q^j}^{ij} \\ \dots & \dots & \dots & \dots \\ x_{Q^j1}^{ij} & x_{Q^j2}^{ij} & \dots x_{Q^jh}^{ij} & \dots x_{Q^jH_Q^j}^{ij} \end{Vmatrix},$$

where i, j, q, h are the instantaneous numbers of the corresponding state, system, electrical signal and electrical signal component; Q^j is the number of electrical signals characterizing the j-th system; x_{qh}^{ij} is the value of /18
the h-th component of the q-th electrical system in the j-th system with

the i-th state; H_q^j is the number of components of the q-th electrical signal. In the general case

$$H_1^j \neq H_2^j \neq H_3^j \neq \ldots \neq H_q^j \neq \ldots \neq H_{Qj}^j;$$

A^{ij} is the i-th state of the j-th system.

Let us expand the ν-dimensional vector, writing it in the form

$$A^{ij} = \| x_{qh}^{ij} \| = \vec{x}^{ij};$$

$$\nu = \sum_{q=1}^{Qj} H_q^j.$$

Then, the i-th state of the whole organism A^i is represented by a μ-dimensional vector

$$A^i = \left\| \sum_{j=1}^{N} x_{qh}^{ij} \right\| = \vec{x}^i, \tag{1.2.1}$$

where N is the number of systems in the organism being tested;

$$\mu = \sum_{j=1}^{N} \sum_{q=1}^{Qj} H_q^j. \tag{1.2.2}$$

As a result, each state $A^1, A^2, \ldots, A^i, \ldots, A^M$ corresponds to its own vector $\vec{x}^1, \vec{x}^2, \ldots, \vec{x}^i, \ldots, \vec{x}^M$, the coordinates of which are components of the electrophysiological signals.

§1.3. Structural Diagram of "Man and Machine" System with Control of Functional State of Human Operator

The problem of objective testing and control of the functional state of the human operator is a very pressing and complex problem. This problem has been the subject of many works which have described testing and control of the functional state of man (so-called autostimulation) performed by devices which can be reduced to single loop closed control systems (Beschereva, Usov, 1960; Zhukov, 1963; Ivanov, Simonov, 1965; Walter, V. I., Walter, W. G., 1949; Shiptan, 1949; Hewlett, 1951; 1951; Mullholland, Runnals, 1962; Dusaily, 1963; Mullholland, 1962, 1964, etc.). One characteristic feature of a system of this type is that the object controlled is man himself, while control of his functional state and testing are performed by means of bioelectrical processes accompanied with involuntary reactions of the organism. The same principle can be used to to construct improved devices with adaptive properties (Simonov, Temnikov, 1965). /19

One essential characteristic of another type of "man and machine" system is their construction according to a well known diagram (Figure 3, 4) in which man is not the controlled object. An example of one such system might be bioelectronic prostheses, in which signals arising during voluntary motor reactions are used (Korbinskiy et al., 1961; Battye, Nightingale, Whillis, 1955).

Both of these types of systems have their defects. Systems of the first type, designed for investigation of various aspects of the functional state of man, are hardly suitable for practical application, since man is the controlled object, closed upon himself through a special device, thus forming a closed control system without any exit to the outside world, in the direct sense of the word.

One defect of the second type of system is its failure to consider perturbations acting on man as the controlling link in the system, which can influence the reliability of the entire system in an important way.

In this connection, there is considerable interest in a combination of the two types of systems to a single "two-loop" system, which does not have these defects.

One typical diagram of a "man and machine" system is the closed control system (Fel'dbaum, Dudykin, Manovtsev, Mirolyubov, 1963) (see Figure 3). Goal function W arrives at the control link from without, containing information on the instructions given to the system concerning the required nature of the process being controlled. The form of the goal function is the form of the control (stabilization, programmed control, tracking). Under the influence of perturbations (noise) Z, the controlled process may assume a character other than that defined by the goal function. In correspondence with information X concerning the course of the process and the goal function, the control link applies influence Y to the control object, designed to decrease the deviation resulting from perturbation Z.

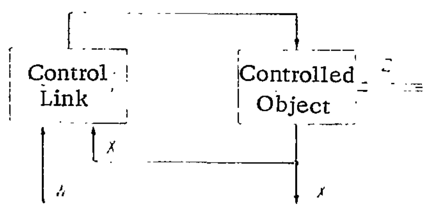


Figure 3 (Explanation in Text)

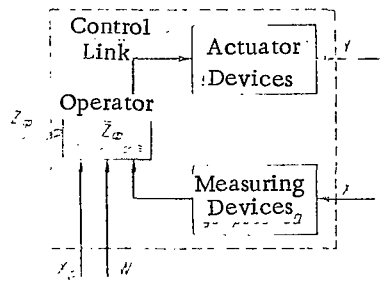


Figure 4 (Explanation in Text)

The diagram shown on Figure 3 does not consider perturbations (Z_ϕ, $\tilde{Z}_\phi$) acting on the control link, or the influence of these perturbations on the quality of control of the object.

Figure 4 shows the control link in expanded form, combining the actuator devices serving to act on the object and the measuring devices designed for transmission of information concerning the state of the object (human operator) performing the operation in correspondence with goal function W. The characteristics of the control link in the presence of a human operator are essentially dependent on the functional state of the /20
operator (Vinogradov, 1958; Denisov, Kuzminov, Yazdovskiy, 1964; Kosilov, 1957; Rubinshteyn, 1958; Gorbov, 1963; Furevich, Edel'man, 1965; Zarakovskiy, 1966; Nebylitsyn, 1961; Lomov, 1966, etc.), resulting from perturbations of the external (Z_ϕ) and internal ($\tilde{Z}_\phi$) medium. Therefore, it seems to us that continuous consideration of the functional state during control of the process X, requires that the diagram (Figure 3) be supplemented and converted to a "two-loop" closed control system (Figure 5). In this system, the first "loop" is the previous system (see Figure 3), while the second "loop" differs from the first in that the controlled object is the operator himself, and the controlled process is his functional state (A). X_ϕ is the information, characterizing in some way the functional state of the operator according to expression (1.2.1), and is represented in the form $X_\phi = \vec{x}^i$; Y_ϕ is the controlling action on the operator, bringing him to the required state. In the case of nonpermissible deviation of the functional state from the required state, action Y_ϕ, depending on the nature of the process being controlled X either provides

for cessation of control of process X, or replacement of the operator. W_ϕ is the goal function for the "loop" controlling the functional state of the operator.

Thus, a "two-loop" system, in addition to the controlling actions of man on machine, calls for controlling influences of the machine on the man in order to increase the reliability of the "man and machine" system as a whole. Systems of the "man and machine" type with functional control of the state of the operator can be broadly used in various branches of the national economy. For example, in railroad transport a demand is already being felt for automatic warning devices to increase the safety of rail travel, and in some cases to reduce operational expenditures (reduction of number of servicing personnel) without decreasing the reliability of transport operations. The requirement, in particular, to have a warning device, which could automatically and timely decide that it was necessary to interfere into the operation of the "man and machine" system, since it is very difficult for man himself to evaluate his own objective capabilities and interrupt his own control function, giving over his functions to a replacement or safety automatic device, has resulted in the development of the "two-loop" system (Luk'yanov, Frolov, 1966). /21

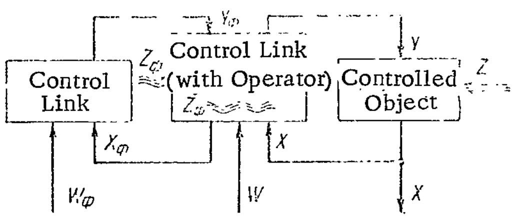


Figure 5 (Explanation in Text)

The state of the operator resulting from the complex and multifaceted interaction of the organism with the surrounding medium is objectively reflected in physiological processes. The great variety of interactions of the organism and medium with various perturbing influences leads to significant fluctuations in physiological processes; therefore, in studying their parameters we must consider them to be random functions of time. When certain conditions are observed, the probability characteristics of these random fluctuations can be estimated only for a limited number of processes. In order to produce the probability characteristics necessary for realization of a "two-loop" system, it is desirable to use sources of information which are closely related to the principal structures participating in the operator activity of man. The leading organ, which generalizes, coordinates and controls all systems of the organism is the central nervous system (CNS). It has been established that the influence of the cerebral cortex extends not only to activities of the internal organisms, but to such complex internal processes as the permeability of cell membranes, chemical regulation of heat liberation, cellular respiration, the metabolism of gases, proteins, fats, carbohydrates, mineral salts and enzymes. Testing of biochemical processes in the intact organism is practically impossible, and can be used only in a limited manner (chemical analysis of the composition of air exhaled, urine, glandular secretions, etc.). Accessible sources of information concerning the state of the human operator, as we have noted, include the bioelectric processes: the electroencephalogram (EEG), the electrocardiogram (EKG), the electromyogram (EMG), the skin-galvanic reaction (SGR), etc., as well as nonelectrical characteristics of respiration, blood pressure, body temperature, speech, etc.

If the number of possible operator states is large, a large number of bioelectrical processes, signal components and knowledge of probability characteristics of the corresponding random processes may be required. In many cases, the matter becomes practically impossible. Considering this, as well as the fact that one of the most important manifestations of changes in the concordance between mental processes and functions during the course of human activity is the change in attention characteristics, as well as the fact that different operators withstand critical conditions in different ways (Lomov, 1963), two characteristic operator states were taken for investigation: the state of attention and the state of emotional stress. /22

The present work is dedicated to the investigation and possible methods of processing of various signal components characteristic for these states.

We note in conclusion that the methods of testing the human operator state analyzed can be extended to more complex cases of human activity (Miles, 1965; Morozov, 1967).

CHAPTER 2

EXPERIMENTAL METHODOLOGY

ABSTRACT. The authors describe four series of experiments which were undertaken to analyze the influence of emotionality and stress on operator efficiency. The experiments included reception of a light signal, the response being to press a button immediately upon receipt of the signal, testing attention stability; selection of an audible signal, consisting of a series of numbers, in which the test subject was to recognize when a number was being repeated; search was visual signal, involving recognition of and counting of certain types of patterns on a pattern-filled secrre; and electron beam control, in which an electron beam was to be made to match a curve on a oscilloscope by manual control.

Based on the works of Soviet and foreign scientists (Anokhin, 1964, 1966; Simonov, 1965a, b; 1966a, c; Linsley, 1960), we can devise an approximate system describing the formation of the states being investigated, their influence on the organism and on the various electrophysiological processes which characterize the vital activity of the organism (Figure 6). /23

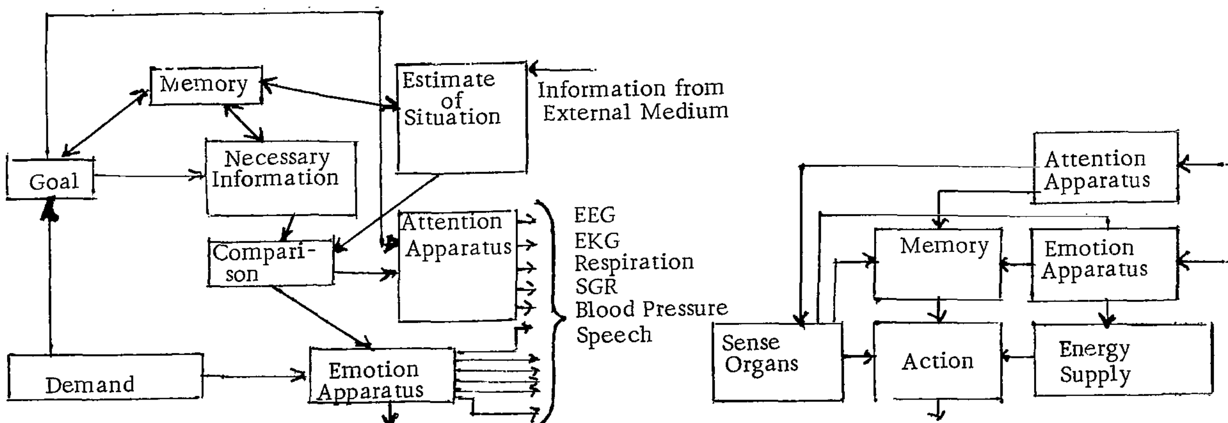


Figure 6 (Explanation in Text)

Above (see Introduction) we noted that emotion is a function of the magnitude of demand and the pragmatic deficit of information. Using the apparatus of memory, containing information both inherent and accumulated on the basis of experience, the demand arising in the organism (see Figure 6, A) is transformed into a concrete goal. Memory also contains information concerning necessary actions in regard to the goal, allowing methods of satisfaction of the demand to be predicted.

The compensatory mechanism of emotion is activated when a comparison of the difference between information prognostically necessary and information available at the moment (evaluation of situation) to the subject arises.

Emotions have varying influences on the vital activity of the organism (see Figure 6, B): they increase the tone of the higher cerebral segments and sense organs, influencing the lower stages of the organizational hierarchy, "opening" the storehouse of subconscious memory which we attribute to the determination of intuitive solutions, increase the functional capabilities of the muscles, and stimulate the energy supply for action. This is expressed in those changes which occur in the electro- /24
physiological indicators and processes of speech formation (the frequency of the pulse and respiration, and intonation characteristics of speech, etc. change). The operation of the attention apparatus is also determined by the degree of information concerning a certain goal (demand) and the significance of the goal itself (see Figure 6, A).

However, as was stated above, emotion is always a function of pragmatic uncertainty: emotion is stronger, the less the information the human has concerning satisfaction of the demand. In contrast to emotion, attention is more concentrated the narrower, the more complete the information concerning where, when and what signals should appear, how and when the operator should react to it when it does appear.

When a demand arises and is made concrete into a goal, a restructuring of the nervous and mental process occurs, with a concentration of mental activity in the required direction. Changes of this process when attention is present represent primarily an increase in the sensitivity of those analyzers through which the information arises (or through which its arrival is expected), necessary to achieve the goal. At the same time, the remaining channels for information reception are blocked, the operation of the organs of memory become more purposeful, the readiness of the actuator apparatus to reaction to signals received is increased, expressed, for example, in a decrease in latent periods (see Figure 6, B).

The appearance of attention as a state of the central nervous system should be noticeable in changes in the bioelectrical activity of the CNS. Also, modern psychology believes that attention reflects the direction of the entire complex activity of the organism, and therefore doubtless participation in the physiological mechanisms of attention of changes in a number of vegetative functions (Griew, Davies, Treacher, 1963; Walter, R. D., Jeager, 1956; Mundy-Castle, 1957; Oswald, 1957; Mirtschew, Penov, 1963; Slatter, 1960; Travis, Ohanian, 1954).

The dynamics of development of the states analyzed can be characterized, on the one hand, by actions on the surrounding medium, and on the other hand by continuous comparison of necessary and available information concerning the satisfaction of demands (instantaneous analysis of the situation).

We have modeled conditions of certain situations of attention and emotional stress in the laboratory in order to produce the signal changes in the processes being investigated. In a number of practically important cases, the work of the operator in the "man and machine" system can be reduced to functions of tracking, search and selection of signals, and to functions of control. This activity is usually not related to great physical stress, but requires considerable stress of the attention and precise, rapid reactions to incoming signals. The perception of incoming information is performed primarily through the visual analyzer (Parin, Bayevskiy, 1966). /25

Considering these facts, four series of experiments were undertaken, in one of which the information arrived through the auditory analyzer, and in the three others -- through the visual analyzer.

A. Reception of light signal. Flashes of light of 10^{-4} sec in length were applied to the closed eyes of the subject at random time intervals between 4 and 16 sec. The values of the intervals differed by 1-2 sec and were evenly distributed. When a flash appeared, the test subject, according to the instructions, was to press a button immediately. The specific feature of this work is that success depends primarily on the stability of attention.

The work (state B) continued 5-7 min, alternating with pauses (state D) of 3-5 min length.

B. Selection of an audible signal. Using a tape recorder, the test subject was exposed to a sequence of numbers from 0 through 9, evenly distributed, at intervals of 1.5 sec. The numbers were selected in this series with equal probability and repeated. The mean interval between pairs of identical numbers was 15 sec. When a repeated number was detected, the test subject was to press a button.

C. Search for a visual signal. Sets of rings, consisting of 510 rings in each frame, were projected onto a 0.7 × 0.7-m screen at a distance of 2 m from the test subject. Each ring had a notch at one of eight positions. The test subject was instructed to count rings with notches oriented in a given direction as rapidly as possible without mistakes. At the same time, when a proper ring (signal) was found, a button was to be pressed immediately. The sets of rings, modifications of the correction tables of Genkin, Medvedev and Shek (1963), were composed and presented in a sequence designed to exclude the possibility of the influence of memory on the results of the experiment. Frames showing rings located in rows were "working" frames. Between the "working" frames, a pause of 2-3 min was allowed, during which time a set of rings in columns was shown, which were not to be counted. In order to observe the changes occurring during changes in the state (B, D), the sets of rings were automatically changed. The appearance of a frame with rings located in rows was the signal to begin counting. Figure 7 shows four frames with sequential sets of rings. The second and fourth frames are "working" frames, the other two with rings in columns are "nonworking" frames.

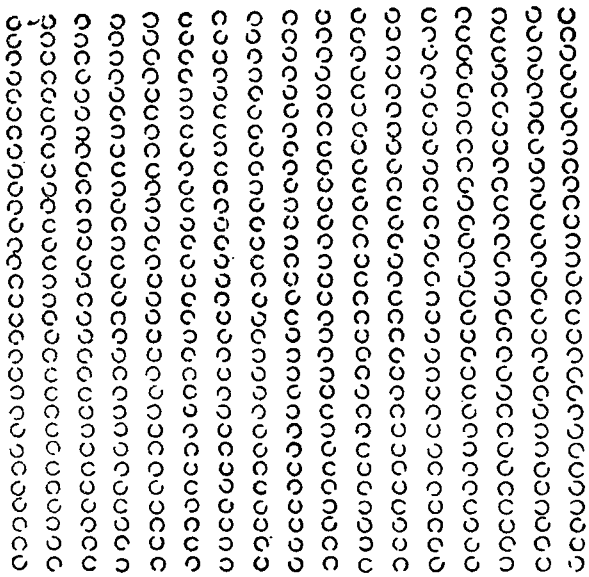
D. Electron beam control. In order to control the electron beam, an indicator device with two curves, described by first and second order equations and located on the screen symmetrically relative to the scan axis, was used. The test subject was to make the electron beam trace one of the curves on command, using the control lever. The rate of movement of the beam across the abscissa was constant (N = 4.7, 9.5 or 19.0 cm/sec). /28

At the same time the electron beam began its horizontal scan, synchronized by a pulse of the recording apparatus, a command was given to the test subject in the form of a light signal indicating the curve along which the electron beam was to be guided. The selection of the curves was random, the probability of selection of the upper curve being equal to the probability of selection of the lower curve. Samples of recordings of electrophysiological processes during the state of attention (A) and rest (R) for one test subject are shown on Figures 8 and 9 respectively.

Any model of the emotional stress should contain the principal elements of the overall formation of emotions (see Introduction; Figure 6). Similar to the state of attention, models should call for action, the solution of a certain problem by man. However, in contrast to the state of attention, where uncertainty related to performance of the assignment is reduced to a minimum (instruction, preliminary training), modeling of the state of emotional stress should include elements of pragmatic uncertainty in the realization of the necessary actions for satisfaction of demands. The creation of the required demands becomes difficult under laboratory conditions. /29

We used two main methods: either the introduction of various degrees of punishment and reward (avoidance of punishment and achievement of reward being the demand) related to the solution of the problem, or the creation of a game situation, arousing the interest of the test subject in fulfillment of the experimentor's assignment.

The essence of the experiments performed consisted on the one hand in studies of the emotional reactions of men under conditions of stimulus by signals, some of which had emotional significance, and on the other hand in the production of a game situation in which the test subject was supposed to conceal his selective attitude toward a certain signal from among those presented, while the task of the experimentor was to detect his emotional reactions to the significant signals. The form of performance of the first portion of the experiments, primarily dedicated to investigation of electrophysiological indicators, was as follows. /30



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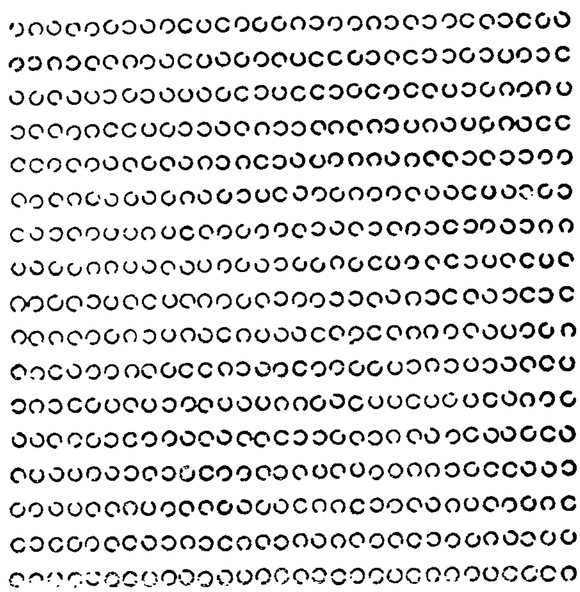

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Figure 7 (Explanation in Text)

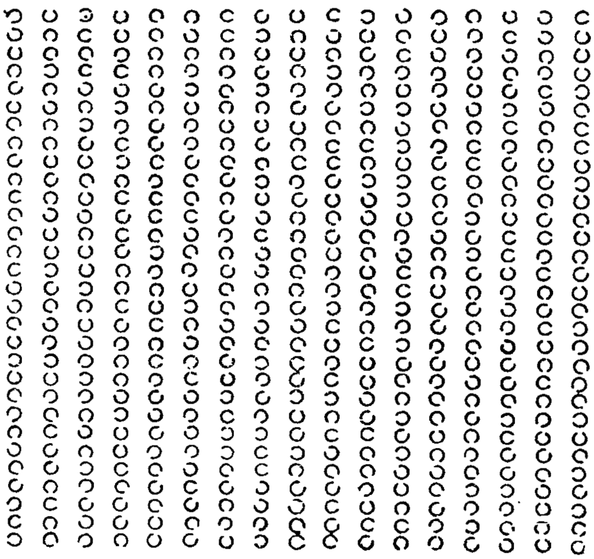

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Figure 7 (continued)

Figure 8. Fragments of Electrophysiological Processes Recorded on Electroencephalograph. State of Attention. 1, Readings of integrator (analysis of EEG); 2, EEG (parietal-occipital lead); 3, α rhythm; 4, PG; 5, SGR (normal gain); 6, SGR (gain increased to twice normal); 7, Channel recording response motor reaction to significant signal; 8, EKG (non-standard lead: left leg-right leg)

A. A number of randomly located "points" was projected on a screen 0.7 × 0.7 m, among which a key light spot moved according to a certain rule. The emotional significance of the signal increased as the key spot approached a certain point and reached a maximum when the key spot corresponded to the given point (possibility of electrical skin stimulus). Changes in the electrophysiological indicators resulting from changes in the space-time characteristics of the signal were studied.

B. Using a motion picture projector, a repeated series of geometric symbols (six figures) was shown, one or two of which were emotionally significant.

The location of the signals from series to series could be either random or regular, and the time of exposure to each (duration of exposure of a frame) varied between 10 and 40 sec. Between each two signal frames there followed a blank frame, the time of which was usually 10 sec. As a result of the experiment, changes in the parameters of the electrophysiological processes were studied. The repetition of input signals allowed the method of accumulation to be used to separate significant signals in the electrophysiological indicators (mainly on the basis of six to ten realizations).

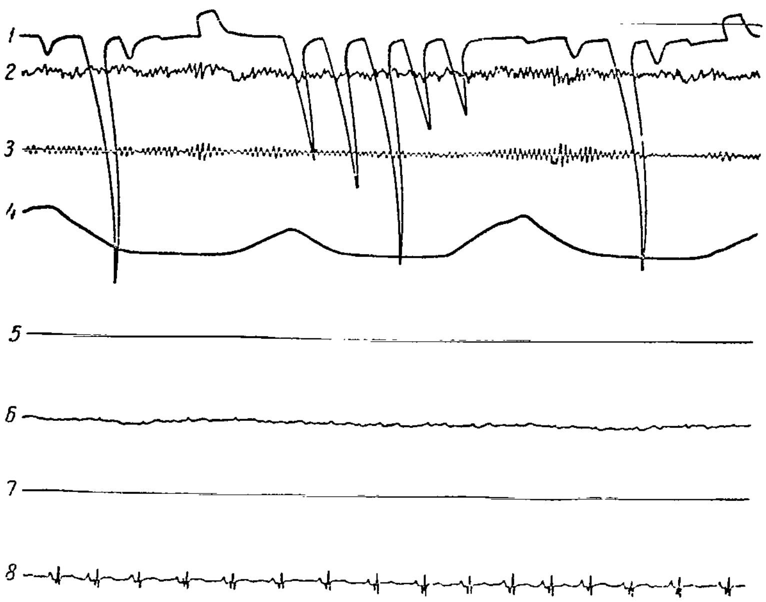


Figure 9. Fragments of Electrophysiological Processes Recorded on Electroencephalograph. State of Operative Rest. Key same as on Figure 8.

Before each experiment, neutral signal stimuli were shown for 3-5 min. Figures 10 and 11 show models of the recordings of electrophysiological processes in the state of rest and emotional stress for the same test subject (R indicates the change in circumference of the thorax during respiration[1], $\alpha(t)$ -- the α rhythm in the frequency range 8-13 Hz).

The second portion of the experiments was dedicated to analysis of intonation of the speech at various levels of emotional stress. In order to achieve this purpose, it was decided to use the principle of modeling of human emotions by acting (Simonov, 1962). We did not assign each actor the character of the intonation to be modeled ("happiness," "alarm," etc.), but rather the situation (suggested situations after K. S. Stanislavskiy), considering that the situations would arouse the corresponding emotional experiences in the actors. Although certain differences between actors' emotions and natural emotional reactions had been established earlier in special experiments (Simonov, Valuyeva, Yershov, 1964), it could be hoped that the high /31

[1] In the experiments, the value of R was determined by the increase in the circumference of the thorax during the process of respiration relative to its value with the breath held.

corticalization of speech functions would make these differences less expressed than in the sphere of vegetative components. It is well known that the intonation coloring of speech is a very reliable means of communications. The main portion of the experiments was performed using 15 actors and students from the studio of Moscow's Sovremennik Theater.

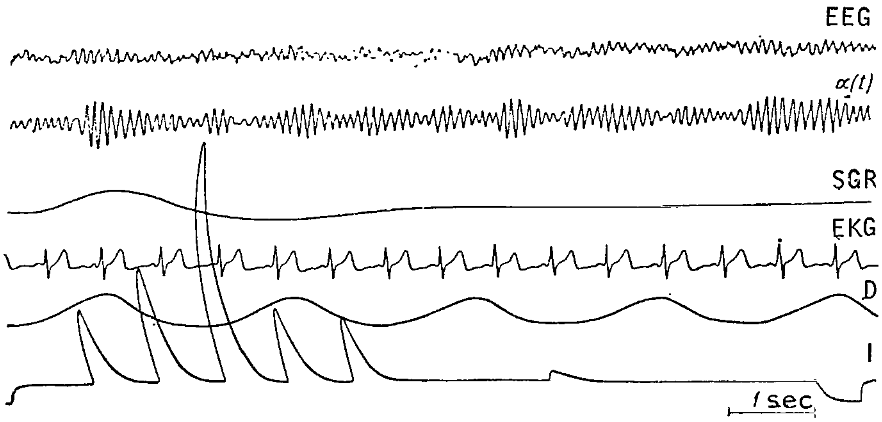


Figure 10 (Explanation in Text)

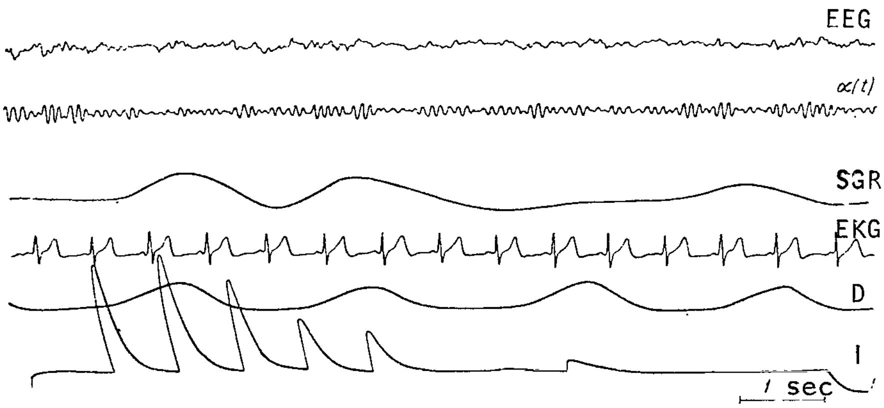


Figure 11 (Explanation in Text)

The experiments were specialized and constructed on the basis of imitation of radio conversation of the actor with the "earth," with the actor in various flight situations. The text of the conversations included expressions broadly used in flight communications: "This is Almaz" (call sign), "Roger," "Good," etc.

The nature of the situations themselves was related either to success in fulfillment of a task (positive emotions), or to the appearance of unexpected /32 complications on board the aircraft, increasing danger (negative emotions). The recording of electrocardiograms during the recording of speech reactions on magnetic tape made it possible to determine whether true emotional stress was actually aroused or whether the actor was really reproducing the intonations purely artificially, while remaining quite calm. The clear changes in cardiac rhythm in the overwhelming majority of cases indicate the correctness of the first assumption (Figure 12) and make the results of the experiments comparable to natural reactions[1].

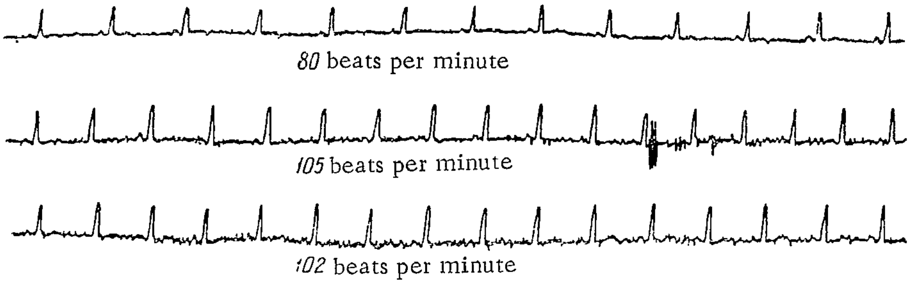


Figure 12. Fragments of Electrocardiograms for Certain States. Top to bottom: "Rest," "Joy," "Alarm"

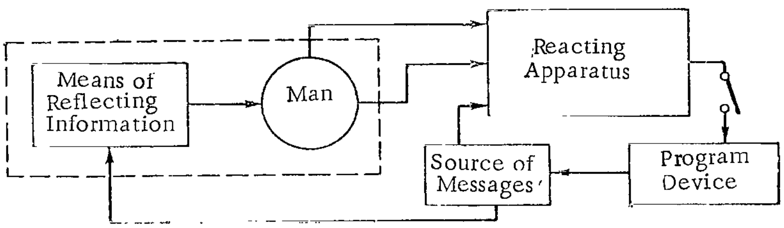


Figure 13 (Here and in Following Illustrations, Explanation in Text)

The block diagram of the experiments, collection and primary processing of information is shown on Figure 13. The test subject was placed in an isolated chamber, along with the means for reflecting information (movie screen, light and sound apparatus, cathode ray indicator, etc.). The message source used was

[1] The authors express their deep gratitude to National Artist of the USSR V. O. Toporkov, chief director of the theater O. N. Yefreymov, director P. M. Yershov and their colleagues for their help in the performance of this portion of our work.

a motion picture projector, tape recorder and various stimulators which were combined with the recording apparatus when necessary through the program device.

The recording of electrophysiological processes was performed using a stripchart recorder and a 17-channel Nihon Kohden type ME-171D electroencephalograph with spectral band analyzer. Magnetic tape recordings were made using a special tape recorder transforming the amplitude modulation of the signal to frequency modulation. /33

In order to record the electroencephalogram (EEG) the parietal-occipital lead was used, while the skin galvanic reflex (SGR) was recorded using a lead from the back and palm surface of the hand. Since the processing of electrocardiograms (EKG) primarily involves the investigation of information consisting of changes in pulse frequency, any of the three standard leads could be used to record the EKG. Changes in the circumference of the thorax during respiration and plethysmograms (reaction of constriction and expansion of vessels) were recorded using special pickups manufactured by the ALVAR firm.

In all, approximately 300 experiments were performed, involving the participation of 53 subjects of both sexes and not over 35 years of age. The duration of each experiment was 1.5-3 hr[1].

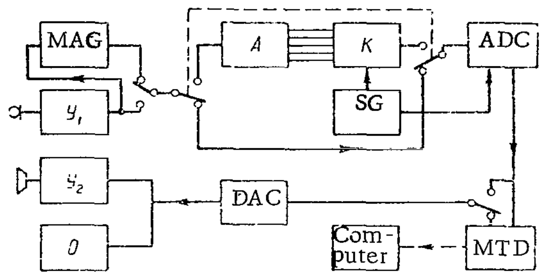


Figure 14

Secondary processing of signals (estimation of distribution, spectral-correlation characteristics) was performed using Ural-2 and M-20 digital computers with an analog-digital converter manufactured at the Institute of Mathematics, Siberian Affiliate, Academy of Sciences USSR (Figure 14), and a band spectral analyzer of the Danish firm "Bruel Kjaer."

[1] The authors express their deep gratitude to E. K. Sepp for his constant help in performing the experiments.

The quantized signal from the output of amplifier Amp_1 or tape recorder MAG could be sent immediately to the analog-digital converter ADC. This converter quantizes with an accuracy of 0.8% (eight binary digits, including the sign bit, corresponding to a dynamic range of 41.9 db). The quantization frequency is established by the synch pulse generator SG and may vary from zero to 30 kHz. At the output of the converter, the codes are recorded on magnetic tape using a special tape recorder (MTD of M-20 computer). At the computer /34 center, the tapes could be read on a similar tape drive and input to the machine. Input quality control was performed by reading the codes from the MTD through a digital-analog converter DAC and listening (speech) through amplifier Amp_2 or visual checking on the screen of oscillograph O of the continuous signal produced. The apparatus allows slowly changing signal parameters (envelope) to be input to the machine from the outputs of five-channel analyzer A through commutator K.

The processing was based on the specific features of the operation of the speech-forming channel, certain model concepts and the possibility of using the results of the theory of stable processes for analysis of electrophysiological signals, which greatly facilitates the production of quantitative estimates for the signals being investigated. In the general case, biological processes reflecting changes occurring in the human organism under the influence of factors in the surrounding medium are described by random, unstable processes. However, in a number of practical applications, including the performance of experiments in which the influence of the surrounding medium can be limited and stabilized and the state of the test subject is determined basically by the experiment itself, with otherwise equivalent conditions, observation intervals can be selected in which the properties of the electrophysiological processes change comparatively slowly with time, so that the processes themselves can be considered stable over time intervals sufficient for calculation of the necessary characteristics (Gnedenko, Fomin, Khurgin, 1962; Denier van der Gon, Strackee, 1966; Genkin, 1965).

In the following sections, we will analyze certain problems involved in the processing of speech and various electrophysiological signals for the states of attention and emotional stress. In the latter case, the main portion of the work, related to analysis of electrophysiological indicators, is dedicated to the study of comparatively weak emotional reactions (relative changes in pulse frequency not exceeding 10-15%).

Problems of the investigation of speech signals have been analyzed, as was noted above, for various emotional situations related to the development of success or the appearance of factors complicating the task.

PART I

UTILIZATION OF PHYSIOLOGICAL SIGNALS FOR EVALUATION OF OPERATOR ATTENTION

CHAPTER 3

SIGNALS OF THE STATE OF ATTENTION

ABSTRACT. Various types of signals of the state of attention are analyzed; the signals analyzed include the alpha rhythm and other electroencephalographic characteristics. The values of a random sequence of readings of the alpha rhythm integrator are subjected to mathematical analysis to determine distribution, confidence limits, etc.

§3.1. State of Attention and Operative Quiet

All processes of cognition, be they preception or thought, are directed toward a certain object reflected in them. The relationship of subject to object is expressed in attention. Attention is usually phenomenologically characterized by excessive direction of the consciousness toward a given object which is then perceived with particular clarity and sharpness. The selective direction as a central phenomenon in attention is accompanied by an increase in the threshold sensitivity of certain analyzer mechanisms, the manifestation of which is so-called sensitization. Analysis of sensitization in a broader sense, not only as a change in the thresholds of sensitivity of the sense organs, but as an increase (and corresponding decrease) in receptiveness to impression, thoughts, etc. allows us to extend the concept of attention to mental processes as well. The basic expansion of the concept of sensitization can be the generality of the mechanisms of these phenomena which are related, according to this assumption, to the trophic influence of the vegetative nervous system on the central nervous system (Rubenshteyn, 1946). /35

Attention (in the broad sense of this word) occurs in operator activity, requiring, in addition to precise operation of the actuator apparatus, the performance of logic operations as well.

The activity of the operator in the "man and machine" system consists of several, generally interrelated stages: perception, analysis of incoming information and performance of action according to decision made. The specific gravity of each component may vary and is determined by the conditions and nature of the task being performed. In nondeterministic systems, the influx of information arriving at the operator may be represented in the form of a random function of one or several arguments, the information on the /36
properties of which is accumulated during the training process. Here, due to the limited capabilities of man, an idea of the probability properties of the random function is formed by averaging. Any averaging always involves a loss of information, so that each time, regardless of the duration of the training stage, there is a difference between the mean a priori data on the flow of information and the current information, which is but one possible realization of the random function.

Thus, in nondeterministic systems, according to the concepts of P. V. Simonov, the state of attention should be emotionally colored and more stable, than in deterministic systems. The emotional coloring is looked upon as one of the lower stages of emotional stress, characterized by uncertainty in the flow of incoming information to the extent to which it is observed as realizations of the random function, whose probability properties are known, follow each other. In contrast to emotional stress resulting from various types of stress factors, emotional coloring, against a background of which the operator activity occurs, is relatively stable with time.

The work of the operator usually does not involve physical stress, but requires considerable attention stress for the performance of logic operations and achievement of rapid, precise reactions to incoming signals. In the following, the state of a human involved in operator activity, which, as was already noted, has considerable emotional coloring, will be represented by the state of attention (A), which will be assumed unchanged as long as the factors of the surrounding medium remain unchanged. The state of operative rest (R) will be taken to mean the state of a human in a working situation in a state of attentive rest, not immediately involved in operator work.

§3.2. Physiological Characteristics of Attention

Attention is the most global characteristic of the active state of man; it is required with any purposeful activity regardless of which analyzers (visual, auditory, etc.) participate in the activity; therefore, characteristics of attention can be determined in changes not only of specific, but of nonspecific reactions of man.

Concerning the state of attention, i.e. the selective readiness to react in a certain manner to a certain signal, we can judge first of all, as is frequently done, from the reactions themselves: their presence (signal not missed), the length of the latent period, the threshold, indicating sensitivity of the analyzer, etc. It has been shown that a warning of upcoming /37 stimulus to which an answer should be given, leads to a reduction in latent periods and a lowering of thresholds (Maruseva and Chistovich, 1954).

Also, attention as a nervous-mental process should be reflected through the functional state of the CNS in its electrical activity. Here we must base ourselves on the statement of N. Ye. Vedenskiy concerning the correspondence of electrical and functional changes in the tissues and organs, the basis of practical electrophysiology, in particular for electroencephalography and electrocardiography (Golikov, 1960; Livanov, 1955; Rusinov, Smirnov, 1957).

A characteristic bioelectric reaction of the cerebral cortex to a stimulus is the "response elicited"[1]. The concentration of attention to stimuli leads to an increase in the amplitudes of these responses and to a change in their configuration (Garcia-Austt, Bogacz, Vanzulli, 1964; Haider, Spong, Lindsly, 1964, etc.). An objective review of the specifics of elicited potentials in man in the state of attention was published by N. I. Chuprikova (1967). A number of works has been dedicated to the characteristic slow negative wave ("expectation wave") which arises between warning and start signals (Walter, 1965; Rebert, McAdem, Knott, Irwin, 1967).

An increase in the amplitudes of elicited responses may be related to the activating influence of reticular formation. According to the data of S. P. Narikashvili (1963), direct electrical stimulus of the reticular formation of the brain stem leads not only to an increase in the amplitude of the first responses in the cerebral cortex, but also to their stabilization, i.e. to constancy of the neurons involved in the reaction.

Although changes in elicited potentials are quite interesting for investigation of physiological mechanisms of concentration of attention, the practical significance of these bioelectric changes is not great as yet, since we rarely encounter signals in the sphere of operator activity which are accompanied by clear elicited responses. Theoretically, a system of testing the operator state by recording elicited responses to special series of test stimuli could be among future developments.

As concerns the summary electroencephalogram with surface leads, in addition to investigation of the EEG itself, which has a tendency to depression in response to various stimuli, there is considerable significance in the study of the rhythms, components of the EEG which are physiological characteristics of the functional state of the brain. Among these rhythms, particular significance must be given to the α rhythm, characteristic for the /38 EEG of a mature, healthy man in the conscious state with closed (Kozhevnikov, Meshcherskiy, 1963) and open eyes (Mullholland, 1965). It is assumed that the α rhythm is the rest rhythm which is desynchronized when the brain is excited, and synchronized when inhibition occurs. This general statement can hardly be be made concerning the nature of the α rhythm during mental loading, with concentrated attention. Still, most investigators note a tendency to

[1] We retain here the terminology used in physiology.

depression (desynchronization) of the α rhythm as attention is increasingly concentrated (Gastaut, Bert, 1954; Walter, 1959; Lorens, Darrow, 1962, etc.). If the probability of appearance of an expected state signal is great, the number of spontaneous depressions of the α rhythm decreases (Yui Vyen Chao, 1964). The worsening of attention with the appearance of α rhythm depression and desynchronization of EEG has been reported by Adrian (1947), Gastaut, Bert (1954), Kugler (1963). Mundy-Castle (1957) speaks out against the direct relationship between attention and depression of the α rhythm. Exaltation of the α rhythm in response to auditory and tactile stimulation, as well as in case of expectation of a signal and mental stress were observed by Durup, Fessard (1935), Darrow (1947), Travis, Barber (1938), Kreitman, Show (1965). The contradiction among the proponents of depression and the proponents of exaltation of the α rhythm during concentration of attention was studied by Travis and Ohanian (1954), who reported that depression of the α rhythm is related to directed thinking, exaltation to generalized thinking.

Together with the investigations of changes in the amplitude of the α rhythm during concentration of attention, a smaller number of works is encountered dedicated to study of other parameters of the main EEG rhythm. For example, M. N. Valuyeva (1965) described the phenomenon of stabilization of the maximum amplitude of the α rhythm during expectation of a significant stimulus, while Darrow (1947) under similar conditions noted an increase in frequency. A. A. Genkin (1963), in investigations of the mental activity of man, noted a change in the asymmetry of phase durations in the EEG. Analyzing various phases of individual cycles of activity of fluctuating processes occurring in physiological systems as corresponding to qualitatively different functional states, Genkin (1965) gives considerable significance to the level of asymmetry of phases as characteristics of physiological processes.

In the experiments of D. M. Grinberg and A. A. Zubkov (1963), mental work led to appearance of rhythms of higher frequency (β and γ) or to an increase in their frequency and amplitude with simultaneous decrease in the frequency and amplitude of the α rhythm. In many cases during activization of attention, no changes in the frequency spectrum of the EEG are noted at all, including in the α range (Artemeva, Khomskaya, 1966).

The mutually contradictory nature of available data indicates that the /39
state of attention apparently cannot be unambiguously characterized by information produced only by analysis of EEG, in connection with which supplementary sources of information are required.

One of the manifestations of attention is an increase in sensitivity of analyzers. The important role of vegetative changes in changes in sensitivity to stimuli has been noted by Ye. N. Sokolov (1958), Graham and Keen (1966), etc. Mirtschew and Penov (1963) note that changes in the functional state influence first the vegetative nervous system, then the electrical activity of the brain.

Griew, Davies and Treacher (1963) consider a decrease in the pulse frequency to be a characteristic of attention. They found a correlation between the variability of the pulse frequency and the number of errors committed by test subjects. It should be noted that an increase in the significance of the stimuli expected, as a rule, leads to an increase in pulse frequency rather than a decrease. Also, the nature of the cardiac reaction (increase or decrease in frequency) depends to a considerable extent on individual specifics of the operator.

Even the briefest review of available data indicates that the problem of reliable, objective characteristics of concentration of attention remains open and requires further study.

Attention, accompanied by adaptation of the entire organism to optimal perception of certain stimuli and most precise reaction to them, naturally, attracts all of the vitally important systems of the organism into the adaptation process through the central nervous system: the cardiovascular, respiratory, nervous-muscular and other systems are all involved. A change in the functional state of systems is reflected in their electrophysiological characteristics: EEG, EKG, PG, EMG, as well as the skin-galvanic reactions.

§3.3. Certain Parameters of Physiological Processes and Their Interpretation

Biological processes reflecting the interaction of the human organism with the medium are quite complex and fluctuating in nature. Even with seemingly identical external conditions, fluctuations of these processes are quite varied in magnitude and form. In connection with this, when biological processes are investigated their characteristics must be assumed to be random functions of time, and the processes themselves analyzed as sequences of locally stable processes, i.e. as processes whose properties change with time so slowly that the processes in the observation interval can be considered /40 stable (Genkin, 1965; Denier van der Gon, Strackee, 1966). The possibility of this approach is indicated by B. V. Gnedenko, S. V. Fomin, Ya. I. Khurgin (1962), who note that in many cases the biological processes, including spontaneous activitiy of various sectors of the central nervous system, can be looked upon with sufficient accuracy as stable processes, and can be approached from the point of view of the theory of stable random processes. The possibility of using the theory of stable processes for the analysis of biological processes greatly facilitates the production of quantitative estimates of the processes being investigated.

The random function which describes a physiological process can be looked upon as an infinite set of random quantities (Pugachov, 1957) where the i-th random quantity is represented by a set of possible values of the physiological indicator, one of which can be observed at the i-th moment of time. The physiological indicator may be, for example, the interval of the cardiac, respiratory or other cycle; the amplitude of the EEG, reflecting the degree of excitation of the nervous structure; the amplitude of the PG, characterizing the depth of respiration; the amplitude of the plethysmogram, indicating the degree of filling of the blood vessels, etc. An exhaustive characterization of the set of random quantities is their distribution. The performance of calculations using the distribution rules of a set of random quantities is frequently quite difficult, so that in practical applications, greater attention has been given to their numerical characteristics. The simple but in many practical cases quite satisfactory numerical characteristics of random processes include mathematical expectation, dispersion and correlation function. The mathematical expectation is a certain mean level, about which the concrete realizations of a random process fluctuate (by realizations, we mean the concrete form taken by a random process as a result of an experiment). The distribution of fluctuations of concrete realizations of a random process about the mathematical expectation is characterized by the dispersion. The correlation function reflects the degree of dependence between preceding and subsequent values of a random process separated by time interval τ.

Highly organized biological systems, in order to provide optimal fulfillment of metabolic reactions in the organism, have an interesting ability to retain relative constancy of the internal medium and certain physiological functions (blood circulation, body temperature, etc.). If the conditions of the external medium are changed, when perturbations arise the /41 central nervous system perceives them through nervous and neurohumoral paths and acts on all functions of the organism so as to retain the parameters of the internal medium unchanged, by switching individual systems of the organism to new modes, new levels of functioning. For example, when the percent content of oxygen in the air being inhaled is changed, the CNS switches the respiratory system to a new level of functioning in order to maintain constant the gas composition of the internal medium, for example by deeper breathing. The external manifestation of the changes in level of functioning of the respiratory system will be an increase in the difference Δ_m between the maximum and minimum values of the PG of the respiratory cycle. In this example, Δ_m is an objective indicator, adequately reflecting the level of activity (depth of respiration) of the respiratory system. With unchanged external perturbation, the depth of respiration, and consequently the value of Δ_m, should remain constant; however, in actuality, due to various types of random factors, it fluctuates. In connection with this, in order to evaluate the functional level of the system it is desirable to select a characteristic which would be independent or only weakly dependent on random factors. For ergodic processes, one such characteristic is the mean value, determined over a rather extensive period of time; the greater the time interval, the less the dependence of a given characteristic on random influences. With unlimited increases in the interval, the averaged mean value of the ergodic process approaches its limiting value, the mathematical expectation. Thus, the mathematical expectation of an ergodic process is an indicator characterizing the level of the functional state of the system being investigated.

An analogous characteristic in the mean is the dispersion of a process, which reflects the degree of fluctuation of values of the physiological indicator about the mathematical expectation. The relationship of dispersion to mathematical expectation can be used to characterize the stability of the functioning of a system, lower values of the ratio corresponding to higher stability.

Numerically, dispersion is equal to one of the values of the correlation function, namely the value with the argument of the correlation function equal to zero. In comparison with the dispersion in this case the correlation function is a more general probabilistic characteristic, establishing the relationship between preceding and subsequent values of the process, for which it is calculated. For a broad class of ergodic processes, the absolute value of correlation function decreases without limit to zero as argument τ approaches infinity (Livshits, Pugachov, 1963) so that we can always /42 indicate a time interval τ_0, after which the correlation function will be practically equal to zero.

For a random process with normal distribution, the practical absence of correlation indicates the practical absence of any relationship between non-correlated readings of the process. This means that the state of the system up to moment in time t_k, characterized by value of the physiological indicator $\phi(t_k)$ has no practical influence on the state of the system at moment in time $(t_k + \tau_0)$, characterized by $\phi(t_k + \tau_0)$, and in this sense the values of $\phi(t_k)$ and $\phi(t_k + \tau_0)$ are considered independent. For a process with monotonous correlation function, the dependence between readings $\phi(t_k)$ and $\phi(t_k + \Delta t)$ increases with decreasing Δt, where $0 \leqslant \Delta t < \tau_0$.

From the point of view of information theory, a decrease in Δt means a decrease in the quantity of information produced by observation at moment $t_k + \Delta t$ and added to the information available at moment t_k. This is explained by the fact that with decreasing Δt with otherwise equivalent conditions, the probability of an essential deviation of $\phi(t_k + \Delta t)$ from $\phi(t_k)$ is decreased; in other words, readings separated by interval Δt will correspond in value to the expected and predicted values on the basis of information on the correlation function and $\phi(t_k)$ most frequently. In connection with this, a problem arises: is it not possible, without essential losses of information, to replace continuous testing of a process being investigated with discrete testing in order to reduce the experimental material and, consequently, the volume of computational work, which is a factor of no little importance in practical applications. Actually, for many processes this can be done. Thus, for random processes with exponential correlation function $B(\tau) = Ae^{-\alpha\tau}$ in determining the mathematical expectation m from experimental data by adding evenly spaced values of the process, interval Δt must be selected equal to time t_M, over which the normalized correlation function decreases to 0.2. Replacement of continuous testing of a random process with discrete testing with interval t_M means that the error in determination of the mathematical expectation increases, but by not more than 10%, if the condition $\alpha T \geqslant 50$ obtains where T is the length of the interval over which determination of the mathematical expectation is performed.

If a random process has nonmonotonously decreasing normalized correlation function $R(\tau)$ and if the intervals Δt between evenly spaced values of the random process is selected not greater than the time required for $R(\tau)$ to attenuate to a value of 0.2 (it is assumed that for τ greater than this time, attenuation $|R(\tau)| \leqslant 0.2$), the error in the determination of m will also be within acceptable limits.

It will be shown in the following that the processes investigated are of this type: either the correlation function is near exponential, or the correlation function is nonmonotonous, but without sharp jumps. /43

When it is necessary to determine the dispersion of a random process on the basis of experimental data by adding evenly spaced values, the discretization step Δt should be selected equal to the time in which the normalized correlation function $R(\tau) = Ae^{-\alpha\tau}$ decreases to 0.45, in which case the error due to replacement of the continuous random process with the discrete process will also not exceed 10% (Livshits, Pugachov, 1963).

Sequence of Readings of α Rhythm Integrator

The behavior of random quantities and random functions can be described by their probability characteristics. An idea of these characteristics can be produced on the basis of processing of the necessary experimental data produced as a result of performance of observations of random quantities and functions of interest.

In the following, in contrast to the theoretical or true probability characteristics, their estimates produced on the basis of experimental data will be called statistical or sampling characteristics.

The basis of the methods of determination of estimates is the law of large numbers, in correspondence to which when a large number of experiments has been performed, possible deviations of the mean value of results of experiments from the corresponding mathematical expectation of the parameter investigated frequently become small, i.e. the estimates approach the true values of the numerical characteristics of the distribution.

The performance of a large number of experiments involves considerable expenditures of time and funds, and in many cases difficulties of principle: for example, when the object of investigation is a biological system, it is natural to want to produce statistical characteristics as close as possible to the corresponding probability characteristics, from the least possible quantity of experimental data.

§3.4. Correlation Properties of Sequence of Readings of α Rhythm Integrator

In beginning our analysis of the correlation properties of a random sequence of α rhythm integrator readings, let us agree to represent the random sequence of readings of the α rhythm integrator by $Z(t_k)$, understanding by $Z(t_k)$ the function, the values of which at discrete moments in time t_k ($k = 1,2,\ldots,n$) are random quantities. The sequence of concrete readings of readings of the α rhythm integrator produced as a result of performance of the i-th experiment will be represented by $\zeta_i(t_k)$ or, in expanded form, /44

$$\zeta_i(t_k) = \{\zeta_i(t_1), \zeta_i(t_2), \ldots, \zeta_i(t_k), \ldots, \zeta_i(t_n)\}.$$

The correlation function is one of the main probability characteristics of the random process. It is well known that the assignment of the correlation function alone fully characterizes the stable Gaussian process; in many cases, this is also sufficient for a description of properties of the process with distribution other than normal (Kharkevich, 1965; Levin, 1966). For a discrete process, an estimate of the correlation function $B(\tau_s)$ could be the statistical correlation function $B^*_0(\tau_s)$, determined by the following formula:

$$B_0^*(\tau_s) = \frac{1}{n-s}\sum_{k=1}^{n-s}[Z(t_k) - m_Z][Z(t_{k+s}) - m_Z], \tag{3.4.1}$$

where m_Z is the mathematical expectation of the random sequence $Z(t_k)$; τ_s is the time shift; $\tau_{k+s} - \tau_k = \tau_s = \Delta t \cdot s$; n is the number of terms in the random sequence $Z(t_k)$; k is the number of term $Z(t_k)$.

In practice in determining the statistical correlation function of the random process, its mathematical expectation is not known; therefore, in formula (3.4.1), m_Z is replaced by the statistical mathematical expectation m^*_Z. The estimate of the correlation function then takes on the form

$$B_0(\tau_s) = \frac{1}{n-s}\sum_{k=1}^{n-s}[Z(t_k) - m_Z^*][Z(t_{k+s}) - m_Z^*], \tag{3.4.1'}$$

where

$$m_Z^* = \frac{1}{n}\sum_{k=1}^{n} Z(t_k).$$

Here, if n is great enough, the statistical correlation function $B^*_0(\tau_s)$ (3.4.1') will differ little from the true correlation function $B(\tau_s)$ (Livshits, Pugachov, 1963). Dividing expression (3.4.1') by $B^*_0(0)$, we produce estimate $R^*_0(\tau_s)$ of the normalized correlation function $R(\tau_s)$:

$$R_0^*(\tau_s) = \frac{n \sum_{k=1}^{n-s} [Z(t_k) - m_Z^*][Z(t_{k+s}) - m_Z^*]}{(n-s) \sum_{k=1}^{n} [Z(t_k) - m_Z^*]^2}. \tag{3.4.2}$$

In order to calculate the values of the normalized statistical correlation function using concrete experimental results, we use the formula /45

$$R^*(\tau_s) = \frac{n \sum_{k=1}^{n-s} [\zeta(t_k) - m_\zeta][\zeta(t_{k+s}) - m_\zeta]}{(n-s) \sum_{k=1}^{n} [\zeta(t_k) - m_\zeta]^2}, \tag{3.4.3}$$

where

$$m_\zeta = \frac{1}{n} \sum_{k=1}^{n} \zeta(t_k).$$

For sequence $Z(t_k)$, the normalized statistical correlation functions $R^*(\tau_s)$ were calculated from the relatively long realizations $\zeta(t_k)$, the quantity of readings n being over 50. The statistical characteristics $R^*(\tau_s)$, calculated using formula (3.4.3), are presented on Figures 15-18.

We can see from these figures that the statistical characteristics $R^*(\tau_s)$ for each person in the state of "operative rest" (see Figures 15 and 17), as well as in the state of "attention" (see Figures 16, 18) have considerable qualitative similarity.

One specific of the characteristics $R^*(\tau_s)$ is their rapid attenuation with increases in the argument. An unlimited decrease of the modulus of the correlation function of the stable random process as argument $(\tau) \to (\infty)$ indicates the ergodic property of the process, allowing averaging using the /47
set of realizations to be replaced by averaging using one realization over a sufficiently long, but finite, time interval.

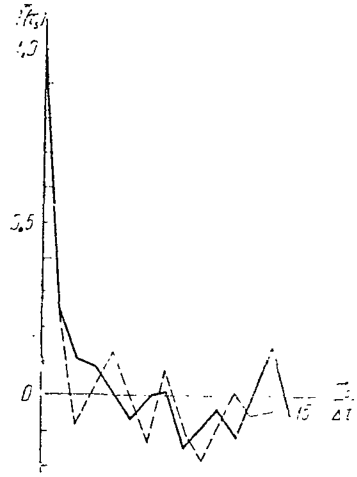


Figure 15

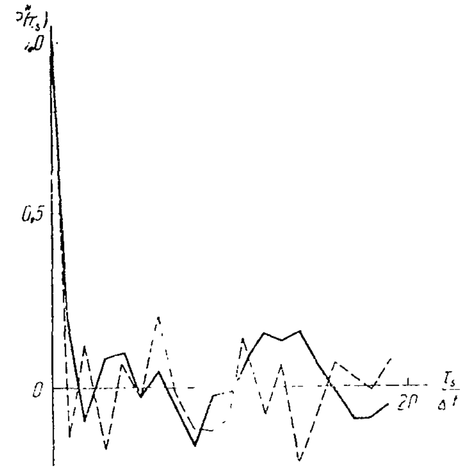


Figure 16

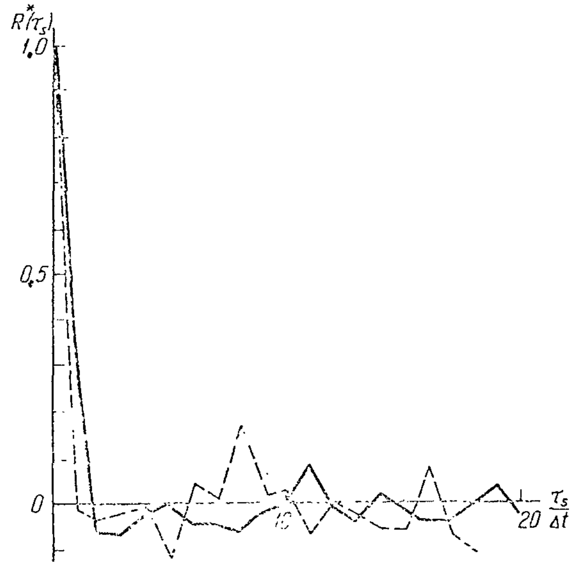


Figure 17

In the cases analyzed, attenuation of $R^*(\tau_s)$ occurs so rapidly that neighboring readings of realization $\zeta(t_k)$ can be considered noncorrelated where $A \approx 0.2$. The sense of constants A will be explained below (3.4.6).

It is considered that readings $\zeta(t_k)$ and $\zeta(t_k + \tau)$ are practically noncorrelated if $\tau > \tau_0$, where τ_0 (correlation interval) is determined by (3.4.4), (3.4.5):

$$\tau_0 = \frac{1}{B(0)} \int_{-\infty}^{\infty} B(\tau)\, d\tau \qquad (3.4.4)$$

or

$$\tau_0 = \int_{-\infty}^{\infty} R(\tau)\, d\tau \qquad (3.4.5)$$

(Kharkevich, 1965).

Determination of (3.4.4) and (3.4.5) is convenient in those cases when the integrand is not negative. In those cases when the correlation function fluctuates about the abscissa, the correlation time is defined as the value of the argument for which the absolute value of the normalized statistical correlation function is equal to fixed quantity A and where $\tau > \tau_0$ remains less than A:

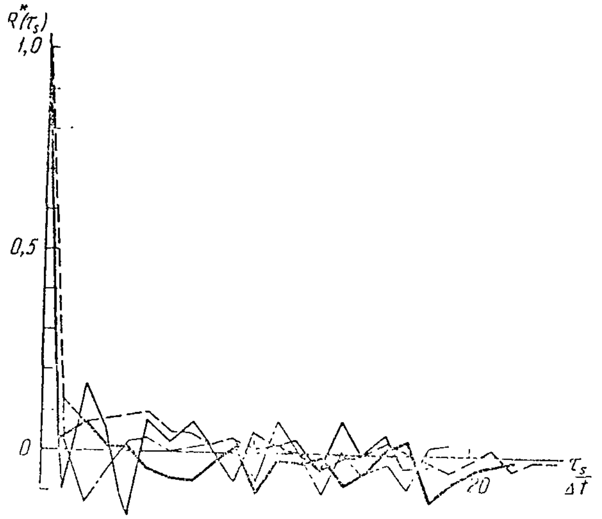


Figure 18

$$|R(\tau)| < A \text{ when } \tau > \tau_0, \qquad (3.4.6)$$

where

$$\tau_0 = \max \tau = \varphi(y)\,|_{y=A};$$

$\phi(y)$ is the inverse function of modulus $R(\tau)$.

In order to increase the reliability of the value of normalized statistical correlation function for determination of the values of the correlation interval (3.4.6), we use data from several realizations produced in m independent experiments. The expressions for statistical correlation function $B^*_m(\tau_s)$ and statistical dispersion D^*_m for calculation using the data of m realizations of random sequence $Z(t_k)$ has the form (Livshits, Pugachov, 1963):

$$B^*_m(\tau_s) = \frac{\sum_{i=1}^{m} \sum_{k=1}^{n_i - s} [Z_i(t_k) - m_Z][Z_i(t_{k+s}) - m_Z]}{\sum_{i=1}^{m} n_i - ms}; \tag{3.4.7}$$

$$D^*_m = B^*_m(0) = \frac{\sum_{i=1}^{m} \sum_{k=1}^{n_i} [Z_i(t_k) - m_Z]^2}{\sum_{i=1}^{m} n_i}. \tag{3.4.8}$$ /48

Let us calculate the normalized statistical correlation function

$$R^*_m(\tau_s) = \frac{B^*_m(\tau_s)}{B^*_m(0)} = \frac{B^*_m(\tau_s)}{D^*_m}. \tag{3.4.9}$$

We introduce the symbols:

$$\sum_{k=1}^{n_i - s} [Z_i(t_k) - m_Z][Z_i(t_{k+s}) - m_Z] = C_i; \tag{3.4.10}$$

$$\sum_{k=1}^{n_i} [Z_i(t_k) - m_Z]^2 = \mathcal{E}_i. \tag{3.4.11}$$

Let us substitute (3.4.7) and (3.4.8) into (3.4.9) considering (3.4.10) and (3.4.11):

$$R^*_m(\tau_s) = \frac{\sum_{i=1}^{m} n_i \sum_{i=1}^{m} C_i}{\left[\sum_{i=1}^{m} n_i - ms\right] \sum_{i=1}^{m} \mathcal{E}_i}. \tag{3.4.12}$$

From the expression

$$D^*_i = B^*_i(0), \quad R^*_i(\tau_s) = \frac{B^*_i(\tau_s)}{B^*_i(0)},$$

$$B^*_i(\tau_s) = \frac{1}{n_i - s} \sum_{k=1}^{n_i - s} [Z_i(t_k) - m_Z][Z_i(t_{k+s}) - m_Z]$$

for the statistical correlation function and statistical dispersion, calculated using the i-th realization of random process $Z(t_k)$, we find

$$\mathcal{E}_i = D_i^* n_i;$$
$$C_i = (n_i - s) D_i^* R_i^* (\tau_s).$$

Let us substitute the values produced for C_i and $\mathcal{E}_i$ in (3.4.12) and, after transformation, we produce

$$R_m^* (\tau_s) = \frac{\sum_{i=1}^{m} \lambda_i^{(n-s)} D_i^* R_i^* (\tau_s)}{\sum_{i=1}^{m} \lambda_i^{n} D_i^*}, \tag{3.4.13}$$

where /49

$$\lambda_i^n = \frac{n_i}{\sum_{i=1}^{m} n_i}; \quad \lambda_i^{(n-s)} = \frac{(n_i - s)}{\sum_{i=1}^{m} n_i - ms}$$ is the weight coefficient

With identical intervals of determination of realizations of random process $Z(t_k)$, equal to $n \cdot \Delta t (n_1 = n_2 = \ldots = n_m)$, the expression for the normalized statistical correlation function $R^*(\tau_s)$ will have the form

$$R^* (\tau_s) = \sum_{i=1}^{m} \lambda_i^D R_i^* (\tau_s), \tag{3.4.14}$$

where

$$\lambda_i^D = \frac{D_i^*}{\sum_{i=1}^{m} D_i^*}.$$

On Figures 19-22, the solid lines show the normalized statistical correlation functions of random sequence $Z(t_k)$ for the state of operative rest (Figure 19, 21) and attention (Figure 20, 22) of two test subjects (V. R. and

L. L.), calculated using the general formula (3.4.13) utilizing realizations presented on Figures 15-18. A comparison of characteristics $R^*(\tau_s)$ for the state of operative rest (see Figure 19, 21) and the state of attention (see Figure 20, 22) shows that the random sequence $Z(t_k)$ in the state of operative rest has a tendency to greater correlation than the state of attention, the readings $\zeta(t_k)$ and $\zeta(t_{k+1})$ being practically noncorrelated in the sense of condition (3.4.6) where $A \leqslant 0.1$.

An idea of the reliability and accuracy of the normalized statistical correlation function $R^*(\tau_s)$ can be composed by calculating, for example using the values of $R^*(\tau_i)$ of the function $R^*(\tau_s)$, the confidence boundaries defining the intervals in which the unknown true values of the normalized correlation function $R(\tau_i)$ of random sequence $Z(t_k)$ are to be found with the required probability (reliability). For a symmetrical confidence interval, its width 2ε is defined by the condition

$$P\{|R^*(\tau_i) - R(\tau_i)| \leqslant \varepsilon\} = a, \tag{3.4.15}$$

where P{ } is the probability of fulfillment of the inequality included in the braces; a is the value of probability characterizing the reliability of the approximate equality $R^*(\tau_i) \approx R(\tau_i)$; ε is the confidence interval boundary.

With known dispersion D_R for $R^*(\tau_i)$ and fixed probability a, the value of ε is determined by the formula /51

$$\varepsilon = z_a \sqrt{D_R}, \tag{3.4.16}$$

where z_a is a quantity corresponding to the fixed level of probability.

For certain quantities a, we present below values $z = z_a$ for Z, following the normal distribution

$$(w(z) = (2\pi)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} z^2\right\}:$$

a . . .	0,6827	0,7287	0,7699	0,8064
z_a . . .	1,00	1,10	1,20	1,30
a . . .	0,8385	0,8664	0,8904	0,9109
z_a . . .	1,40	1,50	1,60	1,70
a . . .	0,9281	0,9426	0,9545	0,9643
z_a . . .	1,80	1,90	2,00	2,10

More detailed information concerning the table of values of the probability integral

$$\Phi(z) = \frac{2}{\sqrt{2\pi}} \int_0^z e^{-\frac{z^2}{2}} dz; \quad 0 \leqslant \Phi(z) \leqslant 1$$

can be found in the works of V. V. Nalimov (1960), A. K. Mitropol'skiy (1956), I. V. Dunin-Barkovskiy and N. V. Smirnov (1955), etc.

For an ergodic, random process, any realization has properties of the entire set, and therefore the result of time averaging performed on one realization corresponds with the corresponding mean for the set at any moment in time. In particular, for the ergodic process we have

$$M[\{X(t) - m_X\}\{X(t+\tau) - m_X\}] =$$
$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T [X(t) - m_X][X(t+\tau) - m_X]\,dt,$$

where the left portion of the equality contains the mathematical expectation of the product of two centered random quantities as values of the ergodic random process X(t) for moments of time t and t + τ; the right portion contains the time average with respect to one realization of the product of values of the process X(t), differing in time by t = τ (Levin, 1966).

Thus, if the ergodic random process X(t) is represented by a continuum of random quantities $X(t_1)$, $X(t_2)$, ..., $X(t_m)$ or more briefly, X_1, X_2, ..., X_m, then the sequence of readings $R(\tau_1)$, $R(\tau_2)$, ..., $R(\tau_k)$ of the normalized correlation function R(τ) can be interpreted in the form of the series $\rho_{i(i+1)}$, $\rho_{i(i+2)}$, ..., $\rho_{i(i+\nu)}$, ($\nu = k - i$) paired correlation coefficients of random quantities (X_i, X_{i+1}), (X_i, X_{i+2}), ..., $(X_i, X_{i+\nu})$. /52

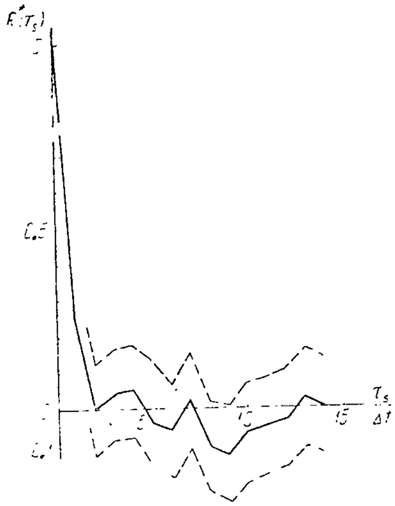


Figure 19

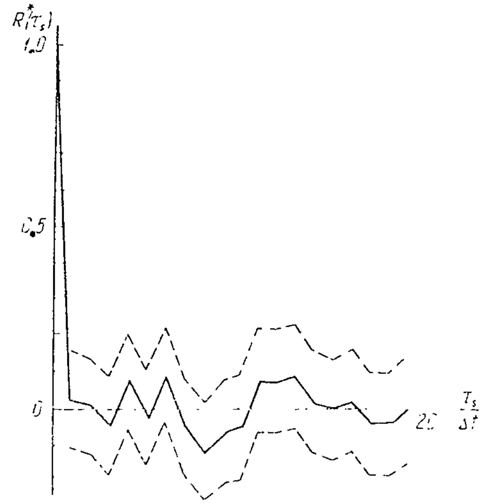


Figure 20

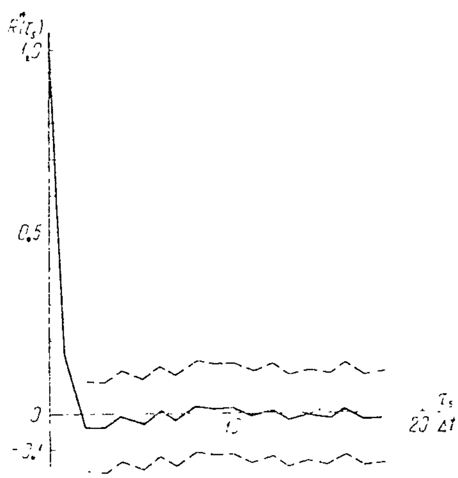


Figure 21

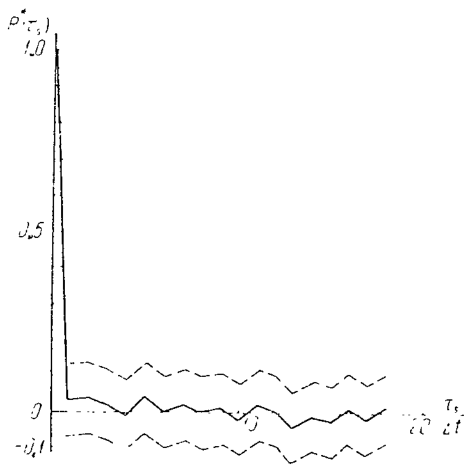


Figure 22

The sampling characteristics, calculated from experimental data, due to the time limitations of actual observations, will of course be random in nature and will differ from the true characteristics of the process being investigated. Thus, the value of the normalized statistical correlation function $R^*(\tau)$ with a change in (τ) greater than (or equal to) the correlation interval (τ_0) may be other than zero. In order to estimate the reliability and precision of the values of $R(\tau)$, we construct the confidence boundaries, considering the ergodic properties of random process $X(t)$, using the confidence boundaries for the sampling correlation coefficient r.

For pairs of random quantities distributed normally, the expression for dispersion D_r of the statistical correlation coefficient r has the following form with an accuracy to terms on the order of $n^{-3/2}$ (Kramer, 1948)

$$D_r = \frac{(1 - \rho^2)^2}{n}; \tag{3.4.17}$$

where ρ is the true correlation coefficient.

The value of ρ is not always known; therefore, in place of expression (3.4.17), we use either formula (3.4.18) with a sufficiently large sample

$$D_r \approx \frac{(1 - r^2)^2}{n}, \tag{3.4.18}$$

or use the upper estimate $D_{r_{max}}$ of dispersion D_r of the statistical correlation coefficient r, produced from (3.4.17): $\rho^2 > 0$, therefore where $\rho = 0$, $D_r = D_{r_{max}}$;

$$D_{r_{max}} = \frac{1}{n}. \tag{3.4.19}$$

It should be noted that estimate (3.4.18) is reliable only when the sample, distributed normally, has volume n very large and ρ^2 not near unity. If the value of n is only moderately large and if ρ^2 is near unity (an indication of this is that r^2 is near unity), dispersion (3.4.17) of the sampling correlation coefficient r may differ significantly from the precise value of dispersion for r (van der Waarden, 1960). V. I. Romanovskiy (1947) recommends that formula (3.4.18) be used for $n \geqslant 50$.

Let us rewrite (3.4.15) considering (3.4.18) and (3.4.16) /53

$$P\left\{R^*(\tau_i)-\frac{z_a}{\sqrt{n}}[1-R^{*2}(\tau_i)]\leqslant R(\tau_i)\leqslant R^*(\tau_i)+\right.$$
$$\left.+\frac{z_a}{\sqrt{n}}[1-R^{*2}(\tau_i)]\right\}=a. \tag{3.4.20}$$

Equation (3.4.20) allows us to construct confidence boundaries for $R(\tau_i)$. From (3.4.20), we produce the following expressions for the upper and lower confidence boundaries:

$$R(\tau_i)=R^*(\tau_i)-\frac{z_a}{\sqrt{n}}[1-R^{*2}(\tau_i)]; \tag{3.4.21}$$

$$R(\tau_i)=R^*(\tau_i)+\frac{z_a}{\sqrt{n}}[1-R^{*2}(\tau_i)]. \tag{3.4.22}$$

Figures 19-22 show the confidence boundaries calculated by formulas (3.4.21) and (3.4.22) with the dotted lines. The confidence zone bounded by $R_l(\tau_i)$ and $R_u(\tau_i)$ should be understood as the zone with random upper and lower boundaries which covers the true normalized correlation function with probability a. Figures 19-22 show the confidence zones constructed for $z_a = 1.5$, which corresponds to confidence probability a = 0.8664 (see p. 36).

Let us now look into the sense of the note made earlier concerning the practical noncorrelation of neighboring readings of the process investigated, based on the definition of correlation time using formula (3.4.6) where $A \leqslant 0.1$. Putting forth the zero hypothesis H_0 of independence of readings ($H_0:\rho = 0$), let us construct the critical area in the form

$$|R^*(\tau_i)|>z_a\frac{[1-R^{*2}(\tau_i)]}{\sqrt{n}} \tag{3.4.23}$$

with level of significance q = 1 - a. In statistical analysis, with a certain degree of arbitrariness, only those divergences, the probability of which q < 0.01 are considered significant, assuming that from the practical point of view the appearance of events with this low probability can be ignored (Nalimov, 1960). Probability q is called the level of significance. The lower the level of significance, the lower the probability that the hypothesis will be negated, when it is true; however, with decreasing level of significance, the area of permissible values expands and the probability of the zero hypothesis being accepted when not true increases.

If the value of r produced from experimental data falls in the critical area, hypothesis $H_0:\rho = 0$ should be negated with level of significance q.

For the value $z_a = 1.5$, corresponding to a more rigid level of significance $q = 1 - a = 0.1336$, according to (3.4.23), we can calculate the critical values for $R^*(\tau_i)$. Considering small (beginning with the third) values of the normalized statistical correlation functions, in place of /54

$$\frac{z_a}{\sqrt{n}}[1 - R^{*2}(\tau_i)]$$

we calculate

$$z_a\sqrt{D_{r_{\max}}} = \frac{z_a}{\sqrt{n}},$$

using the upper estimate of dispersion of the statistical correlation coefficient (3.4.19). For the state of attention (see Figure 20, 22), the critical values are 0.135 and 0.1 respectively. The values of $R^*(\tau_i)$, as we see, do not exceed the critical values and, consequently, the hypothesis of independence of readings of random sequence $Z(t_k)$ is expected as noncontradictory to the experimental data with level of significance 0.13. For the state of operative rest (see Figure 19, 21), the critical values, equal to 0.12 and 0.125 respectively, are slightly increased by the second readings $R^*(\tau_i)$, indicating weak correlation of neighboring readings of random sequence $Z(t_k)$.

For the state of attention, the note indicating noncorrelation of readings $Z(t_k)$ (see Figure 20, 22), separated by distance τ where A = 0.1 (3.4.6), means that the experimental data do not contradict the affirmation of absence of correlation connections between readings separated by interval τ, with sufficiently high levels of significance 0.1336 (see Figure 20) and 0.2501 (see Figure 22). For operative rest, hypothesis $H_0:\rho = 0$ is also not negated where q = 0.003 (see Figure 19) and q = 0.0674 (see Figure 21). The practical absence of correlation dependence between readings of random sequence $Z(t_k)$ distributed normally, which will be demonstrated below, means that the readings are practically independent and that a sufficient statistical characterization of the random sequence can be its univariate distribution rule.

§3.5. Estimate of Univariate Distribution Rule for Values of Random Sequence of Readings of α Rhythm Integrator

It was noted earlier that the statistical characteristics based on the usage of limited experimental material from observations are approximate estimates of the true values of the parameters being studied. In connection with this, it is quite important to know the degree of possible deviations of statistical characteristics from the true values of the corresponding probability characteristics and the probability with which the true value of the probability characteristic will be within a given interval relative to the /55 statistical characteristic, or the interval relative to the statistical characteristic within which, with a given probability, the true value of the probability characteristic will be found. In order to produce confidence probabilities or intervals, it is necessary to know the distribution of the statistical characteristic in question.

The assumption of the form of the distribution is made on the basis of certain theoretical ideas concerning the process being investigated which always require later checking. Thus, in statistical analysis of new experimental material, first of all the necessity arises of estimating the degree of approximation of the experimentally observed distribution to a certain hypothetical distribution. This empirical material, always based on a limited number of experiments, can be matched to a set of theoretical distributions more or less satisfactorily describing the observations.

However, in many cases the hypothesis of normal distribution is accepted. On the one hand, this is done since at the present time the mathematical apparatus for random processes whose values are normally distributed is most fully developed, while on the other hand random quantities encountered in practice do frequently follow the normal distribution rule. Therefore, in those cases when no prior knowledge concerning the distribution of values of random processes being investigated is available, preference is naturally given to the normal distribution.

In order to estimate the degree of approximation of the statistical distribution to the theoretical, the conformity criterion is used, the most common of which is the Pearson criterion (χ^2 criterion) (Kramer, 1948) or the Kolmogorov-Smirnov criterion. The usage of the Kolmogorov[1] criterion is distinguished by its simplicity, although it is applicable in the case when the hypothetical distribution is fully known from theoretical considerations, i.e. when we know not only the form of the distribution function, but all of the parameters which it includes. The measure of the divergence of

[1] The Smirnov criterion is similar to the Kolmogorov criterion. The Kolmogorov criterion compares empirical and supposed theoretical integral distributions. The Smirnov criterion compares two empirical integral distributions (van der Waarden, 1960).

statistical and theoretical integral distributions used is the greatest value of absolute magnitude of difference D_q of the statistical and theoretical integral distribution functions:

$$D_q = \frac{\lambda}{\sqrt{n}}, \tag{3.5.1}$$

where $\lambda > 0$ is the argument of the Kolmogorov function; n is the number of observed values of random quantity X. /56

The experimental value D of quantity D_q is determined by the formula

$$D = \max |F^*(x) - F(x)|, \tag{3.5.2}$$

where $F^*(x)$ is the statistical integral distribution function, F(x) is the theoretical integral distribution function.

With sufficiently large n, the following approximate relationship is correct:

$$P\{D < D_q\} \simeq K(\lambda) \tag{3.5.3}$$

or

$$P\{D\sqrt{n} < \lambda\} \simeq K(\lambda), \tag{3.5.4}$$

where $K(\lambda)$ is the Kolmogorov distribution function, which is the limit distribution of random quantity $D\sqrt{n}$ as $n \to \infty$.

The values of the Kolmogorov function are presented in the tables (Arley and Bukh, 1951; Gnedenko, 1961; Gnedenko, Belyayev, Solov'yev, 1965).

If we fix the level of significance $q = P\{D > D_q\}$, from the conditions of the problem being solved with respect to $K(\lambda) = 1 - q$, we determine the value of λ. If it is found as a result of experimentation that $D\sqrt{n} < \lambda$, this indicates conformity of the statistical distribution to the theoretical distribution.

When the Kolmogorov criterion is used, it is assumed that the theoretical function F(x) is continuous, while the statistical function $F^*(x)$ is constructed using nongrouped data. In practical applications, for reasons of simplicity of calculations, we must use an approximation grouping values of the random quantity into short intervals. The grouping of data, as well as the absence of information on parameters of theoretical distribution F(x) before experimentation when this criterion is used frequently lead to overly high conformity (Nalimov, 1960). In some works (Kutay, Kordonskiy, 1958, etc.) it is suggested, on the basis of practical experience, that the divergence between statistical and theoretical distributions be considered insignificant where $q \geqslant 0.6$ if the parameters of the theoretical distribution are not known in advance. Ya. B. Shor (1962) allows good agreement of functions $F^*(x)$ and $F(x)$ where $q > 0.3$-0.4.

The χ^2 criterion is more cumbersome, but allows determination of parameters of theoretical distribution F(x) using experimental data. The number of parameters used for the theoretical distribution in the χ^2 criterion is considered by introduction of a correction in the form of a corresponding decrease in the number of degrees of freedom of the χ^2 distribution in order to avoid an elevated estimate of conformity. As the measure of permissible divergence between statistical and theoretical distribution rules, a value of χ^2_q is used, the experimental value of which χ^2 is determined by the formula /57

$$\chi^2 = \sum_{i=1}^{l} \frac{(m_i - np_i)^2}{np_i}, \tag{3.5.5}$$

where l is the number of positions into which all experimental values of quantity X are divided; n is the number of observed values of random quantity X; m_i is the number of values of random quantity X in the i-th position; p_i is the probability that X will strike the interval of the i-th position, calculated for the theoretical distribution.

As $n \to \infty$, distribution χ^2, regardless of the distribution of random quantity X, approaches the χ^2 distribution with $k = l - r - 1$ degrees of freedom, where r is the number of parameters of the theoretical distribution calculated from the experimental data.

For a fixed level of significance q and k degrees of freedom, using the tables of values of probability $P\{\chi^2 > \chi^2_q\}$ (Smirnov, Dunin-Barkovskiy, 1959; Nalimov, 1960; Gnedenko, 1961) we can find χ^2_q. If the value of χ^2 calculated using formula (3.5.5) is less than χ^2_q, the divergence of experimental data from the hypothetical assumption concerning the distribution is not essential.

The Pearson criterion must be used with some caution, since it is based not on a strict rule, but on an approximation, which can be used only in the case when the number of positions is not less than five and when for all positions the condition $np_i \geqslant 5$-10 is fulfilled. Positions with $np_i < 5$-10 should be joined together. In testing the hypothesis of normality, it is desirable that the number of observations be no less than 50 (Nalimov, 1960; Smirnov, Dunin-Barkovskiy, 1959).

Due to the absence of information on parameters of the theoretical distribution for random sequence $Z(t_k)$, the χ^2 criterion was used to estimate values of the statistical distribution. The results of the estimates showed that the data produced for various persons both in state R and in state A did not contradict the hypothesis of the distribution of the values of $Z(t_k)$ according to the normal rule with level of significance $q < 0.01$.

§3.6. The Problem of the Normal Distribution of a Random Sequence of Readings of the α Rhythm Indicator

The α rhythm, converted into $Z(t_k)$, is a complex oscillation of "sine-wave" form with frequency f_0 about 10 oscillations per second (Kozhevnikov, Meshcherskiy, 1963),,the amplitude of which is modulated by oscillations /58 fluctuating in amplitude and frequency (Figure 23, 24). On the average, the frequency of the modulating oscillation is several times less than f_0. The spectral band width of the detected α rhythm is approximately

$$\Delta f \approx \frac{1}{\tau_0},$$

where τ_0 is the correlation time for the α rhythm. Where τ_0 = (0.5-1) sec, $\Delta f \approx$ (1-2) Hz.

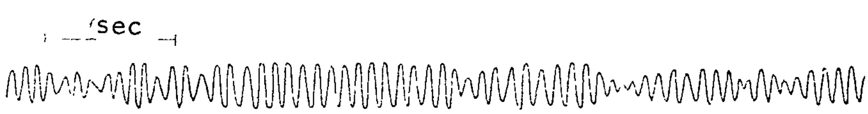


Figure 23

Figures 25 and 26 show the smoothed spectral estimates $F_T(f)$ of the α rhythm (see §4.1) for the state of rest and the state of attention of the test subject respectively, calculated from realizations of the α rhythm (see Figure 23, 24) 17 sec in duration with a maximum shift of the correlation function of 5 sec. We can see from these graphs (Figures 25, 26) that the greatest values of spectral density of the α rhythm, corresponding to the condition $F_T(f) > 1/2 \max F_T(f)$, is concentrated in the frequency band Δf, which is 1.5-2.0 Hz in width.

The pass band of the integrating device Z is $\Delta F \approx 0.5/T$, where T is the integration time (§4.2). Where T = 10 sec, $\Delta F \approx 0.05$ Hz.

Figure 24

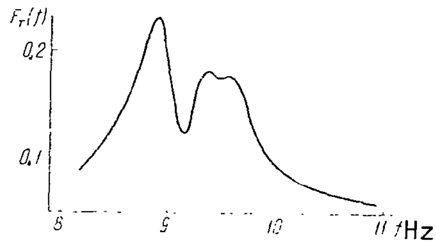


Figure 25

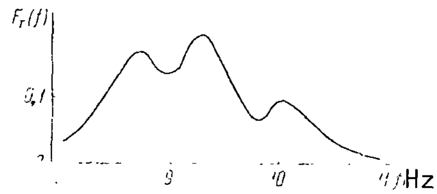


Figure 26

We know that in those cases when the frequency band of the process at the /59
input of the linear system is much broader than the pass band of the system, the process at the output always has a tendency to normalization, i.e. its distribution function rather closely approximates the normal rule (Levin, 1966). In other words, for normalization of the process the condition $\Delta f/\Delta F \gg 1$ must be fulfilled. We have $\Delta f/\Delta F = (1\text{-}2)/0.05 = 20\text{-}40$.

These considerations increase the correctness of statistical conclusions of the normal distribution of random sequences of readings of the α rhythm integrator.

§3.7. Estimate of Signal Characteristics of Attention

In analysis of statistical normalized correlation functions, we noted some difference of these functions with various states of the operator; therefore, it seems natural to us to compare the statistical distribution

$w^*_1(\zeta)$ during the attention state and $w^*_2(\zeta)$ during the operative rest state in order to determine the characteristics which differ the state of attention from the state of operative rest, if such characteristics are present in process $Z(t_k)$.

As a zero hypothesis H_0, to be tested, we will assume that realization $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_m)$, produced during the attention state, and realization $\zeta' = (\zeta'_1, \zeta'_2, \ldots, \zeta'_m)$, in the operative rest state, are extracted from the same set of realizations and, therefore, their distributions are identical:

$$H'_0: w_2(\zeta) = w_1(\zeta). \tag{3.7.1}$$

In order to test (3.7.1), let us use the Kolmogorov-Smirnov equation (§3.5). As applicable to this criterion, the zero hypothesis H'_0 will be represented in the form H_0:

$$H_0: F_2(\zeta) = F_1(\zeta), \tag{3.7.2}$$

where $F(\zeta) = \int_{-\infty}^{\zeta} w(\zeta)\,d\zeta$, is the integral distribution rule.

Using the statistical integral distributions $F^*_2(\zeta)$ and $F^*_1(\zeta)$ for ζ and ζ' respectively, let us determine D (3.5.2):

$$D = \max |F^*_2(\zeta) - F^*_1(\zeta)|. \tag{3.7.3}$$

With a level of significance $q = 0.01$, let us calculate for D the critical value of D_q, defined by formula (3.5.4), in which we substitute $m \cdot m'/(m + m')$ in place of n. Then

$$D_q = 1.63 \sqrt{\frac{m + m'}{m \cdot m'}}. \tag{3.7.4}$$

As our calculations have shown, hypothesis (3.7.2) cannot be accepted, even with a lower level of significance, $q = 0.001$. For these states, the differences in the distributions of $Z(t_k)$ can be determined completely, of course, by direct comparison of their distribution rules. However, with the great variety of possible deviations of one distribution from another, considerable difficulties arise, related not so much with manifestation of the divergences, as with comparison with the divergences determined for other /60 pairs of distributions. Therefore, it is important in application to be able to describe the distribution at least in general terms, by several simple numerical characteristics and to go over from performance of calculations using the distributions to operations with the numerical characteristics, allowing considerable simplification of computational work.

The mathematical expectation m_z and dispersion D_z are numerical characteristics completely defining the univariate normal distribution of the random sequence $Z(t_k)$. For the states of attention and operative rest, let us estimate the essence of the difference of the distributions of sequence $Z(t_k)$ using the statistical parameters $m^*_{Z_1}$, $D^*_{Z_1}$ and $m^*_{Z_2}$, $D^*_{Z_2}$.

§3.8. Attention and Dispersion of Random Sequence of Readings of α Rhythm Integrator

One of the characteristic properties of the brain is its spontaneous, so-called background activity. The background activity, as an external manifestation of the initial state of the nerve centers, is formed as a result of the constant stimulation of these centers, particularly by the influx of background afferent impulses, which is one of the leading factors in the formation of background activity. The constant presence of "spontaneous" activity with various states of the living brain (sleep, waking, activity, rest) indicates that this is a form of activity which, while constantly remaining at a level and fluctuating about it, at the same time under normal conditions at each given moment is always in an equilibrium state. It cannot constantly increase or decrease since both an increase or decrease leads to the development of a pathological state and in the final analysis is incompatible with life (Livanov, 1965). In the EEG, background activity appears as the form of randomly alternating sectors of depression and synchronization of varying lengths. The concept of synchronization is related to high amplitude, rhythmic electrical oscillations recorded from the surface of the cerebral cortex, the result of synchronous activity of a considerable number of nerve cells with identical lability and forming definite spatial systems in the /61
cerebral cortex. The broader these systems and the greater the number of neurons taking part in them, the greater the territory of the cerebral cortex from which the synchronous rhythms can be recorded and the greater the amplitude of these rhythms (Livanov, 1960). When synchronous activity of nerve cells is disrupted, a decrease in the amplitude of oscillations occurs, the electroencephalogram is depressed, indicating an increase in the level of excitation of nervous structures. These changes in the parietal-occipital lead are expressed most strongly in oscillations in the α frequency range, the amplitude of which usually undergoes a characteristic modulation, i.e. more or less periodic increase and decrease with a mean frequency of 0.2-2 Hz (Kozhevnikov, Meshcherskiy, 1963). The "frequency" of the modulation depends on the functional state of the brain. Yui Vyen Chao (1964) and M. B. Mikhalevskaya (1966) showed that the number of spontaneous depressions of the α

rhythm depends on the probability of appearance of the signal expected. In connection with this, with an equal distribution of signals, which was the case in our experiments, the "frequency" of depression and, subsequently, the α rhythm envelope, should remain relatively constant, different than that observed in the state of operative rest. In this case, with otherwise equal conditions, we can expect the phenomenon of stabilization of readings of the α rhythm integrator. If the signal arriving from the detector to the input of the integrator can be represented in the form

$$u(t) = V_0 + V \sin(\omega t + \varphi), \tag{3.8.1}$$

where V_0 is a constant component; V is the amplitude; ϕ is the initial phase; ω is the mean angular "frequency" of depressions of the α rhythm envelope, at the output of the integrator we will have a sequence of readings $\zeta(t_1),\ldots,$ $\zeta(t_k),\ldots,\zeta(t_n)$:

$$\zeta(t_k) = \int_{t(k-1)}^{t_k} u(t)\,dt \tag{3.8.2}$$

or where $t_k - t_{k-1} = T$ - const

$$\zeta(t_k) = \int_0^T [V_0 + V\sin(\omega t + \varphi)]\,dt, \tag{3.8.3}$$

where T is the integration time.

Solving (3.8.3), we produce

$$\zeta(t_k) = V_0 T + 2\frac{V}{\omega}\sin\left(\omega\frac{T}{2}\right)\sin\left(\omega\frac{T}{2} + \varphi\right). \tag{3.8.4}$$

Since the "frequency" of depressions fluctuates about its relatively constant mean value, the value of the initial phase ϕ takes on a random value /62 for the i-th observation interval. Also, the "frequency" of depressions of the α rhythm at 0.2-2.0 Hz is not necessarily a multiple of the frequency (1/T = 0.1 Hz) of readings of the integrator, so that interval T may not include a whole number of "periods" of depression. For these reasons, fluctuations in $\zeta(t_k)$ will be observed due to ϕ, ω - var where V_0 = const, the magnitude of which will be determined by the second component in expression (3.8.4) and will reach the greatest possible value with modulus $2V/\omega$.

Expression (3.8.5) defines the interval of possible values of $\zeta(t_k)$:

$$\zeta_m(t_k) = V_0 T \pm 2\frac{V}{\omega}. \tag{3.8.5}$$

It is important that fluctuations of $\zeta(t_k)$ are inversely proportional to the "frequency" of spontaneous depressions of the α rhythm. In the state of attention, the angular frequency ω should be higher than in the state of operative rest and, consequently, lower fluctuations in readings of the α rhythm integrator should be expected. The experimental data which we have produced confirm these considerations.

When the two states are compared, in order to eliminate the influence of possible changes in the value of V_0 on fluctuations of $\zeta_m(t_k)$, it is expedient to compare the ratios of components which are functions of ω to components which are independent of the mean "frequency" of the depressions of the α rhythm envelope (3.8.5). These relationships can serve as a measure of stabilization of α rhythm integrator readings.

The decrease in statistical dispersions of the random sequence of α rhythm indicator readings $Z(t_k)$ observed in the attention state must be subjected to testing: is it significant. For this purpose, we use the criterion of the dispersion ratio, which can be used for evaluation of the difference of two statistical dispersions $D^*_{Z_2}$ and $D^*_{Z_1}$ for samples whose elements are independent and are normally distributed with parameters m_{Z_2}, D_{Z_2} and m_{Z_1}, D_{Z_1} respectively. In order to check the equality of statistical dispersions $D^*_{Z_2}$ and $D^*_{Z_1}$ to the zero hypothesis H_0:

$$H_0 : D_{Z_2}/D_{Z_1} = 1,$$

where D_{Z_1}, D_{Z_2} are the dispersions of random sequences $Z(t_k)$ respectively for the states of attention and operative rest with the competing hypothesis H_1:

$$H_1 : D_{Z_2}/D_{Z_1} > 1$$

it is necessary to select the critical value for the ratio $F = D^*_{Z_2}/D^*_{Z_1}$. According to the criterion of the dispersion ratio we select as the critical /63 value $F_q[q;(n_1 - 1), (n_2 - 1)]$, where q is a fixed level of significance; n_1, n_2 are sample sizes. The value $F_q[q;(n_1 - 1), (n_2 - 1)]$ is selected from the table (Smirnov, Dunin-Barkovskiy, 1959; Nalimov, 1960) on the basis of known q, n_1, n_2. In the case where

$$F > F_q[q; (n_1 - 1), (n_2 - 1)],$$

the zero hypothesis is negated with level of significance 2q.

In most cases, the experimental data produced contradict the zero hypothesis H_0 with a level of significance $q = 0.05$ concerning a random decrease in dispersion of the random sequence of readings of the α rhythm indicator for the attention state D_{Z_1} in relation to D_{Z_2} -- for the state of operative rest.

For a normal random sequence $Z(t_k)$, the statistical characteristics m_Z^*, D_Z^* are mutually independent. The fulfillment of condition $D_{Z_2}/D_{Z_1} > 1$ and the independence of the characteristics m_Z^*, D_Z^* indicates stabilization of readings of the α rhythm integrator, resulting from the state of attention of the operator.

§3.9. Estimatation of the Difference in Two Statistical Mathematical Expectations with Various Statistical Dispersions

It was demonstrated in the preceding paragraph that for the state of operative rest and the state of attention, the random sequence of readings of the α rhythm integrator have nonidentical dispersion ($D_{Z_2} \neq D_{Z_1}$). This inequality of dispersions D_{Z_2} and D_{Z_1} does not allow us to use the t criterion which is widely used in practice, to test the essence of the difference of the mathematical expectations of two independent, normally distributed samplings. The necessity of comparison of two samplings with different dispersions arises fairly frequently, although the necessary recommendations for the performance of such comparisons are almost not to be found in the domestic literature. In connection with this, we will turn our attention to the problem of the theory and techniques of performing estimates of the difference $(m_1 - m_2)$ between the mathematical expectations m_1, m_2 of two normal distributions (m_1, D_1), (m_2, D_2) using the statistical characteristics m_1^*, D_1^* and m_2^*, D_2^* of two independent samplings with unknown distribution parameters m_1, D_1, m_2, D_2. The solution of the problem consists in establishment of rules for determination of confidence areas for m_1, m_2 and confidence coefficient a. In the space of parameters, each point with coordinates m_1, D_1, m_2, D_2 corresponds to the joint distribution n_1 of the quantities $Z_1 = \{Z_{11}, Z_{12}, \ldots, Z_{1n_1}\}$ and n_2 of the quantities $Z_2 = \{Z_{21}, Z_{22}, \ldots, Z_{2n_2}\}$. In the $(n_1 + n_2)$-dimensional space of points $\{Z_1, Z_2\} = \{Z_{11}, Z_{12}, \ldots, Z_{1n_1}, Z_{21}, \ldots, Z_{2n_2}\}$ we separate the set S of them /64 defined by the relationship

$$K_1 < C_1 \frac{m_1^* - m_1}{\sqrt{D_{m_1}^*}} + C_2 \frac{m_2^* - m_2}{\sqrt{D_{m_2}^*}} < K_2, \tag{3.9.1}$$

where K_1, K_2, C_1, C_2 are constant numbers; D_i^* is the statistical dispersion of the statistical mathematical expectation m_i^* for the i-th state; $i = 1;2$ (in our case, 1 is the state of attention, 2 is the state of operative rest).

The distributions of the random quantities

$$t_1 = \frac{m_1^* - m_1}{\sqrt{D_{m_1}^*}}, \quad t_2 = \frac{m_2^* - m_2}{\sqrt{D_{m_2}^*}},$$

as we know, are the Student distributions with $(n_1 - 1)$, $(n_2 - 1)$ degrees of freedom respectively:

$$S_{n_1-1}(t_1), \quad S_{n_2-1}(t_2).$$

The random quantities t_1, t_2 are independent; therefore, the joint distribution is written as

$$S_{n_1-1}(t_1)\, S_{n_2-1}(t_2). \tag{3.9.2}$$

Knowing the joint distribution of the quantities $\frac{m_1^* - m_1}{\sqrt{D_{m_1}^*}}$ and $\frac{m_2^* - m_2}{\sqrt{D_{m_2}^*}}$, integrating (3.9.2) with respect to the area

$$K_1 < C_1 t_1 + C_2 t_2 < K_2, \tag{3.9.3}$$

we can determine the probability that the sampling point $\{z_1, z_2\}$ belongs to set S:

$$I = \iint S_{n_1-1}(t_1)\, S_{n_2-1}(t_2)\, dt_1\, dt_2. \tag{3.9.4}$$

Let us transform (3.9.1):

$$\frac{C_1 m_1^*}{\sqrt{D^*_{m_1}}} + \frac{C_2 m_2^*}{\sqrt{D^*_{m_2}}} - K_2 < \frac{C_1}{\sqrt{D^*_{m_1}}} m_1 + \frac{C_2}{\sqrt{D^*_{m_2}}} m_2 < \frac{C_1 m_1^*}{\sqrt{D^*_{m_1}}} + \frac{C_2 m_2^*}{\sqrt{D^*_{m_2}}} - K_1. \tag{3.9.5}$$

Varying the constant quantities C_1, C_2, K_1, K_2, we can select a confidence area for m_1 and m_2 such that the confidence coefficient (confidence probability) defined by (3.9.4), will be equal to the predetermined quantity /65
a. Let us assume

$$C_1 = \sqrt{D^*_{m_1}},\ C_2 = -\sqrt{D^*_{m_2}},\ K_1 = m_1^* - m_2^* - b_2,\ K_2 = m_1^* - m_2^* - b_1,$$

where b_1, b_2 are values determining the fixed interval (b_1, b_2) limiting S. Substituting these values into (3.9.5) and (3.9.3), we produce the confidence area for m_1 and m_2:

$$b_1 < m_1 - m_2 < b_2$$

with confidence coefficient (3.9.4), for the calculation of which, the integration area is determined by the expression

$$m_1^* - m_2^* - b_2 < t_1\sqrt{D^*_{m_1}} - t_2\sqrt{D^*_{m_2}} < m_1^* - m_2^* - b_1. \tag{3.9.6}$$

From this, we can conclude the criterion of significance suggested by Barents and Fischer (Kramer, 1948).

Suppose θ is an angle such that

$$\sqrt{D^*_{m_1}} = \delta \sin\theta, \quad \sqrt{D^*_{m_2}} = \delta \cos\theta,$$

where

$$\delta = \sqrt{D^*_{m_1} + D^*_{m_2}}.$$

From the equality I = q for (3.9.4) extending through the area

$$t_1 \sin\theta - t_2 \cos\theta > d_q,$$

we determine the permissible value of divergence d_q. With fixed level of significance q, quantity d_q is a function of n_1, n_2, θ, which can be calculated if the values of the arguments are known. For the function $d_q = f(n_1, n_2, \theta, q)$ we have the tables (Fischer, Jates, 1953). Table 2 presents values of $d_q(n_1, n_2, \theta, q)$ for q(in %) = 1, 5, $n_1 = n_2$ = 6, 8, 12, 24, ∞ and θ° = 0, 15, 30, 45, 60, 75, 90. Table 3 presents values of $d_q(n_1, n_2, \theta, q)$ for q (in %) = 0.2, 0.5, 1, 2, 5, 10, n_1 = ∞, n_2 = 10, 12, 15, 20, 30, 60, ∞ and θ° = 0, 10, 20, 30, 40, 50, 60, 70,80, 90.

If with a fixed level of significance q the following inequality obtains:

$$d = \frac{|m^*_1 - m^*_2|}{\sqrt{D^*_{m_1} + D^*_{m_2}}} > d_q, \tag{3.9.7}$$

the values of m^*_1 and m^*_2 should be considered different from each other with a level of significance equal to q.

TABLE 2

q, %	n_2	n_1	θ, deg 0	15	30	45	60	75	90
5	6	6	2,447	2,440	2,435	2,435	2,435	2,440	2,447
		8	2,447	2,430	2,398	2,364	2,331	2,310	2,306
		12	2,447	2,423	2,367	2,301	2,239	2,193	2,179
		24	2,447	2,418	2,342	2,247	2,156	2,088	2,064
		∞	2,447	2,413	2,322	2,201	2,082	1,993	1,960
	8	6	2,306	2,310	2,331	2,364	2,398	2,430	2,447
		8	2,306	2,300	2,294	2,292	2,294	2,300	2,306
		12	2,306	2,292	2,262	2,229	2,201	2,183	2,179
		24	2,306	2,286	2,236	2,175	2,118	2,077	2,064
		∞	2,306	2,281	2,215	2,128	2,044	1,982	1,960
	12	6	2,179	2,193	2,239	2,301	2,367	2,423	2,447
		8	2,179	2,183	2,201	2,229	2,262	2,292	2,306
		12	2,179	2,175	2,169	2,167	2,169	2,175	2,179
		24	2,179	2,168	2,142	2,112	2,085	2,069	2,064
		∞	2,179	2,163	2,120	2,064	2,011	1,973	1,960
	24	6	2,064	2,088	2,156	2,247	2,342	2,418	2,447
		8	2,064	2,077	2,118	2,175	2,236	2,286	2,306
		12	2,064	2,069	2,085	2,112	2,142	2,168	2,179
		24	2,064	2,062	2,058	2,046	2,058	2,062	2,064
		∞	2,064	2,056	2,035	2,009	1,983	1,966	1,960
	∞	6	1,960	1,993	2,082	2,201	2,322	2,413	2,447
		8	1,960	1,982	2,044	2,128	2,215	2,281	2,306
		12	1,960	1,973	2,011	2,064	2,120	2,163	2,179
		24	1,960	1,966	1,983	2,009	2,035	2,056	2,064
		∞	1,960	1,960	1,960	1,960	1,960	1,960	1,960
1	6	6	3,707	3,654	3,557	3,514	3,557	3,654	3,707
		8	3,707	3,643	3,495	3,363	3,307	3,328	3,355
		12	3,707	3,636	3,453	3,246	3,104	3,053	3,055
		24	3,707	3,631	3,424	3,158	2,938	2,822	2,797
		∞	3,707	3,626	3,402	3,093	2,804	2,627	2,576
	8	6	3,355	3,328	3,307	3,363	3,495	3,643	3,707
		8	3,355	3,316	3,239	3,206	3,239	3,316	3,355
		12	3,355	3,307	3,192	3,083	3,032	3,039	3,055
		24	3,355	3,301	3,158	2,988	2,862	2,805	2,797
		∞	3,355	2,295	3,132	2,916	2,723	2,608	2,576
	12	6	3,055	3,053	3,104	3,246	3,453	3,636	3,707
		8	3,055	3,039	3,032	3,083	3,192	3,307	3,355
		12	3,055	3,029	2,978	2,954	2,978	3,029	3,055
		24	3,055	3,020	2,938	2,853	2,803	2,793	2,797
		∞	3,055	3,014	2,909	2,775	2,661	2,595	2,579
	24	6	2,797	2,822	2,938	3,158	3,424	3,631	3,707
		8	2,797	2,805	2,862	2,988	3,158	3,301	3,355
		12	2,797	2,793	2,803	2,853	2,938	3,020	3,055
		24	2,797	2,785	2,759	2,747	2,759	2,785	2,797
		∞	2,797	2,777	2,726	2,664	2,613	2,585	2,576
	∞	6	2,576	2,627	2,804	3,093	3,402	3,626	3,707
		8	2,576	2,608	2,723	2,916	3,132	3,295	3,355
		12	2,576	2,595	2,661	2,775	2,909	3,014	3,055
		24	2,576	2,585	2,613	2,664	2,726	2,777	2,797
		∞	2,576	2,576	2,576	2,576	2,576	2,576	2,576

Tr. Note: Commas indicate decimal points.

Let us analyze an example. We have two sequences of readings of the α rhythm integrator for the state of attention $\zeta_1 = \{\zeta(t_1), \zeta(t_2), \ldots, \zeta(t_i), \ldots, \zeta(t_{n_1})\}$ and for the state of operative rest $\zeta_2 = \{\zeta(t_1), \zeta(t_2), \ldots, \zeta(t_j), \ldots, \zeta(t_{n_2})\}$. ζ_1 contains 36 terms ($n_1 = 36$) and ζ_2 contains 51 terms ($n_2 = 51$). The statistical mathematical expectation for ζ_1, ζ_2 are respectively $m_1^* = 62.6$, $m_2^* = 51.6$, and their dispersions are: $D^*_{m_1} = 0.32$, $D^*_{m_2} = 1.20$: /67

$$m_1^* = \frac{1}{n_1}\sum_{i=1}^{n_1}\zeta(t_i); \quad m_2^* = \frac{1}{n_2}\sum_{j=1}^{n_2}\zeta(t_j);$$

$$D^*_{m_1} = \frac{n_1^{-1}}{n_1 - 1}\sum_{i=1}^{n_1}[\zeta(t_i) - m_1^*]^2; \quad D^*_{m_2} = \frac{n_2^{-1}}{n_2 - 1}\sum_{j=1}^{n_2}[\zeta(t_j) - m_2^*]^2.$$

Let us calculate

$$d = \frac{|m_1^* - m_2^*|}{\sqrt{D^*_{m_1} + D^*_{m_2}}} = \frac{|62.6 - 51.6|}{\sqrt{0.32 + 1.20}} = 8.95.$$

Let us determine angle θ:

$$\tan\theta = \sqrt{\frac{D^*_{m_1}}{D^*}} = \sqrt{\frac{0.32}{1.20}} = 0.52; \quad \theta = 27.5°.$$

Using the known $n_1 = 36$, $n_2 = 51$, $\theta = 27.5°$ for q (%) = 1, and using Table 2, let us find the critical value $d_q = d_{1\%}$: where $n_1 = 24$, $n_2 = 24$ and $\theta = 30°$, $d_{1\%} = 2.759$; where $n_1 = 24$, $n_2 = 24$ and $\theta = 15°$, $d_{1\%} = 2.785$; where $n_1 = 36$, $n_2 = 51$ and $\theta = 27.5°$, the value of $d_{1\%}$ is less than 2.785. Thus, we have $8.95 > 2.785$.

TABLE 3

q, %	n_2	θ, degrees: 0	10	20	30	40	50	60	70	80	90
10	10	1,812	1,808	1,794	1,774	1,749	1,721	1,693	1,668	1,651	1,645
	12	1,782	1,778	1,767	1,751	1,730	1,707	1,684	1,664	1,650	1,645
	15	1,753	1,750	1,741	1,728	1,711	1,693	1,675	1,659	1,649	1,645
	20	1,725	1,722	1,716	1,706	1,694	1,680	1,667	1,656	1,648	1,645
	30	1,697	1,696	1,692	1,685	1,677	1,668	1,659	1,652	1,647	1,645
	60	1,671	1,670	1,668	1,665	1,661	1,656	1,652	1,648	1,646	1,645
	∞	1,645	1,645	1,645	1,645	1,645	1,645	1,645	1,645	1,645	1,645
5	10	2,228	2,219	2,194	2,157	2,112	2,063	2,024	1,989	1,967	1,960
	12	2,179	2,171	2.151	2,120	2,083	2,046	2,011	1,984	1,966	1,960
	15	2,131	2,126	2,109	2,085	2,056	2,026	1,999	1,978	1,965	1,960
	20	2,086	2,082	2,069	2,051	2,030	2,008	1.989	1.973	1,963	1,960
	30	2,042	2,039	2,031	2,019	2,005	1,991	1,978	1,968	1,962	1,960
	60	2,000	1,999	1,995	1,989	1,982	1,975	1,969	1,964	1,961	1,969
	∞	1,960	1,960	1,960	1,960	1,960	1,960	1,960	1,950	1,969	1,950
2	10	2,764	2,748	2,704	2,637	2,559	2,481	2,414	2,364	2,335	2,326
	12	2,681	2,668	2,631	2,576	2,513	2,450	2,396	2,356	2,334	2,326
	15	2,602	2,592	2,563	2,520	2.470	2,421	2,379	2,349	2,332	2,326
	20	2,528	2,520	2,498	2,466	2,430	2,394	2,364	2,343	2,330	2,326
	30	2,457	2,452	2,438	2,417	2,393	2,370	2,351	2,337	2,329	2,326
	60	2,390	2,388	2.380	2,370	2,358	2,347	2,338	2,331	2,323	2,326
	∞	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326
1	10	3,169	3,148	3,086	2,993	2,883	2,775	2,684	2,620	2,586	2,576
	12	3,055	3,037	2,985	2,909	2,820	2,733	2,661	2,611	2,584	2,576
	15	2,947	2,932	2,892	2,831	2,762	2,695	2,640	2,603	2,582	2,576
	20	2,845	2,835	2,804	2,760	2,709	2,661	2.622	2,595	2,580	2,576
	30	2,750	2,743	2,723	2,693	2,661	2,630	2,605	2,588	2,579	2,576
	60	2,660	2,657	2,647	2,632	2,616	2,601	2,590	2,582	2,577	2,576
	∞	2,576	2,576	2,576	2,576	2,576	2,576	2,576	2,576	2,576	2,576
0,5	10	3,581	3,553	3,473	3,350	3,2[illegible]3	3,058	2,939	2,859	2,818	2,807
	12	3,429	3,405	3,338	3,237	3,119	3,003	2,910	2,848	2,816	2,807
	15	3,286	3,267	3,214	3,134	3,042	2,954	2,884	2,838	2,814	2,807
	20	3,153	3,139	3,090	3,040	2,974	2,911	2,861	2,829	2,812	2,807
	30	3,030	3,020	2,994	2,955	2,912	2,872	2,841	2,821	2,810	2,807
	60	2,915	2,910	2,897	2,878	2.857	2,838	2,823	2,814	2,809	2,807
	∞	2,807	2,807	2,807	2,807	2,807	2,807	2,807	2,807	2,807	2,807
0,2	10	4,144	4,106	3,999	3,832	3,639	3,425	3,259	3,152	3,103	3,090
	12	3,930	3,898	3,809	3,671	3,508	3,347	3,219	3,138	3.100	3,090
	15	3,733	3,708	3,633	3,528	3,401	3,280	3,185	3,126	3,098	3,090
	20	3,552	3,533	3,479	3,399	3,308	3,222	3,156	3,116	3,096	3,090
	30	3,386	3,372	3,336	3,284	3,226	3,172	3,131	3,106	3,094	3,090
	60	3,232	3,225	3,207	3,181	3,153	3,128	3,110	3,098	3,092	3,090
	∞	3,090	3,090	3,090	3,090	3,090	3,090	3,090	3,090	3,090	3,090

Tr. Note: Commas indicate decimal points.

According to (3.9.7), the difference between m_1^* and m_2^* should be assumed significant with a level of significance $q = 0.01$. The difference between m_1^* and m_2^* is found to be significant also where $q = 0.002$.

The Respiratory System and Attention /69

The respiratory system is designed for the fulfillment of one of the functions important to life of the organism: exchange of gases between the organism and the external medium. Gas exchange between the atmosphere and alveolar air occurs due to the quasi-rhythmic acts of inhalation and exhalation. The "master oscillator" of the respiratory system is a nerve formation in the medulla oblongata, called the respiratory center. The activity of the respiratory center depends considerably on changes in the normal gas composition of the air inhaled, an increase in CO_2 content causing an increase in the amplitude of respiratory movements and pulmonary ventilation, a decrease in the partial pressure of O_2, perceived by the chemoreceptors of the carotid and aortal zones also stimulating the respiratory center, causing a reinforcement of respiration. However, the leading role in the regulation of respiration belongs to the higher centers of the central nervous system, particularly under the complex conditions of adaptive behavior of man under the conditions of operator's activity, under conditions of the influence of various psychological factors, etc. The influence of the higher sectors of the brain on the bulbar respiratory center is expressed as various changes in respiratory rhythm, amplitude of respiratory movement, amount of pulmonary ventilation, etc.

Many and various indicators are used for functional investigation of the respiratory system; however, under the conditions of operator activity, the most accessible are the frequency, depth and nature of breathing.

§3.10. Correlation Properties of the Sequence of Intervals of the Respiratory Cycle

In spite of man's ability to control his respiration voluntarily, the intervals of the respiratory cycle under conditions of operator's activity are rather stable in time. Apparently, this results from the fact that the controlling action of the higher sections of the central nervous system on the adaptive respiratory reaction is significant.

In analyzing the correlation properties of the sequence of intervals of the respiratory cycle $Y(t_k) = \{Y(t_1)...Y(t_n)\}$, the number of terms n in realizations $y(t_k)$ (Figure 27, 28) varied between 100 and 250. Figures 29 and 30 show the normalized statistical correlation functions $R^*(\tau_s)$, calculated using formula (3.4.3) for $n = 375$, which amounts to $t = 526$ sec (state of operative rest, Figure 29) and for n = 473 (t = 552 sec) for the state of attention (Figure 30). The curves $R^*(\tau_s)$ are sharply attenuated and continue to fluctuate weakly about the abscissa, without exceeding the level A = 0.1. /71 The correlation functions $R^*(\tau_s)$ are rather typical for most of the test subjects where experimental conditions were held constant. Insignificant oscillations of the normalized statistical correlation function about zero mean that the random sequence of durations of the respiratory cycle $Y(t_k)$ over the observation interval of up to 500 sec is rather stable. According to (3.4.6), neighboring intervals of the respiratory cycle are practically uncorrelated (A ≈ 0.1); therefore, sufficient statistical characterization of the random sequence of durations of respiratory cycles where it is normal require only that the univariate distribution rule be known.

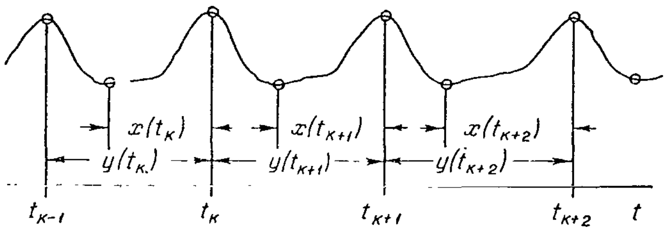


Figure 27

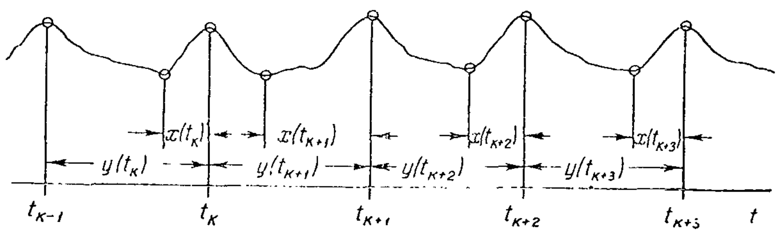


Figure 28

§3.11. Distribution of Random Sequence of Respiratory Cycle Intervals

Due to the absence of information on the parameters of the theoretical distribution of the random sequence of respiratory cycle intervals $Y(t_k)$, in order to test the agreement of experimental data with hypothesis H_0, normal distribution of values of random sequence $Y(t_k)$, the conformity criterion χ^2 was used (§3.5). The results of the evaluation performed show that the experimental data, both for the state of quiet, and for the state of operative rest, do not contradict the zero hypothesis H_0 at level of significance $q < 0.01$. The principal difference in the statistical univariate distribution $Y(t_k)$ lies in the mathematical expectation m_Y. The statistical mathematical expectation m_Y^* of a random sequence of durations of respiratory cycles $Y(t_k)$ for the state of attention $m_{Y_1}^*$ is less than for the state of operative rest $m_{Y_2}^*$. The relative change δ_m in parameter m_Y^* reaches the value 0.32:

$$\delta_m = \frac{m_{y_2} - m_{y_1}}{m_{y_2}}, \qquad (3.11.1)$$

where m_{y_i} is the value of the statistical mathematical expectation of random sequence $Y(t_k)$ for the i-th state.

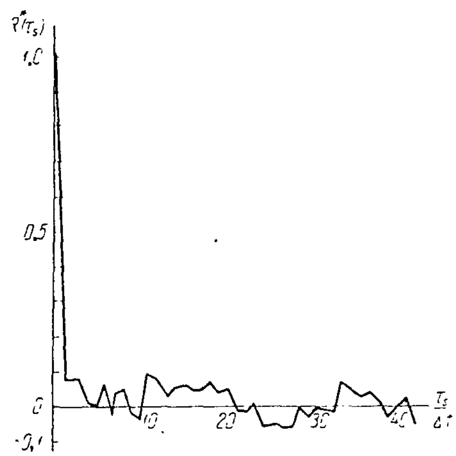


Figure 29

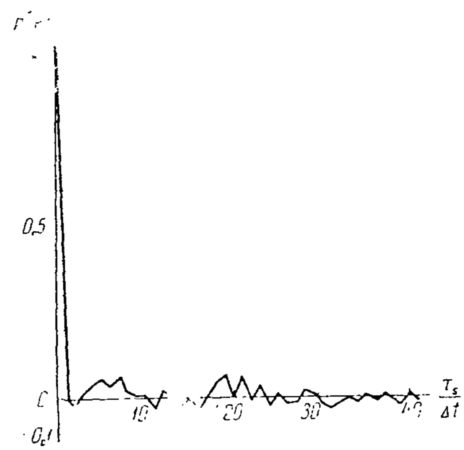


Figure 30

In order to estimate the importance of the difference between states of attention and operative rest on the basis of the mathematical expectation, we can use the Barens-Fischer criterion or the t-criterion (Student criterion), if we assume that the true values of dispersion D_{Y_1}, D_{Y_2} of random sequence $Y(t_k)$ for the state of attention and for the state of operative rest are equal (Smirnov, Dunin-Barkovskiy, 1959; Nalimov, 1960; Kramer, 1948). The t-criterion requires calculation of the following statistics:

$$t = \frac{m_{y_2} - m_{y_1}}{\sqrt{n_1 D_{y_1} + n_2 D_{y_2}}} \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 2)}{(n_1 + n_2)}}, \tag{3.11.2}$$

where n_1 and n_2 are the number of terms of the investigated sequences for the states of attention and operative rest respectively; D_{y_1}, D_{y_2} are the values of the statistical dispersions of the sequences investigated. /72

The hypothesis $H_0: m_{Y_2} - m_{Y_1} = 0$ is assumed if the following inequality is fulfilled:

$$t < t_q\,[q;\ (n_1 + n_2 - 2)], \tag{3.11.3}$$

where $t_q[q;(n_1 + n_2 - 2)]$ is the critical value of the Student distribution for the given level of significance q.

The probability of making the false conclusion $(H \neq H_0)$, i.e. the probability of rejecting hypothesis H_0 when it is actually true, is determined by the expression

$$P\,\{t > t_q\,[q;\ (n_1 + n_2 - 2)]\} = 2q. \tag{3.11.4}$$

The estimate of the difference $(m_{Y_2} - m_{Y_1})$ was performed using both criteria (Student and Barens-Fischer). The experimental data negate hypothesis H_0 of the absence of differences between m_{Y_2} and m_{Y_1} with a level of significance $q > 0.01$.

§3.12. Duration of Respiratory Cycle and Effectiveness of Work

The effectiveness of the work of an operator, the success of his activity is determined by a number of factors, the principal ones of which must be considered natural data, the degree of training, the demands of the work and attention. The dependence of working ability on psychological and functional state (§1.3) makes an estimate of the activity of the operator using quantitative indicators of the work performed without considering the physiological information which would allow us to judge the means and internal stress required to achieve successful performance of the work unreliable.

During the course of training, the organism adapts itself to the new conditions of operator's work, and coordinated interaction of all systems, adequate to the work at hand, is developed. However, the complex process of adaptation of the organism cannot always be completed successfully, even for elementary working conditions. M. Ye. Marshak (1961), in an investigation of the process of establishment of correlations between the working and respiratory movements during the training period to a new type of muscular work, showed that the load on the respiratory system may be inadequate to the muscular work and may be retained for a long period of time, over 3-4 months. Increased pulmonary ventilation is explained by the active participation of nervous mechanisms in the regulation of respiration.

Varying ventilation of the lungs (respiratory frequency) with the same /73
effectiveness (u) of operator work was observed in a group of test subjects in an experiment involving seeking a light signal (Chapter 2). In spite of the rather simple assignment and extended training, continuing for several days, we found no noticeable changes in the test subjects toward any ordering between u and the duration of the respiratory cycle. This fact indicates the complex and extensive adaptive restructuring of the central nervous system which occurs during the process of adaptation of the organism to conditions of new operator's activity.

In another group of test subjects, a rapid (within one hour) establishment of correlation between effectiveness of work u and interval of the respiratory cycle was noted. In each experiment, the estimate of effectiveness of operator work was the value of u (Chapter 2):

$$u = -\frac{N}{T_0}\left\{\frac{N_1 - K}{N}\log_2\frac{1}{8} + \left[1 - \frac{N_1 - K}{N}\right]\log_2\frac{7}{8}\right\}, \qquad (3.12.1)$$

where N is the total number of rings in a frame; N_1 is the number of rings in a frame (with orientation of the notch in the required direction), which the test subject was to seek out; K is the number of rings of N_1 not noticed by the test subject; T_0 was the time expended by the test subject in seeking out (N_1 - K) rings.

After transformation and substitution of N = 510, expression (3.12.1) takes on the form

$$u = \frac{98.43 + 2.807\,(N_1 - K)}{T_0}\left[\frac{\text{bits}}{\text{sec}}\right]. \qquad (3.12.2)$$

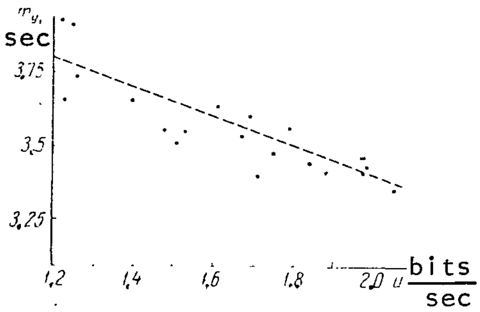


Figure 31

Figure 31 shows points $(u_i, m_{y_1 i})$ for test subject S. G., produced using the data of three experimental days. The abscissa shows the value of u in bits per second, calculated using formula (3.12.2), while the ordinate shows the values of respiratory cycle intervals M_{y_1} averaged over time T_0. The tendency to a linear dependence between the value of m_{y_1} and u confirms the high value of the statistical correlation coefficient: $r_{m,u} = -0.80$. /74

$$r_{m,u} = \frac{\sum_{i=1}^{l}(m_i - \bar{m})(u_i - m_u)}{\sqrt{\sum_{i=1}^{l}(m_i - \bar{m})^2 \sum_{i=1}^{l}(u_i - m_u)^2}}, \qquad (3.12.3)$$

where u_i is the effectiveness of work in the i-th frame, determined using formula (3.12.2):

$$m_u = \frac{1}{l}\sum_{i=1}^{l} u_i; \quad m_i = m_{y_1 i} = \frac{1}{n_i}\sum_{j=1}^{n_i} y_i(t_j); \quad \bar{m} = \frac{1}{l}\sum_{i=1}^{l} m_i;$$

l is the number of frames; $y_i(t_j)$ is the j-th interval of the respiratory cycle in the i-th frame; n_i is the number of intervals of the respiratory cycle in time sector T_i; T_i is the search time in the i-th frame.

The number of frames is not great (l = 23), so that for the true value of correlation coefficient $\rho_{m,u}$, we can find the confidence interval.

If the value of true correlation coefficient ρ is other than zero (an indication of this is nonzero value of the sampling correlation coefficient r), the distribution of the sampling coefficient will be rather complex. With sufficiently large n, this distribution can be approximated by the normal distribution; however, for ρ near ± 1, the approximation is reliable only with very high values of n. In our case, $\rho_{m,u}$ is rather near -1 ($r_{m,u}$ = -0.80), while n is small ($n = l = 23$); therefore, we should not count on normal distribution of r in determining the confidence interval for $\rho_{m,u}$.

Let us use a very practical transformation suggested by Fischer for random quantity r:

$$Z = \frac{1}{2} \ln \frac{1+r}{1-r}, \tag{3.12.4}$$

which does not depend either on ρ or on n. The distribution of random quantity Z approximates the normal distribution very well with dispersion D_z and mathematical expectation m_z practically independent of ρ (Dlin, 1958):

$$m_Z \approx \frac{1}{2} \ln \frac{1+\rho}{1-\rho}, \quad D_Z \approx \frac{1}{n-3}. \tag{3.12.5}$$

Thus, the expression for the confidence interval for m_z will have the form

$$\frac{1}{2} \ln \frac{1+r}{1-r} - z_a \frac{1}{\sqrt{n-3}} \leqslant m_Z \leqslant \frac{1}{2} \ln \frac{1+r}{1-r} + z_a \frac{1}{\sqrt{n-3}} \tag{3.12.6}$$

Several values of z_a are presented in the table (see pp. 36-37). With probability 0.95 (z_a = 2) the confidence interval boundaries for m_z, according to (3.12.6), have the values z_1 = 0.65, z_2 = 1.55. /75

Resolving (3.12.4) relative to r

$$r = \frac{e^{2Z} - 1}{e^{2Z} + 1}$$

and substituting the values of z_1 and z_2 into the solution produced, we can find the confidence interval boundaries for $\rho_{m,u}$:

$$-0{,}91 \leqslant \rho_{m,u} \leqslant -0{,}57.$$

The upper value of the confidence interval differs considerably from zero and, consequently, we can consider the correlation dependence between the interval of the respiratory cycle and the effectiveness of the operator's work established. Having information for calculation of $r_{m,u}$, we can easily find the direct approximate regression of the respiratory cycle interval with respect to operator work effectiveness (Smirnov, Dunin-Barkovskiy, 1959):

$$m \doteq \bar{m} + r_{m,u} \frac{\sum_{i=1}^{l} (m_i - \bar{m})^2}{\sum_{i=1}^{l} (u_i - m_u)^2} (u - m_u) \tag{3.12.7}$$

or

$$m = r_{m,u} \frac{\sum_{i=1}^{l} (m_i - \bar{m})^2}{\sum_{i=1}^{l} (u_i - m_u)^2} u + \left[\bar{m} + r_{m,u} m_u \frac{\sum_{i=1}^{l} (m_i - \bar{m})^2}{\sum_{i=1}^{l} (u_i - m_u)^2} \right]. \tag{3.12.8}$$

In our case, we have

$$m = -0{,}51u + 4{,}42 \text{ (sec)}. \tag{3.12.9}$$

Dependence (3.12.9) is illustrated on Figure 31 by the dotted line. The knowledge of the level of regression allows us to predict values of m for fixed values of u. Let us analyze the case when the test subject is in the state of operative rest. In this case, the value of u is naturally assumed equal to zero. Where u = 0, it follows from expression (3.12.9) that in the state of operative rest the mean value of the respiratory cycle interval should be 4.42 sec. The actual mean value of the respiratory cycle interval for the state of operative rest was found to be 4.30 sec. The slight difference between the predicted and actual values of mean length of respiratory cycle might indicate that dependence (3.12.9) obtains throughout all the range of possible values of u [$0 \leqslant u \leqslant 2.0$], i.e. not only in the interval shown on /76
Figure 31 [1.2; 2.0] but also in the interval [0; 1.2). From this it follows that in many cases a change in the interval of the respiratory cycle is an effective characteristic of the functional state of man, including his state of attention.

§3.13. The Transient Process of the Respiratory Cycle Interval Sequence During a Change in the Functional State of the Operator

In recent years, the attention of investigators has been attracted to the study of physiological problems by methods from the theory of automatic regulation. General methods in the theory of automatic regulation have been rather completely developed only for linear systems, to which the principle of superposition is applicable. According to the principle of superposition, any process in the system formed as a result of simultaneous imposition of several actions of arbitrary form is equal to the algebraic sum of the processes formed when each of these actions is imposed on the system individually. Due to the principle of superposition, the linear theory of automatic regulation allows the investigation of linear systems to be performed quite completely and relatively simply in various areas of application. Biological systems, generally, are nonlinear systems, to which the linear theory of automatic regulation is not applicable, while the theory of nonlinear systems has been insufficiently developed. However, in many cases it is possible to investigate biological systems by linearization of nonlinearities. The ordinary methods for linearization of the characteristics of a nonlinear element consist in representing the process at the output of a nonlinear element within the limits of the actually possible cations on the input in the form of a linear function of the actions. As applicable to a biological system described by nonlinear characteristic $y = f(x)$, representing the reaction y of the system to a stimulus of magnitude x, it can be assumed that the dependence $y = f(x)$ is linearized if in the range of values of stimulant x investigated the change in reaction y of the system can be approximately represented by a linear function of x, i.e.

$$y = f(x) \approx ax + b, \tag{3.13.1}$$

where a and b are constant quantities. The values of constant quantities a and b are determined both by the characteristics of f(x) and by the range of change of stimulus x. Fulfillment of condition (3.13.1) allows the linear theory of automatic regulation to be used for investigation of biological systems.

The behavior of a linear system, its properties can be characterized in various ways: by differential equations, the amplitude-frequency or transient characteristics. The differential equation reflects the dependence of reaction y of the system to magnitude x of the stimulus, on the rate of change (dx/dt) in stimulus x, on the variation in rate dx/dt, etc. The transient characteristic of the system, as the reaction to the system to a sudden change in stimulus x (3.13.2), reveals the dynamic properties of the system: /77

$$x(t)=\begin{cases}0, & t<t_0\\ 1, & t\geqslant t_0,\end{cases} \qquad (3.13.2)$$

where t_0 is the moment of the sudden change in value of the stimulus.

The same properties of the system, expressed in the frequency "language" in contrast to the time "language" of the transient characteristic, are represented by the amplitude-frequency characteristic.

A knowledge of the transient characteristic and the possibility of application of the method of superposition for a linear system allows us to determine the reaction of the system to a stimulus of any form. All three characteristics are interrelated, and if it is identically difficult to produce the necessary information for composition of them, preference is given to one or another depending on the method of analysis being used. In the investigation of biological systems, when information on the structure of the system and the processes which occur within it are quite limited, and the methods of experimental analysis used in technology can be used only partially, the description of the biological system directly by a differential equation, or by the amplitude-frequency characteristic, encounters considerable difficulties. One of the most usable methods for representing information concerning the dynamic properties of a biological system is the experimental determination of its transient characteristic.

In our experiments, the signal (stimulus) x(t) was the operator's work, requiring constant attention from the test subject. As the reaction of the operator to x(t), we used the variations in duration of the respiratory cycle. The magnitude of the respiratory systems reaction to the operator work depends on the level of attention of the test subject on the work being performed (§3.12; 3.14), so that in individual experiments, a difference in level of attention was possible, which was difficult to test quantitatively, but could lead to certain differences in the transient characteristics as concerns absolute values. However, an approximate composition of a diagram of the functional analog of the respiratory system requires only that an idea be known concerning the form of the transient characteristic, without requiring that absolute values of the transient process be known. For each individual, as his state changed, the form of transient characteristic remained near constant. Upon transition from the state of operative rest to the /79
state of attention, the nature of the transient process can be considered identical for various test subjects.

Figures 32 and 33 show two sets of transient characteristics for test subjects S. G. and N. K. upon transition from the state of operative rest to the state of attention; the heavy line shows the transient characteristic T*(t) averaged for the entire set; the lower dotted line shows values of the mean square deviation $S_T(t)$ of individual intervals of the respiratory cycle from T*(t), and the arrow shows the probable beginning of the change of the subject's state. With an unlimited increase in terms, the set of averaged transient characteristics will be a more detailed description of the transient process, approaching the true transient characteristic T(t). Precise expressions describing absolutely all the specifics of the true transient characteristic require complex analytic expressions and in many cases the usage of special functions. Usually, in experimental and theoretical analyses of transient processes, ignoring certain nonessential details of the transient process being investigated, cumbersome precise expressions for T(t) are avoided, being replaced by the approximate expressions for H(t). Attempts are made to make the functions describing the transient characteristics as simple as possible, so that their parameters can be easily determined.

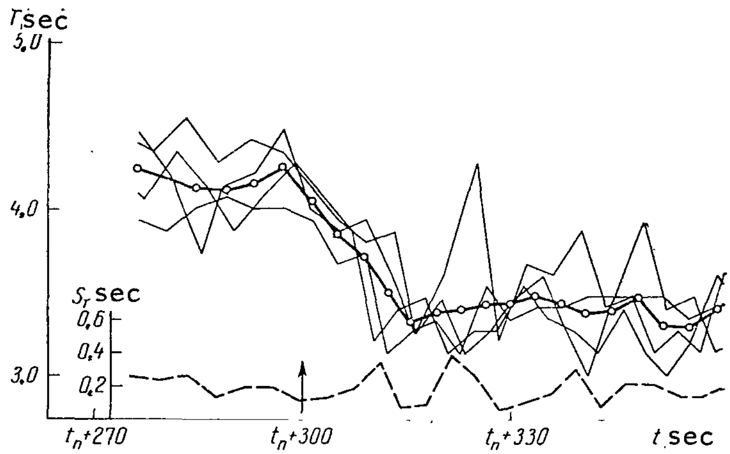


Figure 32

The transient characteristics presented are reminiscent of transient process (3.31.3) in an electrical circuit (Figure 34) with weakly expressed oscillation conditions and predominance of integrating properties:

$$H(t) = K(1 + \alpha t)\, e^{-\alpha t}, \tag{3.13.3}$$

where

$$\alpha = \frac{R}{2L}; \quad R = 2\sqrt{\frac{L}{C}}.$$

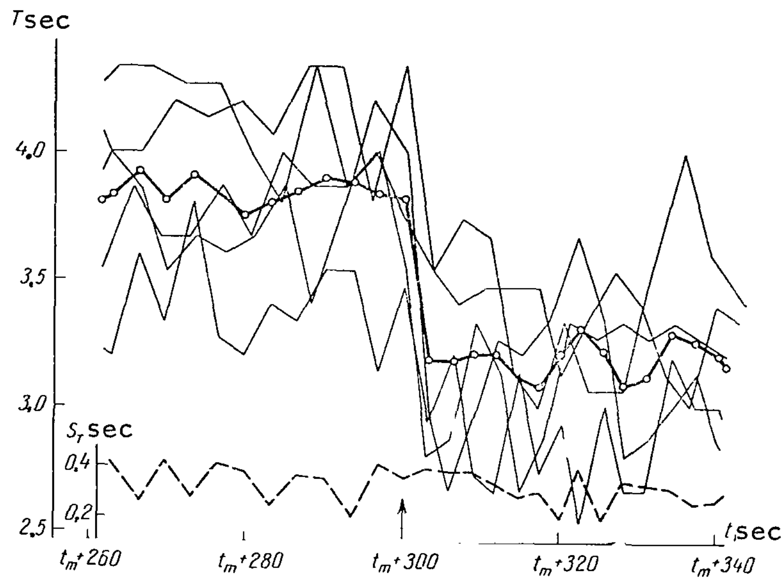


Figure 33

For test subject N. K., the transition from the state of attention to the state of operative rest is accompanied by a transient process (Figure 35) similar to that of an LRC-system (see Figure 34) with the transient characteristic /80

$$H(t) = K[1 - (1 + \alpha t) e^{-\alpha t}]. \tag{3.13.4}$$

Due to the varying adaptation properties of the test subject, the transition from the state of attention to the state of operative rest does not always have the form of (3.13.4).

If we replace the system responsible for the regulation of the duration of the respiratory cycle with its equivalent, determined on the basis of similarity of their transient processes, in our analysis, the oscillation in the transient characteristic (Figure 36) indicates that the transition from the state of attention to the state of operative rest involves a change in the relationships of the values of the LRC-system elements. The LRC system has an oscillating transient characteristic where $R < 2\sqrt{L/C}$:

$$H(t) = K[1 - Ae^{-\alpha t}\cos(\omega t - \psi)]. \tag{3.13.5}$$

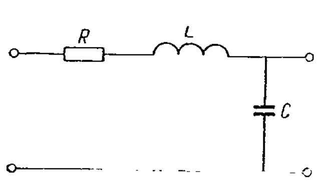


Figure 34

In order to determine the parameters of function (3.13.5), approximating the transient characteristic (see Figure 36), let us use the value of the experimental characteristic T*(t), located in the interval /81

$$[(m_{y_1} + \sqrt{D_{y_1}}), \quad (m_{y_2} + \sqrt{D_{y_2}})],$$

where linear dependence is observed between the duration of the respiratory cycle and the effectiveness of work, related to the attention of the test subject (§3.12). After substituting the values for parameters produced into expression (3.13.5), we produce

$$H_0(t) = 0{,}74[1 - 0{,}95e^{-0{,}018t}\cos(0{,}22t + 0{,}45)]. \tag{3.13.6}$$

The significant excess in the positive fluctuations in the experimental transient characteristic over the negative fluctuations relative to m_{y_2} allows us to assume the presence of nonlinearities in the system. The nonlinear transformation, of the form

$$y = 2{,}4x^3 \tag{3.13.7}$$

in expression (3.13.6) gives rather satisfactory correspondence of $H_1(t)$ (shown by dotted line on Figure 37): /82

$$H_1(t) = 2{,}4[H_0(t)]^3 \tag{3.13.8}$$

with the experimental curve T*(t) -- the continuous line (on Figure 37). Thus, the respiratory system analyzed can be represented (Figure 38) as consisting of a linear portion LRC and a nonlinear portion with characteristic (3.13.7).

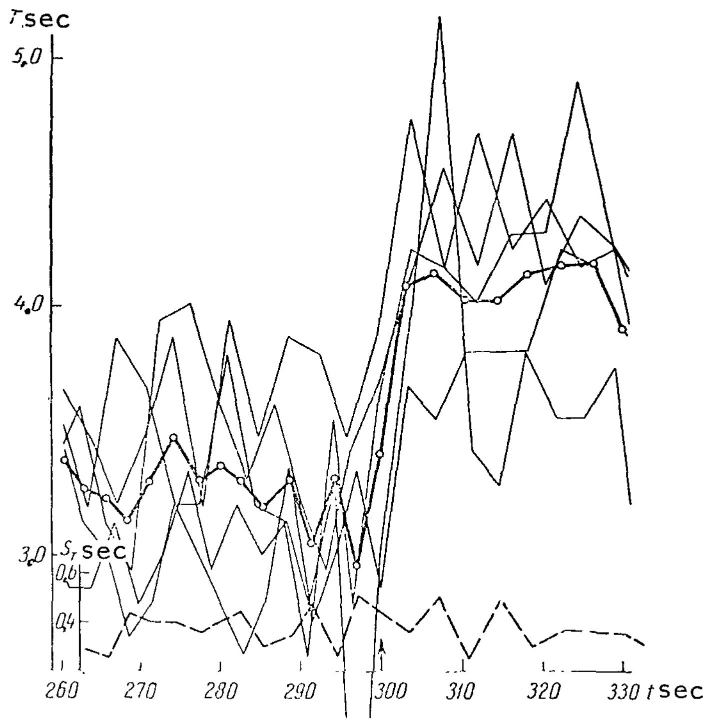


Figure 35

For some test subjects, the change of states causes no essential changes in the character of respiratory frequency, and the following condition is fulfilled for the equivalent circuit of the respiratory system:

$$R \approx 2\sqrt{\frac{L}{C}}. \tag{3.13.9}$$

In this case, the change in intervals of the respiratory cycle occurs within limits such that the "working" sector of the nonlinear characteristic can be considered approximately linear (3.13.1).

For other test subjects, the removal of the controlling action of the higher centers of the brain on the respiratory system resulting from operator's work causes compensatory respiratory activity, and we must change condition (3.13.9) for the equivalent circuit to the respiratory system:

$$R < 2\sqrt{\frac{L}{C}}. \tag{3.13.10}$$

Also, the range of change of durations of respiratory cycles becomes such that the "working" sector of the characteristic of the equivalent circuit to the respiratory system is essentially nonlinear.

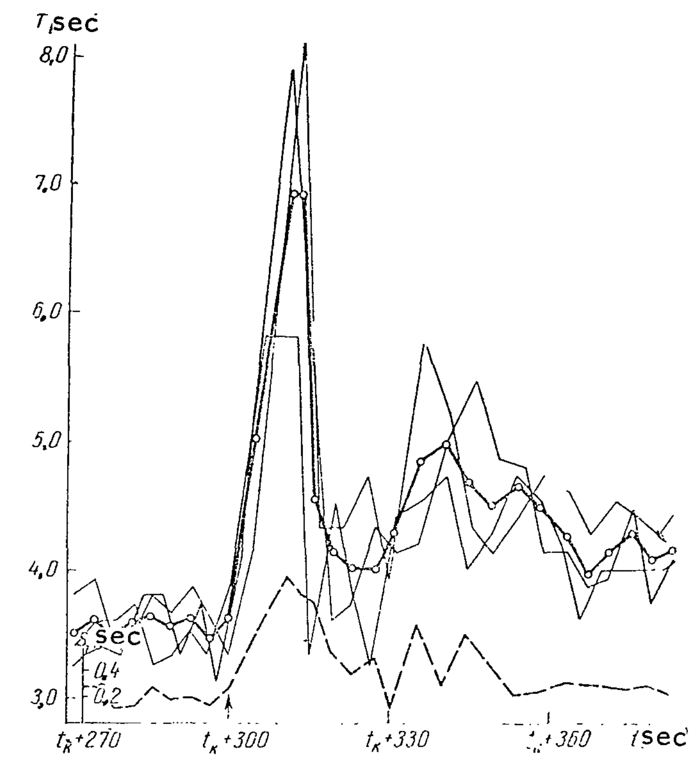


Figure 36

A knowledge of transient characteristics supplements our information on the stable modes of the respiratory system in the states of attention and operative rest in man and makes our concept of the sequence of intervals of the respiratory cycle as a discrete random process more complete. /83

§3.14. Correlation Properties of Sequence of Intervals of Inhalation Phase

Earlier (§3.2), we turned our attention to the significance of the phase of the oscillating process as a parameter of the physiological system. In comparison with the interval of the cycle, the duration of a phase is a more precise physiological characteristic, reflecting the activity of some single structure of the physiological system within the limits of one cycle in the formation of the oscillating process. Thus, during respiration, the rising phase is related to the work of the inspirator sector of the respiratory center, the neurons of which operate by rhythmic discharges, synchronous with

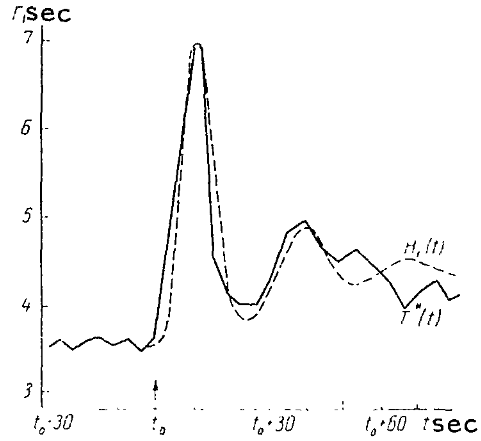


Figure 37

the inhalation phase; the falling phase, the exhalation phase, seize activity of the neurons of the expirator segment of the respiratory center. Relationships of reciprocity are frequently observed between structures in the organism; in particular, the inspirator and expirator segments of the respiratory center show relationships of reciprocal innervation, the essence of which is that excitation of one segment causes inhibition of the other and vice versa, within certain limits. Modern concepts of the integrity of the nervous system have expanded the concept of the respiratory center by the inclusion of cellular accumulations in the area of the pons Varolii, the diencephalon and (in the case of conditioned reflex reactions) the cerebral cortex, which has allowed a clarification and more detailed representation of the process of formation of the respiratory cycle. The rhythmic activity of the respiratory center is constantly influenced by various random factors; therefore, like the intervals of the respiratory cycle, the duration of the inhalation phase undergoes random changes, due to which successive intervals of the inhalation phase can be represented in the form of a sequence of random quantities: $X(t_k) = \{X(t_1), X(t_2), \ldots, X(t_n)\}$. Figures 27 and 28 show models of the recordings of an electropneumogram respectively for the state of operative rest and the state of attention, with the terms $x(t_k)$, $x(t_{k+1})$, ... marking the sequence of intervals of the inhalation phase. On time axis t, the symbols t_{k-1}, t_k, t_{k+1}, ... show the moments of completion of inhalation, where k is the current inhalation number.

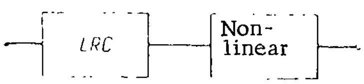


Figure 38

Statistical analysis of random sequence $X(t_k)$ was begun by analyzing its correlation properties. As for the intervals of the respiratory cycle, the statistical normalized correlation functions of the sequence of intervals of the inhalation phase were calculated using formula (3.4.3) for a rather large number of readings, n = 100-250. The statistical normalized correlation functions $R^*(\tau_s)$ for the random sequence of durations of the inhalation phase $X(t_k)$ rapidly attenuate, like the analogous characteristics for the respiratory cycle intervals $Y(t_k)$ and differ little from them. /84

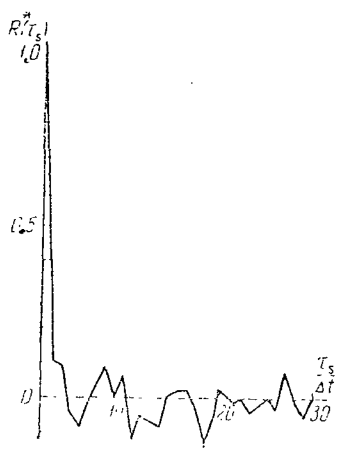


Figure 39

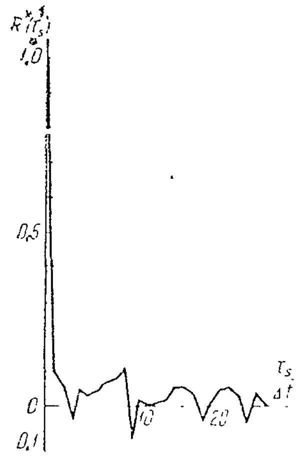


Figure 40

As an example for the identical test subject state, we show on Figure 39 the statistical normalized correlation functions $R^*(\tau_s)$ for a sequence of respiration cycle intervals $y(t_k)$, while Figure 40 shows the same for a sequence of inhalation phase intervals $x(t_k)$. However, in some cases the statistical normalized correlation functions $R^*(\tau_s)$ for $x(t_k)$ are more informative than $R^*(\tau_s)$ for $y(t_k)$.

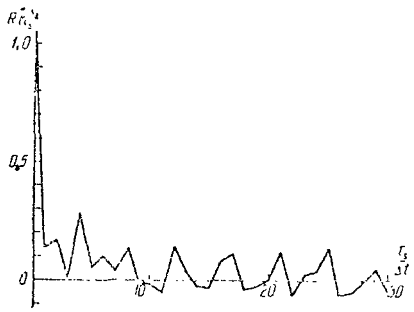


Figure 41

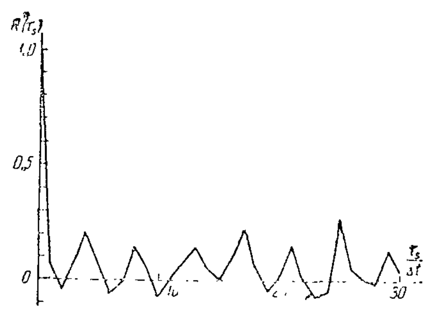


Figure 42

Figures 41 and 42 show $R^*(\tau_s)$ for $y(t_k)$ and $x(t_k)$ respectively. If the final sector ($\tau_s/\Delta t > 10$) of characteristic $R^*(\tau_s)$ still indicates the /85 presence of an oscillating component in sequence $y(t_k)$, the same thing cannot be said on the basis of $R^*(\tau_s)$ in the interval $0 < \tau_s/\Delta t < 10$.

Turning to Figure 42 we can speak with assurance of the presence of an oscillating component in $x(t_k)$. The similarity of the oscillating frequency $R^*(\tau_s)$ for $x(t_k)$ and the oscillating frequency of the tail portion of characteristic $R^*(\tau_s)$ for $y(t_k)$ indicates that there might be a common factor for $y(t_k)$ and $x(t_k)$ causing the oscillating nature of the statistical normalized correlation functions $R^*(\tau_s)$ (see Figure 41, 42). The calculated value of mean oscillating period $R^*(\tau_s)$, near 10 sec, corresponds to period T_c of the synchronizing pulses in the problem of control of an electron beam (Figure 2).

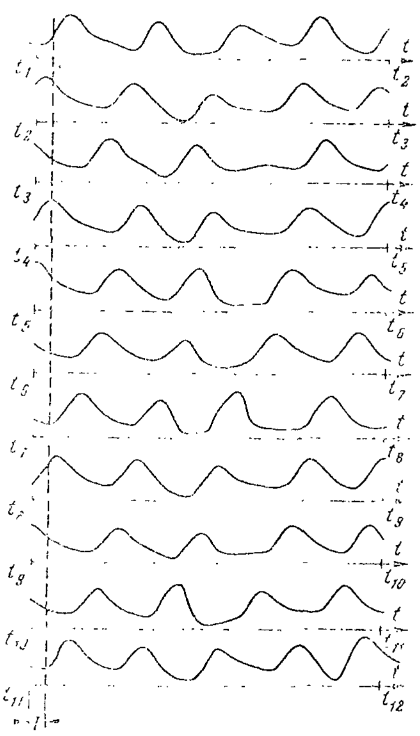


Figure 43

Analyzing sequential sectors of a pneumogram (t_1, t_2), (t_2, t_3), ..., (t_{11}, t_{12}) on Figure 43, each sector being equal to $T_c = 10$ sec and displaced by $T_c/2$ relative to the synchronizing pulses, we can note a sharp transition from short intervals of inhalation phase characteristic for the state of attention (see Figure 28) to longer intervals, characteristic for the state of operative rest (see Figure 27) in the middle portion of the sectors. The increase in the third intervals of the inhalation phase in sectors (t_1, t_2), (t_3, t_4), (t_5, t_6), (t_6, t_7),(t_9, t_{10}), (t_{10}, t_{11}) and the third intervals of the respiratory cycle in sectors (t_2, t_3), (t_3, t_4), (t_4, t_5). If we consider that the third intervals of the respiratory cycle and the inhalation phases correspond to the end of control by the electron beam in the given stage and the transition to the next control stage, separated by a short pause in the work, i.e. with short-term onset of the state of operative rest, characterized, in all probability, by a decrease in the level of attention, a relationship between the state of attention and the intervals of the inhalation phase and respiratory cycle seems quite likely.

§3.15. Estimation of Univariate Distribution of Inhalation Phase Intervals

For a weakly correlated random sequence of inhalation phase intervals, usually normally distributed, the univariate distribution of inhalation phase durations provides a rather complete characterization of $X(t_k)$. For many sequences of inhalation phase intervals, the statistical homogeneous distributions differ essentially from the normal distribution. For distributions other than the Gaussian distribution, the possibility exists of approximate analytic representation using the normal distribution, its derivatives and coefficients of asymmetry and excess. In order to determine deviations of the distribution from the normal or, more precisely, to test the hypothesis $\gamma_1 = 0$ and $\gamma_2 = 3$, let us use the tables of approximate values, which are the percentage points of the Pearson distribution in which the first four moments correspond to the corresponding moments of the distributions γ_1^* and γ_2^*. The values of γ_1 and γ_2 are coefficients of asymmetry and excess respectively, while γ_1^* and γ_2^* are the statistical coefficients of asymmetry and excess, respectively. Assuming the values $X(t_k)$ of the terms of random sequence $X(t_k)$ mutually independent and identically normally distributed with unknown mathematical expectation and dispersion, let us calculate the statistical characteristics as estimates for γ_1 and γ_2: /86

$$\gamma_1^* = \frac{1}{n(\sqrt{D_x})^3} \sum_{k=1}^{n} [x(t_k) - m_x]^3;$$
$$\gamma_2^* = \frac{1}{nD_x^2} \sum_{k=1}^{n} [x(t_k) - m_x]^4,$$

where

$$m_x = \frac{1}{n} \sum_{k=1}^{n} x(t_k); \quad D_x = \frac{1}{n} \sum_{k=1}^{n} [x(t_k) - m_x]^2.$$

Primarily, the values of γ_1^* and γ_2^* do not go beyond the 5% (1%) critical boundaries which correspond to the 10% (2%) summary level of significance for each criterion, and the hypotheses $\gamma_1 = 0$, $\gamma_2 = 3$ are not negated. In some experiments in which results were produced which contradict this hypothesis, individual values of the statistical coefficient of asymmetry reached 0.7-0.8, for the statistical coefficient of excess -- 1.5-2.0.

With a probabilistic investigation of random processes, the normal distribution occupies a particular position. This is explained by the fact that the solution of a number of problems is simplified to a considerable extent when it is possible to consider the distribution rules for the random quantities investigated normal. In connection with this, there is considerable interest in a representation of the distribution rule w(x) for normalized random quantity X (other than the Gaussian rule) in the form of a series based on the normal distribution, the coefficients of which are defined through the numerical coefficients of random quantity Ξ. One such expansion is the asymptotic expansion into an Edgeworth series: /87

$$w(x) = \varphi(x) - \frac{1}{3!}\frac{\mu_3}{\sigma^3}\varphi^{(3)}(x) + \frac{1}{4!}\left(\frac{\mu_4}{\sigma^4} - 3\right)\varphi^{(4)}(x) - \\ - \frac{1}{5!}\left(\frac{\mu_5}{\sigma^5} - 10\frac{\mu_3}{\sigma^3}\right)\varphi^{(5)}(x) + \frac{10}{6!}\left(\frac{\mu_3}{\sigma^3}\right)^2\varphi^{(6)}(x) - \\ - \frac{35}{7!}\cdot\frac{\mu_3}{\sigma^3}\left(\frac{\mu_4}{\sigma^4} - 3\right)\varphi^{(7)}(x) - \ldots \qquad (3.15.1)$$

where

$$\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}};$$

μ_k is the central moment of k-th order of random quantity Ξ; $\sigma = \sqrt{\mu_2}$; x is the value of random quantity $X = (\Xi - m)/\sigma$; m is the mathematical expectation of random quantity Ξ.

Series (3.15.1) converges to w(x) at each point of continuity of function w(x) if the integral

$$\int_{-\infty}^{\infty} e^{\frac{x^2}{4}} w(x)\,dx$$

converges and the function w(x) has limited variation over (-∞, ∞) (Kramer, 1948). Fulfillment of the conditions of convergence of series (3.15.1) to w(x) allows us to estimate the deviation of w(x) from the Gaussian rule. However, in practical applications in most cases we are interested not in the convergence of the expansion, but in rather good approximation to w(x) when two or three components are used. Limiting ourselves to the first three components in series (3.15.1), we produce an approximate description of the distribution rule w(x) in the form

$$w(x) = \varphi(x) - \frac{1}{3!} \cdot \frac{\mu_3}{\sigma^3} \varphi^{(3)}(x) + \frac{1}{4!} \left(\frac{\mu_4}{\sigma_4} - 3 \right) \varphi^{(4)}(x)$$

or, introducing the coefficient of asymmetry and the coefficient of excess using the formulas

$$\gamma_1 = \frac{\mu_3}{\sigma^3}; \quad \gamma_2 = \frac{\mu_4}{\sigma^4} - 3, \tag{3.15.2}$$

in the form

$$w(x) = \varphi(x) - \frac{\gamma_1}{6} \cdot \varphi^{(3)}(x) + \frac{\gamma_2}{24} \varphi^{(4)}(x), \tag{3.15.3}$$ /88

where

$$x = \frac{x(t_k) - m}{\sigma};$$

$x(t_k)$ are the values of random process $X(t_k)$ at moment in time t_k or (for an ergodic process) independent readings on the basis of one of its realizations $x(t_k)$; m and σ^2 are the mathematical expectation and dispersion of random sequence $X(t_k)$.

Using the example with the Reyleigh distribution (3.15.4), let us see how good the approximation of (3.15.3) to the normal rule is:

$$w_p(\zeta) = \frac{\zeta}{a^2} e^{-\frac{\zeta^2}{2a^2}}, \tag{3.15.4}$$

where a is the parameter of the Reyleigh distribution.

First, using expression (3.15.3), let us estimate the deviation Δ of the approximate Reyleigh distribution w(x) from the Gaussian distribution $\phi(x)$:

$$\Delta = w(x) - \varphi(x) = -\frac{\gamma_1}{6} \varphi^{(3)}(x) + \frac{\gamma_2}{24} \varphi^{(4)}(x). \tag{3.15.5}$$

Then, let us calculate the precise deviation Δ^T of (3.15.6) and, comparing it with Δ, determine the magnitude of the divergence between the precise $w_p(x)$ (3.15.10) and approximate (3.15.3) descriptions of the Reyleigh law:

$$\Delta^{T} = w_p(x) - \varphi(x), \tag{3.15.6}$$

where $w_p(x)$ is the normalized Reyleigh distribution.

As we know, the relationship of the distribution rule $w_Z(\zeta)$ of random quantity Z to the distribution $w_X(x)$ of the normalized random quantity $X = (Z - m_Z)/\sigma_Z$ is expressed by the formula

$$w_X(x) = \sigma_Z w_Z(m_Z + \sigma_Z x), \tag{3.15.7}$$

where m_Z and σ_Z^2 are the mathematical expectation and dispersion of random quantity Z. According to (3.15.7), for the Reyleigh distribution we have

$$w_p(x) = \sigma \frac{(m + \sigma x)}{a^2} e^{-\frac{1}{2a^2}(m+\sigma x)^2}, \tag{3.15.8}$$

where m, σ^2 are the mathematical expectation and dispersion of quantity Z with distribution (3.15.4) respectively, determined by the expressions /89

$$m = a\sqrt{\frac{\pi}{2}}; \quad \sigma^2 = a^2\left(2 - \frac{\pi}{2}\right). \tag{3.15.9}$$

Substituting (3.15.9) into (3.15.8), we produce

$$w_p(x) = \sqrt{\frac{4-\pi}{2}}\left(\sqrt{\frac{\pi}{4-\pi}} + x\right)\exp \times$$
$$\times\left\{-\frac{1}{2}\left[\sqrt{\frac{4-\pi}{2}}\left(\sqrt{\frac{\pi}{4-\pi}} + x\right)\right]^2\right\}. \tag{3.15.10}$$

Going over to an estimate of deviation Δ, we note that it can be generally performed at any[1] point (x). For example, it is quite convenient to perform comparison of the ordinates of two normalized distributions at points corresponding to their mathematical expectations. This can be easily seen, using expression (3.15.5), after first transforming it using the following relationship:

$$\varphi^{(n)}(x) = (-1)^n H_n(x)\varphi(x), \tag{3.15.11}$$

where $H_n(x)$ is the n-th power Chebyshev-Hermith polynomial. The Chebyshev-Hermith polynomials $H_n(x)$ for n = 0,1,2,3,4,5,6 have the following values:

$$\begin{aligned} H_0(x) &= 1; \\ H_1(x) &= x; \\ H_2(x) &= x^2 - 1; \\ H_3(x) &= x^3 - 3x; \\ H_4(x) &= x^4 - 6x^2 + 3; \\ H_5(x) &= x^5 - 10x^3 + 15x; \\ H_6(x) &= x^6 - 15x^4 + 45x^2 - 15. \end{aligned} \tag{3.15.12}$$

After transformation of (3.15.5), we have

$$\Delta(x) = \left[\frac{\gamma_1}{6}(x^3 - 3x) + \frac{\gamma_2}{24}(x^4 - 6x^2 + 3)\right]\varphi(x). \tag{3.15.13}$$

The point of interest to us is x = 0. Where x = 0

$$\Delta(0) = \frac{\gamma_2}{8}\varphi(0). \tag{3.15.14}$$

Incidentally, Charlier used the quantity $\Delta(0)/\phi(0) = \gamma_2/8$ as the excess characteristic for $w(x)$ relative to $\phi(x)$.

---

[1] We will return to the problem of estimating two functions in the next paragraph.

For the Reyleigh distribution (3.15.4), the coefficients of asymmetry γ_1 and excess γ_2 are equal to /90

$$\gamma_1 = 2\frac{(\pi-3)}{(4-\pi)}\sqrt{\frac{\pi}{4-\pi}} = 0{,}631;$$
$$\gamma_2 = \frac{(32-3\pi^2)}{(4-\pi)^2} = 0{,}245. \qquad (3.15.15)$$

Substituting the value γ_2 into (3.15.14), we get

$$\Delta(0) = \frac{0{,}245}{8}\cdot\frac{1}{\sqrt{2\pi}} = 0{,}012.$$

$\Delta(0)$, distinguished by its simplicity of calculation, however, cannot satisfy the frequent requirement for estimation of the maximum deviation Δ_m of the investigated distribution from the normal. Well known methods of analysis (3.15.13) indicate $x_0 = -0.64$, corresponding to Δ_m. We note that Kramer (1948) indicated the approximate value of $x_0 = -\gamma_1/2$ (Charlier asymmetry characteristic).

Thus, $\Delta_m = \Delta(-0.64) = 0.059$.

As we see, $\Delta(0)$ may be quite different from Δ_m, by several times.

Let us now calculate the precise value of the maximum deviation Δ_m^T, for which we substitute (3.15.10) and the expression $\phi(x)$ from (3.15.1) into (3.15.6). From the condition $d\Delta^T/dx = 0$, we find the stable point $x = -1.04$, at which Δ^T has its maximum:

$$\Delta^T(-1{,}04) = \Delta_m^T = 0{,}085.$$

In the example with the Reyleigh distribution, we see that the point of the extreme $x_0 = -0.64$ is closer to the maximum point Δ^T, than is the value of x determined from the condition $-\gamma_1/2 = -0.631/2$. However, the relative error δ

$$\delta = \frac{\Delta_m^T - \Delta}{\Delta_m^T}100 \qquad (3.15.16)$$

is also less: for $\Delta(-0.64)$ it is 31%, for $\Delta(-0.631/2)$ it is 46%. Thus, the error resulting from replacement of infinite series (3.15.1) with its first three terms increases by one and one-half times solely due to the fact that the maximum point Δ is determined approximately from the condition $x_0 = -\gamma_1/2$. In order to eliminate this sensible increase in the error in determining the maximum deviation of the distribution being investigated from the normal deviation (3.15.5), the necessity arises of seeking the precise value of the point of the extreme corresponding to Δ_m, which can be easily found using the graphs presented in the next paragraph. /91

In case of an impossibly high value of Δ_m, it can be reduced if we look upon w(x) as the distribution of a sum of independent normalized random quantities.

According to the central limit theorem from probability theory, the distribution $w_n(x)$ of the normalized sum of independent random quantities $X_1, X_2, \ldots, X_n$, with identical, continuous distribution with increasing number of components n, approaches the normal

$$\lim_{n \to \infty} w_n(x) = \varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

or

$$w_n(x) \sim \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

In other words, the deviation $\Delta = w_n(x) - \phi(x)$ approaches zero with increasing n.

For $w_n(x)$, the following expansion into a series is correct:

$$w_n(x) = \varphi(x) - \frac{1}{3!} \cdot \frac{\lambda_3}{n^{1/2}} \varphi^{(3)}(x) + \frac{1}{4!} \cdot \frac{\lambda_4}{n} \varphi^{(4)}(x) - \\ - \frac{1}{5!} \frac{\lambda_5}{n^{3/2}} \varphi^{(5)}(x) + \frac{10}{6!} \cdot \frac{\lambda_3^2}{n} \varphi^{(6)}(x) - \ldots, \tag{3.15.17}$$

where $\lambda_k = \kappa_k/\sigma^k$; σ^2 is the dispersion of the random quantities being added; κ_k is the cumulant of the k-th order of distribution of independent random quantities being added.

Kramer (1948) showed that under rather general conditions, series (3.15.17) is an asymptotic expansion of $w_n(x)$ with respect to powers of $n^{-1/2}$ with a residual term on the order of the first discarded term. Omitting terms of order $n^{-3/2}$ and higher from series (3.15.17) and considering (3.15.2) and the formulas for third and fourth order cumulants:

$$\varkappa_3 = \mu_3;\ \varkappa_4 = \mu_4 - 3\mu_2^2,$$

expression (3.15.17) can be brought to the form

$$w_n(x) = \varphi(x) - \frac{\gamma_1}{6\sqrt{n}}\varphi^{(3)}(x) + \frac{\gamma_2}{24n}\varphi^{(4)}(x) + \frac{\gamma_1^2}{72n}\varphi^{(6)}(x). \qquad (3.15.18)$$

As we can see, the expression for $w_n(x)$ depends both on the parameters of the distribution of the random quantities being added, and on the size of the sample (n).

Thus, difference (3.15.5) /92

$$\Delta = w_n(x) - \varphi(x) = -\frac{\gamma_1}{6\sqrt{n}}\varphi^{(3)}(x) + \frac{\gamma_2}{24n}\varphi^{(4)}(x) + \frac{\gamma_1^2}{72n}\varphi^{(6)}(x) \qquad (3.15.19)$$

for each value of argument x can serve as a measure of the deviation of the normalized distribution $w_n(x)$ of the sum of random quantities from the normalized normal rule $\phi(x)$.

§3.16. Measurement of Maximum Absolute Deviation of Distribution from Normal Distribution As a Function of Sample Size

There are various methods for estimating the two functions $f_1(x)$ and $f_2(x)$. In some cases, the degree of approximation is estimated over the entire interval [a, b] by calculating the mean square deviation:

$$\tilde{\Delta} = \sqrt{\frac{1}{b-a}\int_a^b [f_1(x) - f_2(x)]^2\,dx}\,.$$

In other cases, when $f_1(x)$, $f_2(x)$ are the discrete functions $\phi_1(x_i)$, $\phi_2(x_i)$ or when the functions are fixed by a graph or table, the degree of approximation is estimated by calculating the mean square deviation not over the entire interval [a, b], but only at individual points

$$\tilde{\tilde{\Delta}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[\varphi_1(x_i) - \varphi_2(x_i)]^2}.$$

If the mean square deviation is slight, then for the "overwhelming majority" of values of the argument the absolute value of $|\Delta|$ is also small for the selected interval of x.

A more rigid criterion for evaluation of the degree of approximation is

$$\Delta_m = \max_{a \leqslant x \leqslant b} |\Delta| < \Delta_0; \tag{3.16.1}$$

Criterion (3.16.1) requires that the absolute deviation $|\Delta| = |f_1(x) - f_2(x)|$ in the interval [a, b] not be greater than the permissible quantity Δ_0. Quantity Δ_m can be looked upon as a top estimate of Δ, and is generally simpler than the estimates $\tilde{\Delta}$, $\tilde{\tilde{\Delta}}$.

Using expression (3.15.19), we can find estimate Δ of approximation $w_n(x)$ to $\phi(x)$, omitting the last component and expressing the derivatives of the normal distribution through the Chebyshev-Hermith polynomials $H_k(x)$: /93

$$\Delta = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \left[\frac{\gamma_1}{6\sqrt{n}} H_3(x) + \frac{\gamma_2}{24n} H_4(x)\right]. \tag{3.16.2}$$

The curves described by expression (3.16.2) where $\gamma_1 < 0$ and $\gamma_2 < 0$, where $\gamma_1 < 0$ and $\gamma_2 > 0$, where $\gamma_1 > 0$ and $\gamma_2 < 0$ form figures together with the abscissa; in the general case these figures are symmetrical to the figure where $\gamma_1 > 0$ and $\gamma_2 > 0$. Therefore, in the following the coefficients of asymmetry and excess will always be assumed positive.

In order to determine Δ_m, let us analyze function $\psi(x)$, related to Δ by the relationship

$$\psi(x) = 6\frac{\sqrt{n}}{\gamma_1}\Delta. \tag{3.16.3}$$

Let us find the stable points of the function

$$\psi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} [(x^3 - 3x) + \mu(x^4 - 6x^2 + 3)], \tag{3.16.4}$$

where

$$\mu = \frac{\gamma_2}{4\gamma_1\sqrt{n}}.$$

From the condition $d\psi/dx = 0$, considering $e^{-x^2/2} \neq 0$ where $-\infty < x < \infty$, we have

$$\mu x^5 + x^4 - 10\mu x^3 - 6x^2 + 15\mu x + 3 = 0. \tag{3.16.5}$$

This equation in the general case cannot be solved in the radicals; however, we can see from the equation that the stable points $\psi(x)$ depend on parameter μ. In order to find the boundaries of the areas of existence of extreme values of $\psi(x)$ as the parameter μ is changed from zero to infinity, let us determine the roots of equation (3.16.5) for $\mu = 0$ and $\mu = \infty$. In the first case, we have four roots:

$$x_0 = \pm\sqrt{3 \pm \sqrt{6}},$$

in the second case ($\mu = \infty$) --- five roots:

$$x_\infty = \pm\sqrt{5 \pm \sqrt{10}} \quad \text{and} \quad x_\infty = 0.$$

The maximums of modulus $\psi(x)$ with increasing argument decrease due to the factor $e^{-x^2/2}$, therefore we will eliminate the roots high in absolute value from our analysis. The modulo maximum values of $\psi(x)$ should be expected at those places where first one component $\psi_1(x)$, then the other $\psi_2(x, \mu)$

predominate in the increment of $\psi(x)$. These areas are: $\sqrt{3-\sqrt{6}}<x<0$ and /94
$\sqrt{3-\sqrt{6}}<x<\sqrt{5-\sqrt{10}}$ (Figure 44).

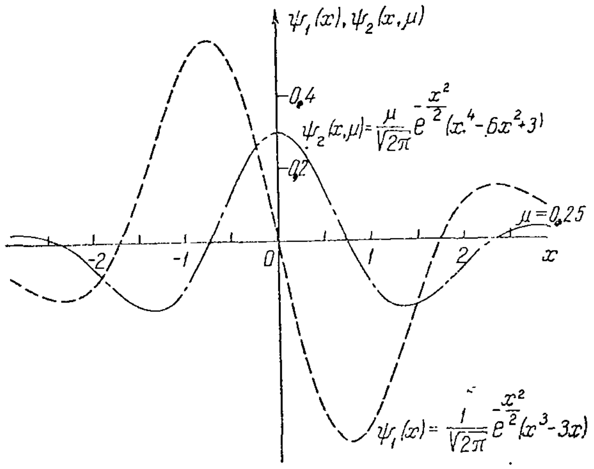


Figure 44

For an approximate solution of equation (3.16.5), let us transform it to a form convenient for solution by a graphic method:

$$(x^2-3)^2-6=\frac{4x(x^2-3)}{x+\frac{1}{\mu}}. \quad (3.16.6)$$

The solutions of equation (3.16.6) for various values of parameter μ are presented on Figures 45 and 46 for negative x in the area $\sqrt{3-\sqrt{6}}<x<0$ and for positive x within the limit $\sqrt{3-\sqrt{6}}<x<\sqrt{5-\sqrt{10}}$ respectively.

In modulus, the negative roots x_- of equation (3.15.6) are less than the positive roots x_+ and, consequently, as was already noted, we should expect

$$|\psi(x_-)|>|\psi(x_+)|.$$

Actually, for example where $\mu = 0.5$ we have: $x_- = -0.34$, $x_+ = 1.07$ and $\psi(-0.34) = 0.806$, $\psi(1.07) = -0.736$,

$$|\psi(-0.34)|>|\psi(1.07)|.$$

Thus, $-\sqrt{3-\sqrt{6}}\leqslant x\leqslant 0$ is the area of existence of the maxima of function $\psi(x)$ for various values of parameter μ.

From the solution (see Figure 45) of equation (3.16.6), we can see that /95
the roots $x_- = f(\mu)$ of the equation are dependent on parameter μ (Figure 47); otherwise, the position of the maxima of function $\psi(x)$ do not remain fixed, but are displaced in the direction of smaller values (x_-) with an increase in parameter μ, while the values of the maxima $\psi(f(\mu))$ of function $\psi(x)$ increase. The function $\psi(f(\mu))$ establishes the dependence between large deviations of the distribution from the normal and generalized parameter μ.

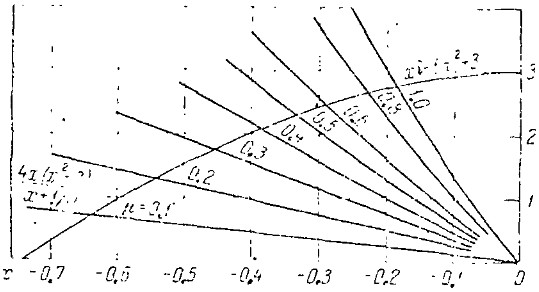


Figure 45

For a clearer idea of the results produced, let us analyze generalized parameter μ, representing it in the form of a function of sample size n with fixed relationships between excess and coefficient of asymmetry and at the same time go over from supplementary function $\psi(x)$ to the /96
function Δ_m/γ_1, dividing $\psi(f(\mu))$ by $6\sqrt{n}$. On Figure 48, the dotted line shows the dependence Δ_m/γ_1 where n = 1, i.e. the dependence of the maximum absolute deviation of the distribution from the normal as a function of the ratio of excess to asymmetry coefficient, for which the scale on the abscissa is divided by 4, since $\gamma_2/\gamma_1 = 4\mu$. The solid lines on the upper portion of Figure 48 show the functions Δ_m/γ_1 for various values of γ_2/γ_1, showing the change in Δ_m/γ_1 as a function of n with fixed ratios of γ_2 and γ_1.

For sequences $X(t_k)$ with a distribution rule having $\gamma_1 = 0.7$, $\gamma_2 = 1.4$, let us determine the maximum deviation Δ_m where n = 1. For this, we calculate the ratio $\gamma_2/\gamma_1 = 1.4/0.7 = 2$ and, using $\gamma_2/\gamma_1 = 2$, n = 1, and the lower graph of Figure 48, we find $\mu = 0.5$, which corresponds to $\Delta_m/\gamma_1 = 0.103$ (upper graph, Figure 48). Finally, we determine $\Delta_m = 1.103 \cdot \gamma_1 = 1.103 \cdot 0.7 = 0.072$. By a similar method we find the maximum deviation of Δ_m in the case of samples with volume n = 8. Δ_m we find to be $0.035 \cdot 0.7 = 0.025$. Thus, the transition from a sample n = 1 to a sample n = 8 leads to a decrease in Δ_m from 0.072 to 0.025.

In the example with the Reyleigh distribution (§3.15) with the same sample (n = 8) we have $\Delta_m = 0.020$, i.e. the increase in the sample to n = 8 leads to a decrease in the maximum deviation of Δ_m from 0.059 to 0.020.

It should be noted that when the generalized parameter μ vanishes ($\gamma_2 = 0$), the dependence of Δ_m/γ_1 on the sample size n cannot be traced from the graph on Figure 48. After determining the value $x_- = \sqrt{3 - \sqrt{6}}$ and substituting it together with $\gamma_2 = 0$ into expression (3.16.2), we produce

$$\Delta_{-}(x_{-}) = \Delta_m = \frac{1}{\sqrt{2\pi}} e^{-\frac{x_{-}^2}{2}} H_3(x_{-}) \frac{\gamma_1}{6} \cdot \frac{1}{\sqrt{n}} = \left(\frac{\Delta_m}{\gamma_1}\right)_{\mu=0} \gamma_1 \frac{1}{\sqrt{n}} = 0{,}0917 \frac{\gamma_1}{\sqrt{n}}, \qquad (3.16.7)$$ /97

where $(\Delta_m/\gamma_1)_{\mu=0}$ is a quantity determined from the graph (see Figure 48).

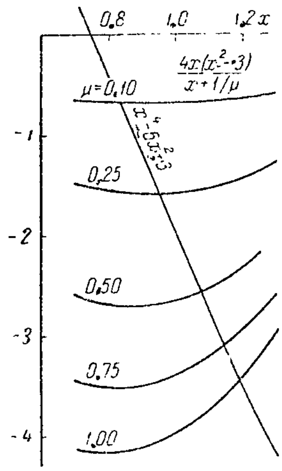


Figure 46

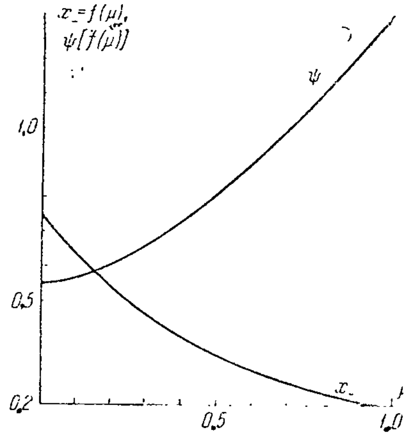


Figure 47

We can see from the expression produced that the value of Δ_m for $\mu = 0 (\gamma_2 = 0)$ is inversely proportional to the square root of the sample size n.

In the case where $\gamma_2 \neq 0$, while the distribution is symmetrical ($\gamma_1 = 0$), the value of generalized parameter μ is infinity (3.16.4). This case, frequently observed in practice, is not shown on Figure 48. Without going into a detailed determination of the abscissa of the maximum $\Delta|_{\gamma_1=0}$ (3.16.2), let us look at function $\psi_2(x, \mu)$ related to $\Delta|_{\gamma_1=0}$ as follows:

$$\Delta|_{\gamma_1=0} = \psi_2(x, \infty) \frac{\gamma_1}{6\sqrt{n}}.$$

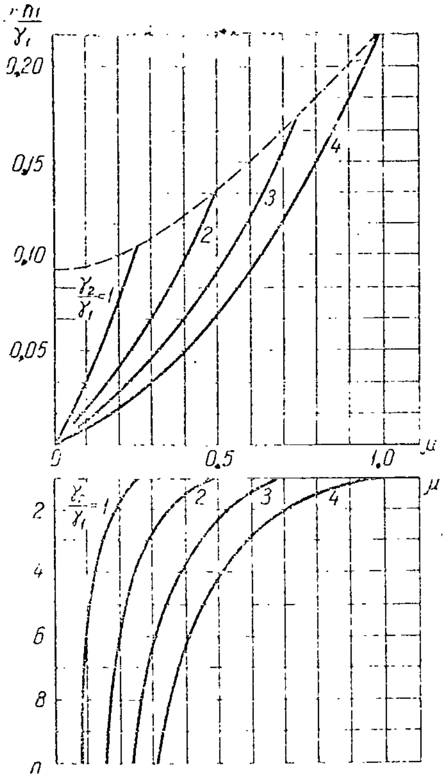


Figure 48

Obviously, the desired point on the abscissa x_- is equal to zero ($x_- = 0$). Substituting the data into expression (3.16.2), we produce

$$\Delta_m = \Delta(x_-)|_{\gamma_1=0} = \frac{\gamma_2}{8n\sqrt{2\pi}} \approx 0{,}05\frac{\gamma_2}{n}. \quad (3.16.8)$$

A comparison of (3.16.7) and (3.16.8) shows that the maximum absolute deviation Δ_m of the symmetrically normalized distribution from the normal decreases more rapidly with increasing sample size n than the similar quantity for the asymmetrical normalized distribution rule. We note in conclusion that for more general cases ($\gamma_1 \neq 0$, $\gamma_2 \neq 0$, $n \geqslant 1$) the dependence of the ratio of maximum absolute deviation of the distribution from the normal to the asymmetry coefficient as a function of the sample size, shown on Figure 48, allows us to determine the maximum absolute deviation Δ_m quite easily with known parameters of the distribution (γ_1, γ_2) and fixed sample size, or, using the parameters of the distribution (γ_1, γ_2) and the sample size (n), to determine approximately the value of the random quantity for which deviation Δ will be its maximum Δ_m. In this latter case, it is necessary to consider the signs of the coefficient of asymmetry and excess, holding the following rule: with identical signs, the stationary points are located in the area of negative values of x, with different signs -- in the area of positive values of argument x.

PART II

THE USAGE OF PHYSIOLOGICAL SIGNALS TO ESTIMATE EMOTIONAL STRESS OF AN OPERATOR

Chapter 4

THE ELECTROENCEPHALOGRAM

ABSTRACT. A general analysis of the electroencephalogram is presented. The various rhythms encounted in the electtoencephalogram are described and characterized statisically. Variations in the typical electroencephalogram with variations in the state of attention are described. Typical EEG integrator output signal are analyzed mathematically.

Introduction

The rhythmic oscillations[1] of the bioelectrical activity are one of the characteristic manifestations of the active state of the central nervous formations. It is assumed that the "spontaneous" rhythmic discharges of nerve cells maintain their tone, indicate readiness for reception of incoming signals and excitability of the synaptic apparatus (points of contact of branches of a nerve cell to branches or bodies of other cellular elements). The influx of signals from the sense organs and internal receptors disrupt and modify the background rhythmic activity. /98

In the organization of the oscillations in biopotentials recorded from the surface of the brain, considerable significance is given to the circulation of nervous excitation through closed loops between various layers of the cerebral cortex, between points on the surface of the cortex and also between the cortex and the lower sections of the brain. Thus, negativation (excitation) of the upper layers of the cortex produces the first half-wave of the surface oscillation in the biopotential, while excitation of the lower layers produces the second half-wave. The shorter and narrower the neuron ring, through which the excitation is transmitted, the more rapid the rhythm;

[1] Rather complex, quasi-periodic processes are involved. We retain here the terminology used in physiology.

the longer the ring, the slower the rhythm (Rysunov, 1954). The discovery of phenomena of reverse inhibition, functioning on the principle of negative feedback, and of specialized inhibiting cells first in the spinal column, then in the higher located sections of the central nervous system (cerebellum, hippocampus) led to the formation of the concept of the decisive role of inhibiting mechanisms in the organization of the cerebral rhythm (Andersen, Rudjord, 1964; Sokolov, 1965, etc.). The study of the regular changes in the /99
α rhythm with increasing ascending excitation allowed P. V. Simonov (1965b) to state the assumption that the α rhythm is an electrographic expression of the activity of mechanisms which, on the one hand, maintain rather high reactivity of the nervous structures, and on the other hand, prevent over-excitation, which would threaten to go over to spasmodic activity ("pulsating inhibition barrier").

In the opinion of Andersen and Sears (1964), the nuclei of the visual tubercle contain no 10-cps rhythm establishing unit, and the frequency of the discharges of cortical neurons is determined by the negative reverse influence of the intermediate inhibiting neurons. It is this mechanism which is the basis of the spindle-shaped activity recorded on the electroencephalogram. A comparison of the discharges of individual cells of the cortex to the EEG has shown their coincidence with the surface-positive half-waves, the inhibiting periods -- with the surface-negative half-waves (Calvet, Calvet, 1963). According to the data of B. I. Kotlyar and V. V. Shul'govskiy (1967), the maximums in the discharges of neuron in the hippocampus correspond to the maximums of negative values of the θ rhythm (oscillating frequency 4-7 Hz). A. I. Roytbak (1963) considers that the α wave is formed as a result of addition of the potentials of branching segments of the cortical dendrite-cells. The formation of δ waves (1.5-3 Hz) is related to the mechanism of slow oscillations of the negative potential, the generation of which, possibly, includes participation of the neuro-glial cells.

Thus, there is reason to look upon the rhythmic oscillations of bio-potentials as a result of interaction of excited and inhibiting nerve elements, which allows us in the first approximation to describe this interaction in the language of models of the Italian mathematician V. Volterra (1931, see Goldacre, 1960). Volterra, in particular, looked upon the hypothetical situation: a lake contains sharks (the consuming form), small fish (the consumed form) and plant food for the small fish. The sharks eat the small fish, the number of small fish decreases, and with time the sharks begin to die of starvation. Then the small fish multiply, the sharks begin to eat them, and the number of sharks increases. The number of sharks and other fish increases and decreases periodically, the period of the oscillations being identical, the only difference being a shift in phase. In order to describe external influences on the system, Volterra suggested several rules. However, in order to use them, we must determine which elements in the system correspond to the sharks, which -- to the small fish and which -- to the plant food.

Let us present several of Volterra's rules.

1. The oscillating period depends on the coefficient of decrease (increase) in elements of each species and the initial number of elements in each.

2. Destruction of consuming elements accelerates the fluctuation; destruction of consumed elements retards the fluctuation. /100

3. An increase in the degree of protection of consumed elements leads to an increase in the number of elements of both species.

4. With simultaneous and equivalent destruction of both species, the ratio of amplitudes of the fluctuation has a tendency to increase.

5. If we attempt to destroy both species uniformly and in proportion to their quantity, the mean number of elements of the consumed form increases, of the consuming form -- decreases.

In the situation which we have analyzed, the influx of information from the surrounding medium and the state of the internal organs can be considered the "food," facilitating the propagation of excitation processes (small fish) in the nerve elements, the development of which is hindered by inhibition processes (hunter fish). From this point of view, the activating and inhibiting influences on the cortex from lower segments of the brain can be equated to increases in the number of excitation or inhibition neurons. Of course, the Volterra rules do not reflect all the complexity of the interaction of the excited and inhibiting nerve elements arising in various segments of the brain as a continuously changing volume of information arrives. However, in many cases they can be useful for conclusions concerning the interaction of mechanisms of inhibition and excitation in various situations, judging from the predominant rhythm of the EEG or, on the other hand, allow a preliminary characterization of the EEG, if we have information on the relationships between processes of excitation and inhibition.

We note that the Volterra rules do not contradict the principal conclusions on the predominance of various EEG rhythms, resulting from analysis of the model presented below (see point "d", p. 35) of the interaction of excitation and inhibition mechanisms in various human emotional states, based on physiological considerations and practical observations.

After briefly presenting contemporary concepts concerning the mechanisms of formation of the EEG, let us go over to a review and analysis of available data on the problem of changes in the summary electroencephalogram under the influence of emotional excitation.

§4.1. Electroencephalographic Characteristics of Emotional Stress

a) δ Waves and the Inhibiting Influence of the Lower Segment of the Brain Stem

In the literature, we could not find any information on manifestations of δ waves (1.5-3 Hz) during emotional stress in healthy persons. Nevertheless, spectral analysis of EEG in the experiments of P. V. Simonov, M. N. Valuyeva and P. M. Yershov (1964) using the G. Walter narrow band analyzer produced by the English "Eddisvan" firm showed a clear increase in summary wave voltage at /101 a mean frequency of 2 and 3 Hz at the moment of expectation of a rather strong pain stimulus. The reinforcement of δ activity is frequently combined with a tendency to increased pulse frequency, which can be looked upon as a characteristic of activation of the parasympathetic segment of the vegetative nervous system. This type of effect is characteristic for the inhibitory system, localized in the lower portion of the brain stem.

The lower (bulbar) inhibiting system has a manifest synchronizing influence on the electrical activity of the cerebral cortex. Most authors believe that this effect is achieved by inhibition of the activating segment of the reticular formation (Morutsyn, 1962). In the opinion of Bloch and and Bonvalet, there is a negative mesencephalo-bulbo-mesencephalic feedback (Bloch, Bonvalet, 1961; Bonvalet, Bloch, 1961). Many experiments have demonstrated the parallelism between vegetative reactions arising during stimulus of the medulla oblongata, and the synchronizing (deactivating) effects of the bulbar segment on the electrical activity of the cortex. Direct stimulation of the caudal portion of the stem produces synchronization of the rhythm in the cortex and a decrease in pulse frequency (Mayorchik, Koreysha, Gabibov, 1962). Stimulation of the baroreceptors of the carotid sinus, decreasing the pulse frequency and blood pressure, results in synchronous, slow waves of high amplitude in the cortex (Hibel, Bonvalet, Dell, 1954; Okudzhava, Meladze, 1964).

These data agree well with the facts concerning the role of the lower inhibitory system in the regulation of emotional reactions. Stimulation of the baroreceptors of the carotid sinus, and also direct stimulation of the bulbo-pontine structures inhibits the anger reaction in decorticized cats (Tsanketti, 1965). The characteristic vegetative manifestations of fear indicate the activation of the parasympathetic segment of the nervous system (decrease in pulse frequency, defecation and nausea), indicate the participation of the synchronizing structures of the medulla oblongata in the passive-defense states.

b) The θ Rhythm and Its Functional Significance

The oscillations at 4-7 Hz are weakly expressed in the electroencephalogram of a healthy, mature man. However, it was noted that this rhythm becomes clear with mental stress (Sorel, 1965) or negative emotions, particularly in children (Walter, W. G., 1953). A similar rhythm is also observed in higher mammals whenever their nervous system undergoes a sufficiently strong load, be it the solution of a difficult problem or injection of certain biologically active substances into the blood stream. In the laboratory of P. K. Anokhin, /102 this rhythm has come to be called the "stress rhythm." The fact that the θ rhythm in the surface electroencephalogram is combined with reinforced impulsation of neurons in the reticular formation stimulated P. K. Anokhin to deny the inhibiting nature of the θ rhythm and to look upon it as a reliable EEG sign of negative emotional excitation (Anokhin, 1963).

Further investigations have shown the complex and contradictory phenomenology of the θ rhythm. First of all, it has been found that the θ rhythm in some areas of the cerebral cortex may exist with manifest desynchronization of the electrical activity of other areas. The reinforcement of the effect of a stimulus which has caused the appearance of the θ rhythm generally leads to its replacement by desynchronization (rapid, low-amplitude oscillations). The θ rhythm increases not only during negative, but also during clearly positive emotions (Valuyeva, 1967). It can be stated in general forms that the θ rhythm is particularly characteristic for the situation of active search for paths of satisfaction of demands which have arisen. Strong and purposeful excitation most frequently results in generalized desynchronization of the EEG.

Among the brain formations located beneath the neopallium, the θ rhythm is particularly characteristic for the hippocampus. It is clearly expressed in those cases when the experimental situation contains elements of pragmatic uncertainty, in the early stages of development of a new skill, and disappears as the problem is solved (Adey, 1960). A high level of development of a skill is accompanied by a decrease in fluctuation of phase variations of EEG waves in the hippocampus (Adey, Walter, D. O., 1963). Grastyan et al. (1965) emphasized that the θ rhythm is recorded in the hippocampus only in the behavioral reaction of approach, search for objects useful for the animal. The reaction of avoidance is accompanied by desynchronization of the hippocampal activity. Data are available indicating that the θ rhythm in the hippocampus is determined by the participation of the medial area of the hypothalamus. Disruption in this area leads to replacement of the θ rhythm by desynchronization (Corazza, Parmeggiani, 1963). Direct stimulation of the hypothalamus causes desynchronization in the cortex and the θ rhythm in the hippocampus.

Apparently, the hippocampus is involved in the formation of the θ rhythm in the higher brain sections as well. Direct stimulus of the hippocampus by signals at a frequency of 50-70 Hz retards the rhythm of the EEG of the cerebral cortex and causes the appearance of high amplitude spindles. The activation reaction threshold in response to stimulus of the reticular formation increases in this case. However, stimulus of the hippocampus by signals of higher frequency, on the order of 300 Hz, leads to desynchronization of the EEG (Zislina, Novikova, Tkachenko, 1963). These EEG effects are interesting to compare with the inhibiting influence of the hippocampus on /103 the reticular formation (Lishak, Grastyan, 1962), the conditioned defense reflexes (Rimble, Kirkly, Stein, 1966) and certain vegetative reactions (Ayrikyan, Gaske, 1967).

Another structure which takes part in the formation of the θ rhythm is the area of the septum. According to the data of I. I. Poletayeva (1967), disruption of the septum in rabbits or its temporary disconnection by novocaine leads to disappearance of the θ rhythm in all leads. What are the functional specifics which characterize the area of the septum? We would like to emphasize at least two: 1) in the septum area in man and in animals are located the centers of positive emotions (Oldz, 1963; Lilli, 1960; Gis, 1963, etc.); 2) the septum has an inhibiting influence on the centers of negative emotions, and its direct stimulus is accompanied by primarily parasympathetic vegetative effects (Allikmets, 1965; Allikmets and Lapin, 1966).

Summing up the entire set of available facts, we can make the following conclusion. The θ rhythm is actually closely connected to the formation of emotional reactions, although only in those cases when the structure of the eomtional state contains rather clear characteristics of the "inhibiting component." This type of situation is generally observed during active search for means of satisfying some demand, where the desire encounters difficulties hindering the solution of the behavioral problem. The θ rhythm can hardly be attributed only to negative emotions, being no less characteristic for positive states. Furthermore, strong negative reactions are more usually accompanied by desynchronization of the EEG than by the well ordered θ rhythm.

c) Changes in α Rhythm

Oscillations of the biopotentials at 8-13 Hz occupy a leading position in the human EEG. According to the data of Saunders (1963), the univariate distribution density of α rhythm values can be characterized by the Gaussian rule. The mean frequency of this rhythm -- 10 Hz -- corresponds to an interval which is quite essential for mental activity. The observations of psychologists and precise experiments on dogs (Papovich, 1957) have shown that the conditioned reflex is formed only if the interval between combinations of stimulants exceeds 100 msec. This interval corresponds to the minimum time necessary for detection and differentiation of a signal (Davis, 1957). A reduction in the duration of signal action leads to the activation of special mechanisms for elongation of the signal, allowing the brain to succeed in performing the operation of analysis (Gershuni, 1963).

Generalizing material accumulated by physiologists, N. Wiener (1963) came to the conclusion that the higher segments of the central nervous system can /104 perceive impulses no more frequently than each 100 msec. In the opinion of Adrian (1954), the physiological mechanism manifested in the oscillations of the α rhythm breaks up and limits the impulsation (signals) arriving from the periphery. In other words, this rhythm reflects changes in excitability of the cortical structures in relation to incoming signals (Walter, W. G., 1954).

Investigators have noted many times the relationship between the α rhythm and the activity analyzer of the brain. Perhaps this factor causes the particularly clear expression of the α rhythm in the posterior areas of the cerebral cortex, where the higher centers of vision -- the leading analyzer system in man -- are localized[1]. Apparently, it is not by chance that the background rhythm is most effectively suppressed (desynchronized) by a burst of light. Disconnection of the visual analyzer is accompanied by attenuation or even disappearance of the ordered α rhythm (Adrian, Matthews, 1934; Novikova, 1960, etc.). The variety of α activity recorded in the zone of the central motor analyzer in the form of so-called arc-shaped or rolandic rhythm is unique, and is pressed by proprioceptive impulsation from the muscles and joints. The appearance of this rhythm in response to emotionally colored stimulus (for example, showing of a motion picture film) probably results from increased muscular stress -- spinal tonus (Sorel, 1965).

Many authors relate the formation of the α rhythm to the inhibiting influence on the cerebral cortex of the nonspecific nuclei of the thalamus (Jasper, 1954; Roytbak, 1958; Serkov, 1963; Narikashvili, Moniava, Arut'yunov, 1965, etc.). However, data are available which contradict this hypothesis. Thus, complete disruption of the right visual tubercle by a tumor did not lead to disruption of the α rhythm or asymmetry of the α rhythm in comparison to the electrical activity of the other hemisphere (Tumskoy, Mayorchik, 1966).

When emotional stress appears, a suppression (activation, desynchronization) of the α rhythm is observed, together with reinforcement of rapid β oscillations at 14-30 Hz (Lindsli, 1960; Gorbov, Myasnikov, Yazdovskiy, 1953; Glass, 1964 and many others). According to the data of Lange, Storm Van Levena and Verre (1962), constant predominance of β activity in the EEG of a subject indicates increased emotionality, stress, disquiet, and lack of confidence. A low α index, increased frequency and low amplitude of the α rhythm indicate chronic predominance of excitation as an individual specific of the nervous system of the person (Teplov and Nebylitsyn, 1963). Puister (1962) considers that persons with depressed, low-amplitude, weakly expressed /105 α rhythm are unsuitable for flying work. A "flat" EEG, decrease in the α index and manifest β activity are characteristic for patients suffering from neurotic diseases, with manifestations of emotional instability and affective explosiveness (Latash, 1963; Bobkova, Mesishchev, 1967).

After the classical works of Bremermegun, Moruzzi and their school, it became obvious that depression of the α rhythm results from the activating influence of the reticular formation in the brain stem. These data with a basis of the "activation theory of emotion" suggested by Lindsli (1960), on the basis of analysis of the reticular formation of the brain stem as the principal substrate of emotional reaction. However, "activation theory" has a number of vulnerable links. First of all, a large number of facts indicates

[1] The close relationship of the α rhythm and functions of the eyes -- their motor control, position, information of visual axes has been indicated by recent works of Mulholand, 1966; Dewan, 1967, etc.

secondary involvement of the reticular formation in the occurrence of emotional reactions which are initiated by specialized centers in the hypothalamus (aggression, fear, satisfaction, etc.), as well as descending commands from the structures of the neopallium, where finer evaluation of the situation at hand occurs. Direct electrical stimulus of the hypothalamus causes clear desynchronization of the EEG (Gellhorn, 1960; Baklavadzhan, 1966, etc.). Furthermore, disruption of the rear segment of the hypothalamus excludes the behavioral characteristics of excitation of the animal, although activation of the EEG remains in this case (Feldman, Weller, 1962). In all probability, the reticular formation more nearly supports regulation of the level of excitation, the level of working ability of the higher segments of the brain than determines the emotional nature of the behavioral reactions (Cardo, 1961).

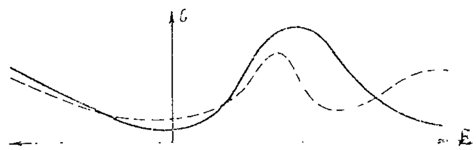
On the other hand, during mental stress (mental calculations), the perception of emotionally colored words, expectation and fear, many authors have recorded not a depression, but a reinforcement of the α rhythm, an increase in its amplitude, increase in the α index (Darrow, 1947; Lorens, Darrow, 1962; Kreitman, Shaw, 1965; Emrich, Heinemenn, 1966). According to the data of P. V. Simonov (1965b), moderate emotional excitation, the degree of which was judged by the increase in pulse frequency, is accompanied by an increase in the summary voltage of the α rhythm, while a further reinforcement of emotional excitation leads to a depression of the rhythm.

How can these facts be understood if we agree with the statement that an increase in amplitude of the α rhythm reflects reinforcement of inhibitory processes (Rysunov and Smirnov, 1957; Felinskaya and Ivanitskiy, 1964, etc.)? Here we must first of all recall the property of weak stimulus to increase the inhibitory processes in the central nervous system. Thus, a weak electrical stimulus of the posterior hypothalamus or reticular formation of the brain stem depresses bioelectrical activity of the cortical structures caused by application of strychnine, while a strong stimulus reinforces it (Gellhorn, /106
1960). Stimulus of the reticular formation of the middle brain in the rabbit by signals at 6 Hz inhibited the activity of 30% of the neurons and reinforced the activity of 22%. At 150 Hz, these figures were 24 and 56% respectively. In other words, an increase in the strength (frequency) of the stimulation leads to an increase in the number of activated neurons and a decrease in the number of inhibited neurons (Krupp, Monnier, 1964).

Using the example of the changes in α rhythm, we will see that emotional stress is a result of dynamic existence and complex interaction of exciting and inhibiting mechanisms. The degree of involvement of these counterdirected mechanisms, their specific gravity, "their balance, depending on the qualitative and quantitative characteristics of a given emotional stress, determine the electroencephalographic expression of the emotion. Rephrasing D. Lindsli, we can state that the emotional reaction is not "activational," but "activation-deactivational" in nature.

d) Hypothetical Diagram of Mechanism of Changes in EEG During Emotional Stress

Thus, we can see that the electroencephalogram recorded at the surface is a result of the complex interaction of excitation and inhibition processes on the cerebral cortex. We assume that in order to clarify the EEG manifestations of various emotional states, it is important to acknowledge not the variations in influences, not the predominance of inhibition over activation or activation over inhibition, but the dynamic existence of inhibitory and activating mechanisms. The transition from strong emotional excitation to rest, drowsiness and sleep cannot be represented as a simple, unidirectional replacement of rapid, low-amplitude oscillations by slow, high-amplitude waves. The activity of the synchronizing mechanisms can increase in parallel with reinforcement of excitation.



Action	Contact Action	Satisfaction	Comfort	Dissatisfaction	Disgust
	Possession	Joy	Quiet	Alarm	Grief
	Defense	Courage	Relaxation	Active Fear, panic	Passive Fear Immobility
	Combat	Enthusiasm	Coolness	Fury	Frenzy, Prostration

Figure 49

On Figure 49 we have attempted to show schematically the interaction of the exciting and inhibiting factors in various emotional states of man. Although this diagram must unavoidably simplify the actual complexity of the phenomena and is hypothetical in nature, it clearly illustrates a very important statement, concerning the absence of direct reciprocity (antagonism) between exciting and inhibiting mechanisms.

The abscissa corresponds to the degree of emotional stress E, which in turn is determined, on the one hand, by the demand and on the other hand by /107 the deficit (negative emotions) or excess (positive emotions) of pragmatic information, characterizing the contrast between prognosis (probability of achievement of a goal, earlier predicted by the subject) and the reality (probability of satisfaction of the demand determined at the given moment).

The investigations of physiologists show that the estimate of the probability of achieving a goal is performed by the brain on the basis of hereditary and individual acquired experience.

The ordinate G corresponds to the degree of stress of the activating, exciting, desynchronizing (solid line) and inhibiting, deactivating, synchronizing mechanisms (dotted line). Let us analyze the electroencephalographic characteristic of various emotional states.

1. For the state of rest, predominance of synchronizing influences is characteristic, corresponding to clearly expressed α rhythm. This state frequently and most easily leads to sleepiness.

2. During positive emotions, excitation is clearly increased, although at the same time an increase in inhibitory influences is observed. This situation frequently manifests itself in periods of exultation (increasing amplitude) of the α waves, as well as a reinforcement of θ activity. However, this does not exclude, particularly during strong positive emotions, phenomena of depression of the α rhythm and reinforcement of rapid β oscillations. It is quite probable that the very combination of activation of exciting and inhibiting mechanisms and the adequacy of the "inhibitory defense" of the brain structures are the basis for the practical harmlessness for the organism of even strong positive emotions. In any case, clinical medicine knows of no neurotic disorders or psychosomatic diseases which arise as a result of too much joy. A feeling positive in its coloration may be dangerous only for persons with deep organic pathology of the cardiovascular /108
system, when any additional stress is undesirable.

3. For negative emotions, appearing in animals with rapid motor activity (in man the external manifestation of emotions is frequently suppressed) shows most typically strong excitation with manifest depression of the α rhythm and an increase in rapid oscillations. It must be emphasized that in the first stages of development of this type of emotion, the inhibitory influences continue to increase, which is manifested in cases of exaltation of the α rhythm, and reinforcement of θ activity. However, in contrast to positive emotions, the stabilizing mechanisms soon predominate over the increasing excitation.

4. Unique relationships are observed in the stage in which negative emotions take on a manifest passive nature (deep grief, strong fear resulting in immobility, etc.). On the background of ever increasing tonus we observe here clear predominance of inhibitory influences with the manifestation of slow waves in the electroencephalogram. There is some foundation for relating this picture to the inhibitory influences of the lower (bulbar) synchronizing system. This idea is supported by the capability of this system to inhibit the reaction of aggression (Tsanketti, 1965) and clear symptoms of activation of the parasympathetic segment of the vegetative nervous system (decrease in pulse, nausea, vomiting, involuntary defecation and urination).

These are the general characteristics of the activation-deactivation interaction with increasing emotional stress. We repeat, our diagram is rather approximate and hypothetical, but, to some extent, allows us to orient ourselves in the contradictory information provided by the physiological literature. We hope that further investigations will allow the phenomenology of electroencephalographic changes during various emotional states as well as the essence of the mechanisms which determine them to be clarified.

e) Results of Experiments

In conclusion, let us analyze certain practical results of spectral analysis of the α and θ rhythms of the EEG, allowing us to evaluate the nature of the behavior of the spectral density of the processes investigated during changes in the degree of emotional stress of the test subjects in the experiment (determination of a significant stimulus).

The sampling correlation functions $B_T(\tau)$ were determined using a discrete analog of the following formula (Pugachov, 1957):

$$B_T(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} [y(t) - y_T(t)]\,[y(t-\tau) - y_T(t)]\,dt,$$

where y(t) is the signal at the output of the corresponding filter of the spectral analyzer, the band pass of which is Δf = (8-13) Hz for the α rhythm or Δf = (4-8) Hz for the θ rhythm; T is the duration of realization of the process; τ is the shift in the correlation function $0 \leqslant \tau \leqslant \tau_{max}$; $y_T(t)$ is the sampling mean value of y(t). /109

Fragments of the EEG with the separated α rhythm signals [α(t)] and θ rhythm signals [θ(t)] for the state of rest and high emotional stress of the operator (expectation of pain stimulus) are shown on Figures 50 and 51 respectively. The index I marks the readings from the integrator.

The correlation function step is determined from the condition that over an interval as long as one period of the highest frequency harmonic observed in the recording of the random signal being investigated, there are not over 6-10 readings.

In order to find the smoothed spectral estimates $F_T(f)$, we used the expression

$$F_T(f) = \frac{1}{\pi B_T(0)} \int_0^{\tau_{max}} p(\tau) B_T(\tau) \cos(2\pi f\tau) d\tau,$$

in which the Hamming function was taken as the weight function p(t), allowing us to produce rather satisfactory results from spectral analysis of stable random processes over rather short observation intervals with finite shift τ (Kagnnan, 1964; Meshalkin, Yefreymova, 1965). Here, the relationship between estimates of spectral density $F^*_T(f)$ for the case $p(t) \equiv 1$ and $F_T(f)$ determined using the Hamming weight function can be described by the formula

$$F_T(f) = 0.23 F^*_T\left(f - \frac{1}{2\tau_{max}}\right) + 0.54 F^*_T(f) + 0.23 F^*_T\left(f + \frac{1}{2\tau_{max}}\right);$$

where the quantity $1/\tau_{max}$ is the resolving capacity of spectral analysis (Viner, 1961).

The graphs of $F_T(f)$ of the α rhythm, constructed for various states of three test subjects, are shown on Figure 52, in which we have used the symbols:

$$\begin{aligned} &F_1(f) = F_{T_1}(f), \quad T_1 = 35 \text{ sec}, \quad \tau_{max} = 6.5 \text{ sec}; \\ &F_2(f) = F_{T_2}(f), \quad T_2 = 10 \text{ sec}, \quad \tau_{max} = 3 \text{ sec}; \\ &F_3(f) = T_{T_3}(f), \quad T_3 = 15 \text{ sec}, \quad \tau_{max} = 4.5 \text{ sec}; \end{aligned}$$

The curves denoted by the figure 1 on the graphs of $F_1(f)$ and $F_3(f)$ and the figure 2 on the graph for $F_2(f)$ are estimates of the spectral density for the states of weak emotional stress of the operators (signals of low significance, carrying no information concerning immediate possibility of painful action), and curve 1 on the spectral picture of $F_2(f)$ characterize the state /111 of rest (eyes open).

These dependences have a manifest spectral peak which primarily determines the "rhythm" of the oscillations (the presence of the "rhythm"), observed visually in various situations. The greater the share of the spectrum which is concentrated in a certain band about the mean frequency and the narrower this band, the more the oscillations of the α rhythm at the output of the spectral analyzer will resemble the oscillations of a sine wave with slowly changing amplitude and phase (see Figure 50). We note that with otherwise equal conditions, estimate $F_1(f)$ calculated for a longer realization has less error than $F_2(f)$ and $F_3(f)$.

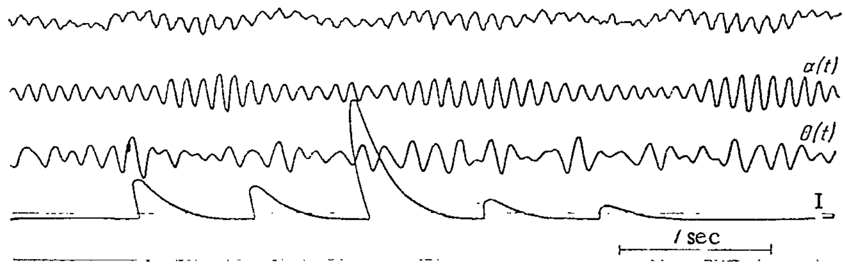


Figure 50

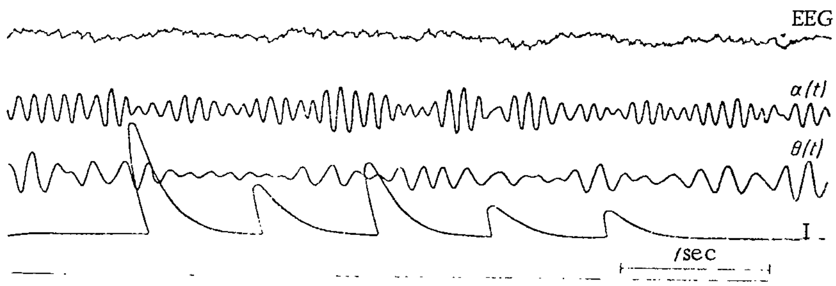


Figure 51

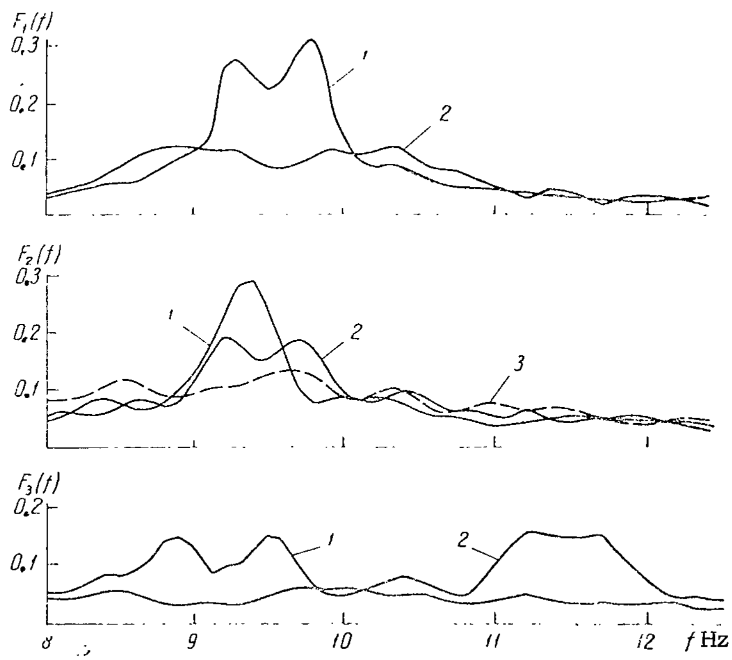


Figure 52

With increasing degree of emotional stress, the spectral dependencies are deformed, the spectral peak has a tendency to "erosion" and its clarity decreases. Curves 2 and 3 on the graphs of $F_1(f)$ and $F_2(f)$ are constructed for comparatively high emotional stress (significant signal, possibility of painful stimulus). In this case, the "rhythm" of the oscillations of the α rhythm is disrupted -- the band width of the process increases (see Figure 51).

The graph $F_2(f)$ illustrates the predominance of the influence of mechanisms of excitation which in the final analysis lead to depression of the α rhythm with rather strong emotional stress.

Characteristic 2 on the graph of $F_3(f)$ is constructed with exaltation of the α rhythm (significant signal), which was judged by the increase in mean amplitude of integrator readings, which is proportional to the dispersion (or the mean square value) of the initial process (see §4.2). The spectral peak was displaced into the area of higher frequencies, causing an increase in the mean frequency of oscillations observed over interval T. Experimental processing of the material showed that with increasing degree of emotional stress (the α rhythm is depressed) the dispersion of the process $y(t)$ decreases[1]. Thus, the usage of the F-distribution for the ratio of sampling dispersions (Nalimov, 1960; van der Waarden, 1960; see also §3.8) during the influence of emotionally significant and neutral signals, indicated a decrease in dispersion for the state of emotional stress with a level of significance of 0.05 in most experiments. An exception, most probably, consisted of those situations in which the possibility of moderate electrical reinforcement at the skin in the experiments was not a sufficiently significant stimulus for the test subject. The number of independent readings was estimated by the expression $N = \Delta f \cdot T$, where the value of Δf, characterizing the energy band of /112
the process with the test subject in the state of rest (20 persons) or in similar situations (for example, showing of neutral signals) was assumed equal to 0.8-1 Hz in correspondence with the time required for attenuation to level 0.1 (estimate of correlation interval τ_k) for normalized correlation functions ($\tau_k \approx 1/\Delta f$). On the other hand, as will be shown below in the analysis of signals at the output of the integrator of bioelectrical activity, in the case $\Delta f \approx 1$ Hz, good correspondence between theoretical and experimental relative errors in calculation of mean square values (dispersion) of the α rhythm in the state of rest of the test subject is produced.

[1] According to the χ^2 criterion, the hypothesis of normal distribution of values of α rhythm $y(t)$ is not contradicted by the experimental data in the overwhelming majority of experiments performed (regardless of the level of emotional stress).

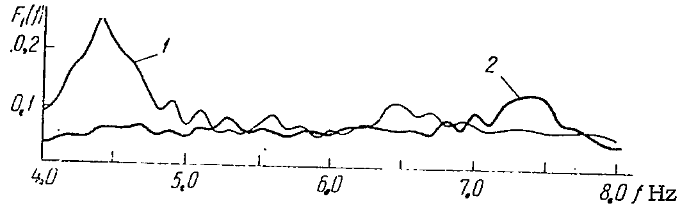


Figure 53

Figure 53 shows estimates of the spectral density $F_1(f)$ of the θ rhythm:

$$F_1(f) = F_{T_1}(f), \quad T_1 = 35 \text{ sec}, \quad \tau_{max} = 6.5 \text{ sec},$$

constructed using the same realizations of the EEG as the graphs of $F_1(f)$ for the α rhythm, the curves 1 and 2 of θ rhythm corresponding to curves 2 and 1 of the α rhythm. Combined analysis of graphs of $F_1(f)$ shows that with sufficiently high emotional stress, the spectral estimate of the θ rhythm has a manifest spectral maximum (4-5-Hz frequency area). In this case, the electroencephalogram begins to show clear visually obvious "rhythm" of the θ waves, whereas the α rhythm is depressed, indicating the continuing predominance of inhibitory influences against the background of comparatively strong excitation (possibility of painful stimulus). With low emotional stress (a state near the state of rest) the spectral peak of the θ rhythm has a weak character (curve 2). In the electroencephalogram, oscillations of the α rhythm are noted, indicated by characteristic 1 (see Figure 52, graph $F_1(f)$). In this case, excitation is low. Inhibitory influences predominate, although they are less strongly expressed than in the reaction of the test subject to a significant signal analyzed above. We note that in many experiments, the spectral picture of the EEG rhythms had a main spectral peak /113 with two maximums and a rather deep depression between them, which is explained, on the one hand, by the corresponding changes in the state of the test subject during the observation interval and, on the other hand, by errors in calculation. However, the nature of the behavior of spectral dependencies for the operator states being investigated was retained in these cases as well.

The changes in the experimental spectral functions of the α and θ rhythms analyzed above are in good correspondence with the electroencephalographic characteristics presented in analysis of the model of the interaction of exciting and inhibiting mechanisms for various emotional states of man (see §4.1, section "d"). The usage of the Volterra rules in this case allows us to estimate the nature of the ratios between inhibitory and excitation mechanisms (judging from the position of the spectral maximum) as applicable to the model of the formation of the rhythmic EEG oscillations (see Chapter 4, Introduction).

Effective calculation of spectral evaluations can only be performed by high-speed computer. In certain cases, coarser characteristics may be sufficient for description of the human states being investigated. Below we analyze the operation of a brain bioelectrical activity integrator, the indications of which are used in primary processing of material, and some problems in the production of information on the nature of the spectral density on the basis of the average number of times the process being investigated crosses the zero level per unit time are analyzed.

§4.2. Analysis of EEG Integrator Output Signals

At the present time, one broadly used method for quantitative evaluation of the EEG in electrophysiology is measurement of the summary electrical activity of the brain U(T). Quantity U(T) is determined by the area bounded by the zero line and the rectified voltage of the electroencephalogram or any of its rhythm over a given time sector T. This indicator has been used in primary processing of data as one of the characteristics of the degree of emotional stress and as an indicator of EEG rhythm behavior (depression, exaltation, etc.) at the output filters of the EEG spectral analyzer (see §4.1, section "e").

Let us analyze a typical block diagram of measurements, presented on Figure 54, where for simplicity we have shown only one of the N parallel processing channels. The electroencephalogram x(t) is fed to a system of linear filters $\Phi \ldots \Phi_{N-1}$ with frequency characteristics $C(\omega) \ldots C_{N-1}(\omega)$, with rather good linearity, the band pass properties of which are determined by the EEG rhythms (Kozhevnikov, Mescherskiy, 1963). The output voltage y(t) of /114 a certain filter Φ is rectified [z(t)] by single or dual half-wave noninertial detector NID and fed to integrating device I. The operating time of the integrator -- T is periodically fixed by circuit P_2, after which the reading u(t) is read and cleared. Reading is performed by device P_1, which scans the outputs of the integrating filters in all channels. In correspondence with the above, the total electrical activity of the brain U(T) in a certain frequency band is determined by the expression (see Figure 54):

$$U(T) = \int_0^T z(t)\,dt. \tag{4.2.1}$$

Below we analyze the dependence of quantity U(T) on the parameters of the measuring circuit and the input process y(t) and estimate the error in its measurements over finite observation interval T_{ob}.

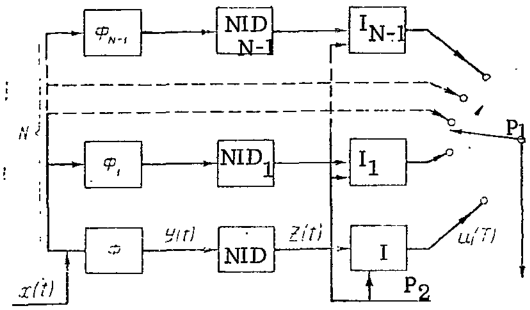


Figure 54

Let us analyze the questions related to the passage of signals through the circuits of the brain summary electrical activity measuring device. Since the circuit consists of N parallel identical measurement channels $\Phi \rightarrow$ NID $\rightarrow$ I $\rightarrow P_1$, analysis is performed in general for one channel, that carrying the α rhythm.

Certain earlier published works (Rozenblit, 1962; Viner, 1963; Saunders, 1963; Grindel', 1965) and our experimental results (see §4.1, section "e") indicate that the α rhythm at the output of the spectral analyzer filter -- $\alpha(t)$ -- can be described for the states of interest to us by a narrow band, Gaussian random process. The energy spectrum of this process in general is concentrated in the area of a certain mean frequency f_c, and the correlation function has a slowly attenuating, oscillating nature. In the following, we will assume

$$\alpha(t) = y(t); \quad \overline{y(t)} = 0,$$

where, as was already noted, y(t) is the result of passage of the EEG through /115
the linear system (filter) with frequency characteristic presented on Figure 55 (the overline represents statistical averaging).

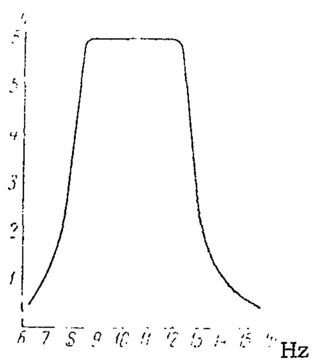


Figure 55

Based on the above, the autocorrelation function and energy spectrum $F_y(\omega)$ of the reaction of the filter y(t) can be represented in the following form (Levin, 1966):

$$B_y(\tau) = B_{y_0}(\tau)\,\varphi(\omega_c, \alpha_\tau);$$
$$F_y(\omega) = C^2(\omega)\,F_x(\omega), \qquad (4.2.2)$$

where the spectral density $F_{y_0}(\omega)$, i.e. the Fourier transform of the slowly changing term $B_{y_0}(\tau)$, is primarily concentrated about the zero frequency, while the spectral density corresponding to the rapidly changing term $\phi(\omega_c, \alpha_\tau)$ is concentrated in a narrow band of frequencies depending on α_τ, about ω_c. Response z(t) of the noninertial detector NID is stationary, if the input action y(t) is stationary and defined by the parameters of y(t) and the nonlinear element. The output signal of the integrator depends, obviously, on the analysis time T and on process z(t). The relationship between the response of the integrating filter u(T) and the perturbation which causes it z(t) can be represented with zero initial conditions by a Duhamel integral in the form

$$u(T) = \int_0^T z(t)\,h(T-t)\,dt = \int_0^T z(T-t)\,h(t)\,dt \qquad (4.2.3)$$
$$h(t) \equiv 0, \quad t<0,$$
$$z(t) \equiv 0, \quad t<0,$$

where h(t) is the pulse transient characteristic related to the stable complex transmission function of the quadrupole (filter) H(jω) by Fourier transforms:

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)\,e^{-j\omega t}dt; \qquad (4.2.4)$$

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)\,e^{j\omega t}dt. \qquad (4.2.5)$$

The modulus and argument of the transfer function

$$H(j\omega) = C(\omega)\,e^{\,j\phi(\omega)} \qquad (4.2.6)$$

are the frequency C(ω) and phase ϕ(ω) characteristics of the filter.

Keeping in mind the finite nature of analysis time T, it is convenient /116 to represent the new pulse characteristic h(t, T) such that it is equal to the reaction of the filter to a unit impulse within the observation interval and equal to zero outside this interval (Davenport, Johnson, Middleton, 1952):

$$h(t, T) = \begin{cases} h(t) & 0\leqslant t\leqslant T \\ 0 & t>T;\ t<0, \end{cases} \qquad (4.2.7)$$

where t is the instantaneous moment of the reading; T is the observation interval.

Under these conditions at moment of reading t = T, expressions (4.2.3), (4.2.4), (4.2.5) can be represented in the following form [for brevity, we assume u(T, T) ≡ u(T)]

$$u(T) = \int_{-\infty}^{\infty} z(t)\, h(T-t, T)\, dt = \int_{-\infty}^{\infty} z(T-t)\, h(t, T)\, dt; \tag{4.2.8}$$

$$H(j\omega, T) = \int_{-\infty}^{\infty} h(t, T)\, e^{-j\omega t} dt; \tag{4.2.9}$$

$$h(t, T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega, T)\, e^{j\omega t} d\omega. \tag{4.2.10}$$

Function H(jω, T) is the instantaneous complex transfer characteristic of the filter, where

$$H(0, T) = \int_{-\infty}^{\infty} h(t, T)\, dt = \int_{0}^{T} h(t)\, dt. \tag{4.2.11}$$

The new pulse reaction h(t, T), as well as the transfer function H(jω, T) combines the effect of averaging of the low-frequency filter and an effect which is the result of the finite nature of analysis time T.

The expressions for the mean value $\overline{u(T)}$ and the dispersion $\sigma_u^2(T)$ of process u(T) at the output of integrating filter I can be produced on the basis of formulas (4.2.7), (4.2.8):

$$\overline{u(T)} = \int_{-\infty}^{\infty} h(T-t, T)\, \overline{z(t)}\, dt = \overline{z(t)} \int_{-\infty}^{\infty} h(T-t, T)\, dt = \overline{z(t)} \int_{0}^{T} h(t)\, dt. \tag{4.2.12}$$

In order to determine dispersion $\sigma_u^2(T)$, we use the equation

$$\sigma_u^2(T) = \overline{[u(T)]^2} - [\overline{u(T)}]^2 \tag{4.2.13}$$

and find the mean square of random quantity u(T): /117

$$\overline{[u(T)]^2} = \iint_{-\infty}^{\infty} h(T-x, T)\, h(T-y, T)\, \overline{z(x)\, z(y)}\, dx\, dy.$$

Performing replacement of variables $\tau = y - x$; $x = x$, we produce

$$\overline{[u(T)]^2} = \iint_{-\infty}^{\infty} h(T-x, T)\, h(T-x-\tau, T)\, \overline{z(x)\, z(x+\tau)}\, dx\, d\tau = \int_{-\infty}^{\infty} \rho_h(\tau, T)\, B_z(\tau)\, d\tau, \qquad (4.2.14)$$

where $B_z(\tau) = \overline{z(x)z(x + \tau)}$ is the autocorrelation function of response z(t) of noninertial detector NID, while the expression

$$\rho_h(\tau, T) = \int_{-\infty}^{\infty} h(T-x, T)\, h(T-x-\tau, T)\, dx \qquad (4.2.15)$$

depends only on the parameters of the integrating filter.

Relationships (4.2.12), (4.2.13), (4.2.14) show that the dispersion and mean value of signal u(T) are determined by the characteristics of the linear system (integrator -- I), and autocorrelation function $B_z(\tau)$. With unlimited increase in τ, random quantities z(t) and $z(t + \tau)$ become statistically independent. The mean value of the product of the statistically independent quantities is equal to the product of the mean values of the cofactors. Consequently,

$$\lim_{\tau\to\infty} B_z(\tau) = B(\infty) = \lim_{\tau\to\infty} \overline{z(t)\, z(t+\tau)} = [\overline{z(t)}]^2. \qquad (4.2.16)$$

Autocorrelation function $B_z(\tau)$ is determined by the parameters of the noninertial nonlinear device NID and action y(t), related to process z(t) by a certain functional dependence characterizing the type of the nonlinear element (see Figure 54).

Let us analyze the noninertial detector NID in the general case as a rectifying device, made up of nonlinear elements with single half-wave ν-th power characteristics:

$$\gamma(y)=\begin{cases} ay^{\nu} & y>0 \\ 0 & y\leqslant 0. \end{cases} \tag{4.2.17}$$

Common examples of such devices include single half-wave ν-th power devices with the characteristic

$$g_1(y)=\gamma(y) \tag{4.2.18}$$

and dual half-wave ν-th power devices with the characteristics /118

$$g_2(y)=\gamma(y)+\gamma(-y)=\begin{cases} ay^{\nu} & y>0 \\ 0 & \\ a(-y)^{\nu} & y<0, \end{cases} \tag{4.2.19}$$

where a is the scale factor, ν is a nonnegative real number. Where ν = 1, on the basis of (4.2.17), we produce, for example, a single half-wave rectifier, and where ν = 2 -- a dual half-wave rectifier [see (4.2.18)].

The autocorrelation function of the response of the nonlinear device z(t), considering the above, can be written in the form

$$B_z(\tau)=\iint\limits_{-\infty}^{\infty} g(y_1)\,g(y_2)\,W(y_1y_2\tau)\,dy_1dy_2, \tag{4.2.20}$$

where $W(y_1y_2\tau)$ is the two-dimensional probability density of random quantities y_1y_2: $y_1(t) = y(t)$; $y_2(t) = y(t - \tau)$.

If the action on the nonlinear element y(t) has a narrow band width, which is of interest in the situations at hand, and if

$$\frac{\Delta\omega_e}{\omega_c}\gg 1,$$

where $\Delta\omega_e,\omega_c$ are the effective width and central frequency of spectrum $F_y(\omega)$, we can assume the energy spectrum of process z(t) to fall clearly into bands: a low-frequency band, adjacent to $\omega = 0$, plus high-frequency bands, concentrated in the areas of frequencies $n\omega_c$; $n > 1$. The correlation function (4.2.20) for each of these bands with normal distribution of y(t) with a two-dimensional characteristic function

$$\Theta_{y_1 y_2}(jv_1 jv_2 \tau) = \iint_{-\infty}^{\infty} W(y_1 y_2 \tau)\, e^{j(v_1 y_1 + v_2 y_2)} dy_1 dy_2 = $$
$$= e^{-\frac{\sigma_y^2 v_1^2}{2}} e^{-\frac{\sigma_y^2 v_2^2}{2}} \sum_{n=0}^{\infty} \frac{(-1)^n R_y^n(\tau)\, \sigma_y^{2n}}{n!} v_1^n v_2^n$$

is determined by well known methods. Considering integrating filter I (see Figure 54) to be such that the high-frequency components of the response z(t) have practically no influence on it (the filter separates a frequency band adjacent to $\omega = 0$), we can represent the expression for autocorrelation function $B_{z_0}(\tau)$ for the low-frequency portion of the spectrum $F_z(\omega)$ in the following form[1] (Middleton, 1947) /119

$$B_{z_0}(\tau)_b = a^2 b^2 \frac{\sigma_y^{2\nu} 2^{\nu-2}}{\pi} \Gamma^2\left(\frac{\nu+1}{2}\right) {}_2F_1\left(-\frac{\nu}{2}; -\frac{\nu}{2}; 1; R_{y_0}^2(\tau)\right), \qquad (4.2.21)$$

where $R_{y_0}(\tau) = \frac{B_{y_0}(\tau)}{\sigma_y^2}$ is the correlation coefficient corresponding to the spectrum $F_y(\omega)$ shifted to the low-frequency area (4.2.2)]; σ_y^2 is the dispersion of process y(t);

$$\Gamma(\nu) = \int_0^{\infty} e^{-t} t^{\nu-1} dt \quad \text{is the } \gamma \text{ function;} \qquad (4.2.22)$$

[1] With a broad band effect, the response spectrum of the nonlinear element is not divided into individual bands. The autocorrelation function $B_z(\tau)$ in this case is determined by expression

$$B_z(\tau)_b = \frac{a^2 b^2}{4\pi} (2\sigma_y^2)^{\nu} \left\{\Gamma^2\left(\frac{\nu+1}{2}\right) {}_2F_1\left[-\frac{\nu}{2}; -\frac{\nu}{2}; \frac{1}{2}; R_y^2(\tau)\right] + \right.$$
$$\left. + (2-b)\, 2R_y(\tau)\, \Gamma^2\left(\frac{\nu}{2}+1\right) {}_2F_1\left[\frac{1-\nu}{2}; \frac{1-\nu}{2}; \frac{3}{2}; R_y^2(\tau)\right]\right\}.$$

$$
{}_2F_1(\alpha, \beta, \gamma, x) = 1 + \frac{\alpha\beta}{\gamma} x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1)} \cdot \frac{x^2}{2!} + \\
+ \frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{\gamma(\gamma+1)(\gamma+2)} \cdot \frac{x^3}{3!} + \dots \qquad (4.2.23)
$$

is the hypergeometric function;

$b = $ 1 -- single half-wave ν-th power nonlinear element;
2 -- dual half-wave ν-th power nonlinear element.

Useful information on the hypergeometric and γ functions can be found in the books of Ye. Yanke and F. Emde (1949) and I. M. Ryzhik and I. S. Gradshteyn (1951).

Assuming in equation (4.2.21) $\tau \to \infty$ and keeping in mind relationships (4.2.12) and (4.2.16), as well as the equation ${}_2F_1(\alpha, \beta, \gamma, 0) = 1$, we produce an expression for the mean value of the output signal u(T) (see Figure 54) as a function of the parameters of process y(t), nonlinear element NID, integrator I and analysis time T:

$$
\overline{u(T)} = \frac{ab}{\sqrt{\pi}} 2^{\frac{\nu-2}{2}} \sigma_y^\nu \Gamma\left(\frac{\nu+1}{2}\right) \int_0^T h(t)\,dt. \qquad (4.2.24)
$$

Thus, the mean value for the set of the response of integrator $\overline{u(T)}$ is proportional to the ν-th power of the mean square value of action y(t) on the input of noninertial detector NID, the coefficient of proportionality being known for moments in time T, in which observations are performed.

The spectrum of the low-frequency component at the output of the nonlinear device $z_0(t)$ can be found on the basis of the Hinchen-Wiener theorem (pair of Fourier transforms): /120

$$
F_{z_0}(\omega) = 2 \int_{-\infty}^{\infty} B_{z_0}(\tau)\, e^{-j\omega\tau} d\tau;
$$
$$
B_{z_0}(\tau) = \frac{1}{4\pi} \int_{-\infty}^{\infty} F_{z_0}(\omega)\, e^{j\omega\tau} d\omega \qquad (4.2.25)
$$

in the case of linear or quadratic noninertial detectors, for example

(Levin, 1966) has the form of a right triangle[1] with base $\Delta\omega_e$ ($\Delta\omega_e$ being the energy band of the input process $y(t)$ with rectangular spectrum).

With increasing ν, the spectrum of low-frequency fluctuations $z_0(t)$ expands. Let us determine now the disperse signal at the output of the integrating filter -- $\sigma_u^2(T)$ [see (4.2.13)]. On the basis of the above, we rewrite (4.2.14) in the form

$$\overline{[u(T)]^2} = \int_{-\infty}^{\infty} \rho_h(\tau, T)\, B_{z_0}(\tau)_b\, d\tau. \tag{4.2.26}$$

Considering relationships (4.2.12), (4.2.13), (4.2.15), (4.2.16), (4.2.26) and the equality

$$[\overline{u(T)}]^2 = [\overline{z_0(t)}]^2 \iint_{-\infty} h(T-x, T)\, h(T-y, T)\, dx\, dy =$$
$$= B_{z_0}(\infty)_b \int_{-\infty}^{\infty} \rho_h(t, T)\, dt,$$

we find

$$\sigma_u^2(T) = \int_{-\infty}^{\infty} \rho_h(\tau, T)\, [B_{z_0}(\tau)_b - B_{z_0}(\infty)_b]\, d\tau = \sigma_{z_0 b}^2 \int_{-\infty}^{\infty} \rho_h(t, T)\, R_{z_0}(t)_b\, dt, \tag{4.2.27}$$

where
$$R_{z_0}(\tau)_b = \frac{B_{z_0}(\tau)_b - B_{z_0}(\infty)_b}{\sigma_{z_0 b}^2}; \tag{4.2.27'}$$

[1] This spectral distribution is precisely fulfilled in the case of a quadratic detector and is fulfilled with good approximation for a linear detector.

$\sigma^2_{z_0 b}$ is the dispersion of low-frequency fluctuations at the output of the nonlinear element.

Substituting the values of $B_{z_0}(\tau)_b$ and $B_{z_0}(\infty)_b$ from equation (4.2.21) into formula (4.2.27), we produce an expression for the dispersion of the output process u(T) as a function of the parameters of the input action y(t), /121 the noninertial detector NID, integrator I and observation time T:

$$\sigma^2_u(T) = a^2 b^2 \frac{\sigma_y^{2\nu} 2^{\nu-2}}{\pi} \Gamma^2\left(\frac{\nu+1}{2}\right) \int_{-\infty}^{\infty} \rho_h(t, T) \times$$
$$\times \left[{}_2F_1\left(-\frac{\nu}{2}; -\frac{\nu}{2}; 1; R^2_{y_0}(\tau)\right) - 1\right] d\tau. \quad (4.2.28)$$

We note that the dispersion of low-frequency fluctuations $\sigma^2_{z_0 b}$ at the output of the nonlinear device is determined on the basis of (4.2.21) and the equation

$$\sigma^2_{z_0 b} = B_{z_0}(0)_b - B_{z_0}(\infty)_b = a^2 b^2 \frac{\sigma_y^{2\nu} 2^{\nu-2}}{\pi} \Gamma^2\left(\frac{\nu+1}{2}\right) \left[\frac{\Gamma(1+\nu) - \Gamma^2\left(1+\frac{\nu}{2}\right)}{\Gamma^2\left(1+\frac{\nu}{2}\right)}\right]. \quad (4.2.29)$$

where the following relationships are considered:

$${}_2F_1(\alpha, \beta, \gamma, 1) = \frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)};$$
$$Re\ \gamma \neq 0, -1, -2, \ldots;$$
$$Re\ (\gamma-\alpha-\beta) \neq 0, -1, -2, \ldots;$$
$$\Gamma(1) = 1.$$

If we consider the band width of the integrating filter small in comparison with the band width of process $z_0(t)$ at the input ($\rho_h(\tau, T)$ changes slowly in comparison with $B_{z_0}(\tau)_b$), we can expand $\rho_h(\tau, T)$ into a MacLauren series:

$$\rho_h(\tau, T) = \sum_k \frac{\tau^k}{k!} \rho^{(k)}(0, T)$$

and limit ourselves to the first term. We then produce

$$\sigma_u^2(T) \simeq \rho_h(0, T) \int_{-\infty}^{\infty} [B_{z_0}(\tau)_b - B_{z_0}(\infty)_b] \, d\tau. \tag{4.2.30}$$

In turn, $\rho_h(0, T)$ characterizes the area of the energy spectrum of the output signal $\zeta(t)$ of the integrator ($\rho_h(\tau, T)$ is the autocorrelation function of signal $\zeta(t)$) when noise $\xi(t)$ with even spectral density equal to 2 is fed to its input (Davenport, Johnson, Middleton, 1952), and the integral

$$\int_{-\infty}^{\infty} [B_{z_0}(\tau)_b - B_{z_0}(\infty)_b] \, d\tau$$

defines the spectral density of the power of low-frequency fluctuations of the response of nonlinear element $z_0(t)$ to the zero frequency (see (4.2.25)).

Considering (4.2.28), we can represent (4.2.30) in the form /122

$$\sigma_u^2(T) \simeq \frac{a^2 b^2}{\pi} \sigma_y^{2\nu} 2^{\nu-2} \Gamma^2\left(\frac{\nu+1}{2}\right) \rho_h(0, T) \int_{-\infty}^{\infty} \times$$
$$\times \left[{}_2F_1\left(-\frac{\nu}{2}, -\frac{\nu}{2}, 1, R_{y_0}^2(\tau)\right) - 1\right] d\tau. \tag{4.2.31}$$

In order to perform concrete calculations using formulas (4.2.24), (4.2.28) and (4.2.31), let us analyze in greater detail the operation of the integrating filter (see Figure 54), in the overwhelming majority of cases made in the form of a commutating RC-circuit (Kozhevnikov, Mescherskiy, 1963).

Let us determine under which conditions equation (4.2.1) is valid, what errors arise during integration and how the frequency characteristic of the integrator depends on the observation time T. Condenser C_1 (Figure 56) is charged over time T through charging resistor R_3, while the charge

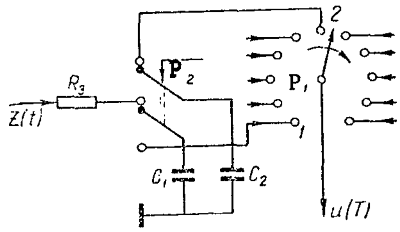


Figure 56

accumulated earlier by condenser C_2 is discharged through switch P_1. After time interval T has passed, discharged condenser C_2 is connected through switch P_2 for charging, and condenser C_1, charged to a certain level u(T) is switched to the terminals of P_1, etc. The charging resistance used is frequently a pentode operating in the anode current cutoff mode and, in many cases, acting also as a one-sided limiter. However, these notes do not influence the essence of the problem under discussion.

The pulse transient characteristic of an RC-circuit can be written in the form

$$h = \alpha e^{-\alpha t},$$

where

$$\alpha = \frac{1}{RC}$$

or, considering the finite nature of analysis time T [see (4.2.7)],

$$h(t, T) = \begin{cases} \alpha e^{-\alpha t} & 0 \leqslant t \leqslant T \\ 0 & t < 0,\ t > T \end{cases} \tag{4.2.32}$$

Analysis of (4.2.7) indicates that in order for mathematical integration to be performed for the response of the nonlinear device z(t) over observation interval T, it is necessary that /123

$$h(t, T) = \text{const} \tag{4.2.33}$$

In the ideal case, h(t, T) = 1. Condition (4.2.33) can be fulfilled with varying degrees of accuracy if the following inequality is satisfied:

$$\alpha T \ll 1, \tag{4.2.34}$$

which gives us, when the exponents are expanded into series,

$$e^{-\alpha T} = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{(\alpha T)^n}{n!} \approx 1 \tag{4.2.35}$$

and, as follows from (4.2.32),

$$h(t, T) \simeq \alpha. \tag{4.2.36}$$

In practice, relationship (4.2.35) is frequently assumed fulfilled when the following inequality is satisfied:

$$\alpha T \leqslant 0{,}1;\ RC \leqslant 10T \tag{4.2.37}$$

i.e. with sufficiently small time of analysis T in comparison with the time constant of the RC-circuit -- τ_{RC}:

$$\tau_{RC} = RC. \tag{4.2.38}$$

Considering (4.2.35), (4.2.36), let us write (4.2.8) in the form

$$u(T) = \alpha \int_0^T z(t)\, dt, \tag{4.2.39}$$

which corresponds, with an accuracy to constant factor α, with formula (4.2.1):

$$u(T) = \alpha\, U(T). \tag{4.2.40}$$

This analysis shows that as conditions (4.2.35) and (4.2.36) are fulfilled, the RC-circuit switched approaches a mathematical integrator. The error in integration can be easily determined by applying constant voltage z(t) = c = const to the input of the circuit.

In this case, in correspondence with expressions (4.2.8), (4.2.32), the output voltage u(T) is defined by the equation

$$u(T) = c\int_0^T \alpha e^{-\alpha t}\,dt = c(1 - e^{-\alpha T}). \tag{4.2.41}$$

when inequality (4.2.34) is fulfilled, considering conditions (4.2.7), (4.2.32) can be limited to the first three terms of series (4.2.35):

$$e^{-\alpha T} \simeq 1 - \alpha T + 0.5\,(\alpha T)^2. \tag{4.2.42}$$ /124

The error ΔS in the representation of even series (4.2.35) of n terms is always less than the first discarded term a_{n+1}:

$$\Delta S = |S - S_n| < |a_{n+1}|,$$

where S and S_n are the general and particular sums of series (4.2.35). In the case at hand

$$\Delta S < \frac{(\alpha T)^3}{6}, \quad \alpha T \ll 1.$$

Substituting the relationship (4.2.42) into formula (4.2.41), we produce

$$u(T) \simeq \alpha c T\left(1 - \frac{\alpha T}{2}\right) = \alpha c T - u\Delta. \tag{4.2.43}$$

With mathematical integration of constant voltage c, quantity u(T) should increase linearly, which is characterized by the first term of the difference in equation (4.2.43). Consequently, the second term of this difference characterizes the error of integration using the RC-circuit. The relative value of integration error is determined by the equation

$$\delta = \frac{\Delta u}{\alpha c T} = \frac{1}{2}\alpha T. \tag{4.2.44}$$

For the case $\alpha T \approx 0.1$ [see (4.2.37)], $\delta \approx 0.05$. The output voltage can be expressed using the relative error δ [see (4.2.43)]:

$$u(T) \simeq \alpha\, c T \simeq 2\, c\delta. \tag{4.2.45}$$

Formulas (4.2.34), (4.2.44) and (4.2.45) reflect the contradictory nature of the demands on the time constant of the integrating circuit τ_{RC} as concerns accuracy and effectiveness.

Let us now determine the complex instantaneous transfer characteristic $H(j\omega, T)$ of the RC filter.

For small observation times, keeping in mind (4.2.9), (4.2.34) and (4.2.36), we produce

$$H_M(j\omega, T) \simeq \int_{-\infty}^{\infty} h(t, T)\, e^{-j\omega t} d\,t \simeq \alpha \int_{0}^{T} e^{-j\omega t}\, dt = \alpha \frac{1 - e^{-j\omega T}}{j\omega T}. \tag{4.2.46}$$

From this last equation, considering (4.2.6), we can find the instantaneous frequency characteristic $C_M(\omega, T)$ of the switchable RC-circuit ($\alpha T \ll 1$):

$$C_M(\omega, T) = |H_M(j\omega, T)| \simeq \alpha T \left| \frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}} \right|. \tag{4.2.47}$$

The instantaneous energy band $\Delta\omega_{eM}$ can be represented in the form /125

$$\Delta\omega_{eM} = \frac{\int_0^{\infty} C_M^2(\omega, T)\, d\omega}{C_M^2(0, T)} \simeq \frac{2}{T} \int_0^{\infty} \frac{\sin^2 x}{x^2}\, dx \simeq \frac{\pi}{T}$$

or

$$\Delta F_{eM} = \frac{0{,}5}{T}, \tag{4.2.48}$$

in which we consider

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

The sign of approximate equality in formulas (4.2.48) is included on the basis of relationship (4.2.36).

In the general case, the expression for the instantaneous combined transfer characteristic $H(j\omega, T)$ of the RC-circuit is determined by the following equation [see (4.2.9), (4.2.32)]:

$$H(j\omega, T) = \alpha \int_0^T e^{j\omega T} e^{-\alpha t}\, dt = \frac{\alpha}{\alpha + j\omega} [1 - e^{-(\alpha + j\omega)T}]. \quad (4.2.49)$$

Where $T \to \infty$, expression (4.2.49) goes over to a stable transfer characteristic $H(j\omega)$:

$$H(j\omega) = \frac{\alpha}{\alpha + j\omega}.$$

Consequently, using the relationship

$$H(j\omega) H(-j\omega) = |H(j\omega)|^2 = C^2(\omega), \quad (4.2.50)$$

we produce a formula for the stable energy band $\Delta\omega_{eRC}$:

$$\Delta\omega_{eRC} = \int_0^\infty \frac{\alpha^2}{\alpha^2 + \omega^2}\, d\omega = \frac{\alpha\pi}{2};$$
$$\Delta F_{eRC} = \frac{\Delta\omega}{2\pi} = \frac{1}{4RC}. \quad (4.2.51)$$

The autocorrelation function of $\rho_h(\tau, T)$ in this case is determined on the basis of (4.2.25) under the condition that noise $\xi(t)$ with even spectral density $F_\xi(\omega) = 2$ acts on the input of the integrator:

$$\rho_h(\tau, T) = \frac{1}{4\pi} \int_{-\infty}^{\infty} 2|H(j\omega, T)|^2 e^{j\omega\tau}\, d\omega = \frac{1}{\pi} \int_0^\infty |H(j\omega, T)|^2 \cos\omega\tau\, d\omega, \quad (4.2.52)$$

where the signal spectrum at the output of the filter $F_h(\omega, T)$ is recorded in the form [see (4.2.2), (4.2.49)]: /126

$$F_h(\omega, T) = F_\xi(\omega) C^2(\omega, T) = 2|H(j\omega, T)|^2.$$

Substituting into formula (4.2.52) the expression for the instantaneous frequency characteristic (4.2.47) in the situation of interest to us (4.2.36), (4.2.39), we produce (Ryzhik, Gradshteyn, 1951)

$$\rho_h(\tau, T) = \frac{2T\alpha^2}{\pi}\int_0^\infty \frac{\sin^2 x}{x^2}\cos\frac{2\tau}{T}x\,dx = T\alpha^2\left(1 - \frac{|\tau|}{T}\right), \tag{4.2.53}$$

where τ changes in the area of positive and negative values and

$$0 \leqslant \left|\frac{\tau}{T}\right| \leqslant 1.$$

Function $\rho_h(\tau, T)$ can be found directly from the equation (4.2.15):

$$\rho_h(\tau, T) = \int_{-\infty}^{\infty} h(v, T)\,h(v-\tau, T)\,dv.$$

Setting up the integration limits in consideration of (4.2.7), we can represent (4.2.15) in the following manner:

$$\rho_h(\tau, T) = \begin{cases} \int_\tau^T h(v)\,h(v-\tau)\,dv & 0 \leqslant \tau \leqslant T \\ \int_0^{T+\tau} h(v)\,h(v-\tau)\,dv & -T \leqslant \tau \leqslant 0 \end{cases} \tag{4.2.54}$$

and $\rho_h(\tau, T) = 0$ where $\tau > T$; $\tau < -T$. The areas of integration for positive and negative values of τ are limited by four lines in the plane $vo\tau$: $v = 0$; $v = T$ (change in argument $h(v)$) and $v - \tau = 0$; $v - \tau = T$ (change in argument $h(v + \tau)$). Substituting $h(v) = \alpha$ into (4.2.54) [see (4.2.36)], we produce

$$\rho_h(\tau, T) = \alpha^2 T\left(1 - \frac{|\tau|}{T}\right),$$

which corresponds to (4.2.53).

We note that these discussions are correct not only in the case of a switchable RC-circuit, as described above, but also when realizations of a signal of length T each are fed to a noncommutated RC-circuit, following each other with an interval no less than the time required for the filter to recover from the preceding action.

Let us now discuss the approximate equation (4.2.30), which was produced on the assumption of slow change in $\rho_h(\tau, T)$ in comparison with $B_{z_0}(\tau)$. /127
Since the correlation time of the fluctuations $z_0(t)$ has the order $1/\Delta f_e$, where Δf_e is the energy band of process y(t), we can fix the conditions of fulfillment of (4.2.30) for $h(T, T) \approx \alpha$ [see (4.2.36), (4.2.39), (4.2.28)] in the form $\Delta F_{eM} \ll \Delta f_e$, consequently, expression (4.2.30) can be used when the following relationship is satisfied [see (4.2.48)]

$$\Delta f_e T \gg 0.5. \tag{4.2.55}$$

In conclusion, let us analyze the case, frequently encountered in practice, of a device designed to measure the summary electrical activity of the brain (see Figure 54) with linear (quadratic) noninertial detector NID and integrator I, made as a commutated RC-circuit, where $\alpha T \ll 1$.

On the basis of formulas (4.2.24), (4.2.28), (4.2.36), (4.2.53), we produce the corresponding expressions.

1. The linear noninertial detector. $\nu = 1$:

$$\overline{u(T)} = \frac{ab}{\sqrt{2\pi}} \alpha T \sigma_y; \tag{4.2.56}$$

$$\sigma_u^2(T) = \frac{a^2b^2}{8\pi} \alpha^2 \sigma_y^2 \int_{-T}^{T} (T - |\tau|) R_{y_0}^2(\tau)\, d\tau =$$

$$= \frac{a^2b^2\alpha^2}{4\pi} \sigma_y^2 \int_0^T (T - \tau) R_{y_0}^2(\tau)\, d\tau. \tag{4.2.57}$$

2. Quadratic noninertial detector. $\nu = 2$:

$$\overline{u(T)} = \frac{ab}{2} \alpha T \sigma_y^2; \tag{4.2.58}$$

$$\sigma_u^2(T) = \frac{a^2b^2}{4} \alpha^2 \sigma_y^4 \int_{-T}^{T} (T - |\tau|) R_{y_0}^2(\tau)\, d\tau =$$

$$= \frac{a^2b^2\alpha^2}{2} \sigma_y^4 \int_0^T (T - \tau) R_{y_0}^2(\tau)\, d\tau. \tag{4.2.59}$$

In concluding equations (4.2.56), (4.2.57), (4.2.58) and (4.2.59), we used representation of the hypergeometric function (linear noninertial detector) by the first two terms of its expansion into a series (4.2.23), giving in this case a good approximation for practical application (Levin, 1966)

$$ {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}, 1, R_{y_0}^2(\tau)\right) \approx 1 + \frac{R_{y_0}^2(\tau)}{4}$$

and the relationship /128

$$ {}_2F_1(-1, -1, 1, R_{y_0}^2(\tau)) = 1 + R_{y_0}^2(\tau);$$
$$\Gamma\left(\frac{1+1}{2}\right) = 1; \quad \Gamma\left(\frac{2+1}{2}\right) = \frac{\sqrt{\pi}}{2}.$$

We note that the mean value of the signal $\overline{u(T)}$ at the output of the integrator after the quadratic detector (4.2.58) is proportional to the energy of the process $y(t)$ (electroencephalogram rhythm) during the analysis time T. If we assume further that condition (4.2.55) is fulfilled, then formulas (4.2.57), (4.2.59) can be written as follows [see (4.2.31)]:

$$\sigma_u^2(T) \simeq \frac{a^2b^2\alpha^2T}{4\pi} \sigma_y^2 \int_0^\infty R_{y_0}^2(\tau)\, d\tau, \qquad \nu = 1; \tag{4.2.60}$$

$$\sigma_u^2(T) \simeq \frac{a^2b^2\alpha^2T}{2} \sigma_y^4 \int_0^\infty R_{y_0}^2(\tau)\, d\tau, \qquad \nu = 2. \tag{4.2.61}$$

Let us find further the relative error

$$\delta_u(T) = \frac{\dot{\sigma}_u(T)}{\overline{u(T)}}$$

of process u(T) at the output of the integrator (for quadratic and linear noninertial detectors). On the basis of (4.2.56), (4.2.57), (4.2.58), (4.2.59), we produce

$$\delta_u(T) = \frac{1}{\sqrt{2}T}\left\{\int_0^T (T-\tau)\, R^2_{y_0}(\tau)\, d\tau\right\}^{\frac{1}{2}}, \quad \nu = 1. \tag{4.2.62}$$

$$\delta_u(T) = \frac{\sqrt{2}}{T}\left\{\int_0^T (T-\tau)\, R^2_{y_1}(\tau)\, d\tau\right\}^{\frac{1}{2}}, \quad \nu = 2. \tag{4.2.63}$$

When (4.2.55) is satisfied, expressions (4.2.62), (4.2.63) are simplified [see (4.2.60), (4.2.61)] and take on the form

$$\delta_u(T) \simeq \frac{1}{\sqrt{2T}}\left\{\int_0^\infty R^2_{y_0}(\tau)\, d\tau\right\}^{\frac{1}{2}}, \quad \nu = 1; \tag{4.2.64}$$

$$\delta_u(T) \simeq \frac{\sqrt{2}}{\sqrt{T}}\left\{\int_0^\infty R^2_{y_1}(\tau)\, d\tau\right\}^{\frac{1}{2}}, \quad \nu = 2, \tag{4.2.65}$$

where the integral /129

$$\int_0^\infty R^2_{y_0}(\tau)\, d\tau \tag{4.2.66}$$

characterizes the correlation time of the low-frequency fluctuations $z_0(t)$ at the output of the nonlinear element, i.e. on the basis of equations (4.2.21), (4.2.27'), (4.2.29) the correlation coefficient $R_{z_0}(\tau)$ is determined for the situations analyzed by relationships

$$R_{z_0}(\tau) = R_{y_0}^2(\tau), \qquad \nu = 2;$$
$$R_{z_0}(\tau) \simeq R_{y_0}^2(\tau), \qquad \nu = 1. \qquad (4.2.67)$$

Assuming for estimation of expressions (4.2.64) and (4.2.65) that the correlation time of the low-frequency fluctuations of the magnitude are on the order of $(\Delta f_y)^{-1}$, where Δf_y is the energy band of y(t), we produce

$$\delta_u(T) = \frac{1}{\sqrt{2\Delta f_y T}}, \qquad \nu = 1; \qquad (4.2.68)$$

$$\delta_u(T) = \frac{2}{\sqrt{2\Delta f_y T}}, \qquad \nu = 2. \qquad (4.2.69)$$

The experimental data show (see §4.1) that the energy band of the α rhythm (y(t)) decreases with a decrease in the degree of emotional stress of the test subject and in the state of rest (eyes open) is approximately equal to 1 Hz (mean estimate for all test subjects participating in experiments). Here, the error $\delta_u(T)$ reaches its maximum (T fixed):

$$\delta_{uM}(T) \simeq 0{,}22, \qquad T = 10 \text{ sec}, \qquad \nu = 1; \qquad (4.2.70)$$
$$\delta_{uM}(T) \simeq 0{,}44, \qquad T = 10 \text{ sec}, \qquad \nu = 2. \qquad (4.2.71)$$

On the other hand, the results of calculations of $\delta_u(T)$ for the state of rest of the operator, performed during processing of realizations of the output process of the integrator (100 experiments, involving 25 persons) did not exceed the quantity $\delta_{uM}(T)$ [see (4.2.70)] in 95 of 100 cases. The volume of each realization averaged 50 independent readings, the integration time T = 10 sec.

The above allowed us to use the expressions presented (4.2.68), (4.2.69) as estimates of the upper limit of relative error $\delta_u(T)$, which could be produced in situations of rest state of the test subject (eyes open).

Let us now determine the nature of the relative error $\Delta_u(T)$, the calculation of $\delta_u(T)$ performed using approximate formulas (4.2.64), (4.2.65). Considering that in the case of linear and quadratic noninertial detectors $\Delta_u(T)$ will be identical, we produced (for example, ν = 1) /130

$$\Delta_u(T) = \left| \frac{\frac{1}{\sqrt{2T}} \left\{ \int_0^T (T-\tau) R_{y_0}^2(\tau)\, d\tau \right\}^{\frac{1}{2}} - \frac{1}{\sqrt{2T}} \left\{ \int_0^\infty R_{y_0}^2(\tau)\, d\tau \right\}^{\frac{1}{2}}}{\frac{1}{\sqrt{2T}} \left\{ \int_0^T (T-\tau) R_{y_0}^2(\tau)\, d\tau \right\}^{\frac{1}{2}}} \right| =$$

$$= \left| 1 - \sqrt{T} \left\{ \frac{\int_0^\infty R_{y_0}^2(\tau)\, d\tau}{\int_0^T (T-\tau) R_{y_0}^2(\tau)\, d\tau} \right\}^{\frac{1}{2}} \right|. \qquad (4.2.72)$$

As was noted above, as the degree of emotional stress decreases, the spectrum of the α rhythm becomes concentrated into a relatively narrow band of frequencies, beyond the limits of which the spectral density falls off rapidly and becomes practically insignificant (Viner, 1963) for the rest state of the subject (eyes closed). Therefore, in order to estimate the behavior of quantity $\Delta_u(T)$, we can use an approximation of the spectral peak of the α rhythm by a П-shaped curve with the same energy band as for the initial spectrum. We note that with fixed σ_y^2 and T, the dispersion at the output of the integrator increases with increasing rectangularity of the spectrum, leading to an increase in the relative error $\delta_u(T)$. Keeping in mind the expression for the correlation coefficient $R_{y_0}(\tau)$ in the case of a rectangular spectrum with energy band $\Delta\omega_y$ [determined from (4.2.25)]:

$$R_{y_0}(\tau) = \frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \qquad (4.2.73)$$

and performing integration, we can write the final answer in the form [see (A.1.5)]

$$\Delta_u(T) =$$

$$+\left| 1 - \left\{ \frac{\pi \Delta\omega_y T}{2\,[\Delta\omega_y T\, \mathrm{Si}\,(\Delta\omega_y T) + \cos\,(\Delta\omega_y T) - (1 + C_0) - \ln\,(\Delta\omega_y T) + \mathrm{Si}\,(\Delta\omega_y T)]} \right\}^{\frac{1}{2}} \right|. \qquad (4.2.74)$$

The graph of the change in relative error $\Delta_u(T)$ as a function of quantity /131

$\Delta\omega_y T$ is presented on Figure 57. The integral in the denominator of expression (4.2.72) is proportional to the dispersion of process u(T) [see precise equations (4.2.57), (4.2.59)] and its value is presented separately in Appendix 1 (A.1.6).

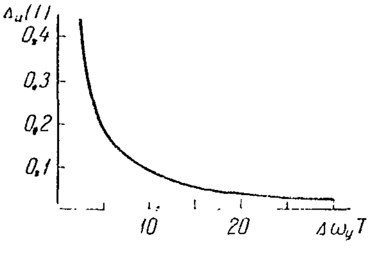


Figure 57

§4.3. Mean Number of Intersections of Zero Level per Unit Time

The results of processing of experimental material presented above (see §4.1) give us reason to believe that spectrum $F_y(\omega)$ of process y(t) (α rhythm, θ rhythm) is deformed (in form, band width and position of spectral peak) as the degree of emotional stress is increased.

Let us analyze the dependence between the spectral density $F_y(\omega)$ and the mean number of intersections n(0) of process y(t) across the zero line per unit time. For an arbitrary spectral form, we have (Tikhonov, 1966)

$$n(0) = \frac{1}{\pi}\left(\frac{\sigma_{y'}^2}{\sigma_y^2}\right)^{\frac{1}{2}} = \frac{1}{\pi}\left(\frac{\int\limits_0^{\infty} \omega^2 F_y(\omega)\,d\omega}{\int\limits_0^{\infty} F_y(\omega)\,d\omega}\right)^{\frac{1}{2}} \tag{4.3.75}$$

where σ_y^2 and $\sigma_{y'}^2$ are the dispersion of y(t) and its derivative y'(t) $(\overline{y(t)} = 0)$. In the case of narrow band processes $(\Delta\omega_y \ll \omega_c)$, the spectrum of which is concentrated about a certain frequency ω_c, by performing replacement of variables $\omega = \omega_c - v$ in equation (4.3.75), we produce

$$n(0) = \frac{1}{\pi}\left\{\omega_c^2 - 2\omega_c \frac{\int\limits_{-\infty}^{\infty} vF_y(\omega_c - v)\,dv}{\int\limits_{-\infty}^{\infty} F_y(\omega_c - v)\,dv} + \frac{\int\limits_{-\infty}^{\infty} v^2F_y(\omega_c - v)\,dv}{\int\limits_{-\infty}^{\infty} F_y(\omega_c - v)\,dv}\right\}^{\frac{1}{2}}, \tag{4.3.76}$$

where the upper limits of integration are extended to infinity. Since the energy spectrum $F_y(\omega)$ is concentrated in a narrow band of frequencies about $\omega = \omega_c$, spectrum $F(\omega_c - v)$ is located in the low-frequency area. If, furthermore, $F_y(\omega)$ is symmetrical relative to central frequency ω_c, expression (4.3.76) takes on the form /132

$$n(0) = \frac{1}{\pi}\left\{\omega_c^2 + \frac{\int_{-\infty}^{\infty} v^2 F_y(\omega_c - v)\,dv}{\int_{-\infty}^{\infty} F_y(\omega_c - v)\,dv}\right\}^{\frac{1}{2}}, \tag{4.3.77}$$

and the integral

$$\int_{-\infty}^{\infty} v F_y(\omega_c - v)\,dv = 0.$$

Consequently, the quantity n(0) is determined by the ratio of dispersions of process y(t) and its derivative and depends both on the form of the spectrum and on its location in the frequency area [see (4.3.75), (4.3.76), (4.3.77)]. It can be shown, for example (Tikhonov, 1966) that for a fixed spectral characteristic $F_y(\omega)$ of the normally distributed stable random process y(t), the mean number of intersections of the zero line per unit time n(0) increases with increasing mean frequency of the spectrum -- ω_c and its energy band -- $\Delta\omega_y$. If the parameters ω_c and $\Delta\omega_y$ do not change, the value n(0) will be higher, the less the rectangularity of the spectrum, as a result of which the influence of high-frequency components increases. We note that the mean number of intersections of the zero line N(0) in analysis time T can be determined from the formula

$$N(0) = Tn(0) = \frac{T}{\pi}\left\{\frac{\sigma_{y'}^2}{\sigma_y^2}\right\}^{\frac{1}{2}}. \tag{4.3.78}$$

Thus, in our case, the problem of estimation of n(0) or N(0) is reduced to the problem of estimation of the dispersion over a finite observation interval T. The relative error in measurement of the process at the output of the integrator in the general case is represented by the expression [see

(4.2.12), (4.2.15), (4.2.26)]:

$$\delta_u(T) = \left\{ \frac{\iint\limits_{-\infty}^{\infty} R_z(x)\, h(v, T)\, h(v+x, T)\, dx\, dv}{[\overline{z(t)}]^2 \iint\limits_{-\infty}^{\infty} h(v, T)\, h(v, T)\, dv\, dx} \right\}^{\frac{1}{2}}.$$

For known characteristics of input action z(t), we can find an optimal form h(x, T) for which $\delta_u(T)$ will be minimal. In the case when the autocorrelation function $R_z(x)$ is not precisely known, the optimal smoothing filter is a filter, the impulse reaction of which is equal to a finite, constant quantity over the integration interval (Davenport, Johnson, Middleton, 1952; Fel'dman, 1962). These conditions are satisfied, for example, by an integrating filter with an impulse characteristic of the following form (ideal integrator) /133

$$h(t, T) = \begin{cases} \frac{1}{T} & 0 \leqslant t \leqslant T \\ 0 & 0 > t;\ t > T \end{cases} \qquad (4.3.79)$$

and by the commutating RC-circuit analyzed in §4.2 (approximately, to the extent to which inequality $\alpha T \ll 1$ is fulfilled). A block diagram of the device for measuring the dispersion of process y(t) and its derivative $y_1(t) = y'(t)$, keeping in mind that which we have stated above and the fact that the mathematical expectation of the response of the quadratic detector is proportional to the dispersion of the input action, is shown on Figure 58 (see §4.2). Signal y(t) and its derivative at the output of the differentiating circuit DC pass through the quadratic noninertial devices NID and arrive at integrating filters I. In the general case, the autocorrelation function $R_z(\tau)$ of the response z(t) of nonlinear element NID is unknown; therefore, the pulse characteristic of the integrator is determined by expression (4.3.79). If we further subdivide one output of smoothing filter $u_1(T)$ by that of the other u(T), we produce an estimate of the quantity n(0).

Further discussions are performed where possible in general form, although where it is required to make the problem more concrete, references are made to the characteristics of the α rhythm.

Input action y(t), a result of the passage of the EEG through the corresponding filter of the spectral analyzer with its rectangular frequency characteristic (see Figure 47) will be assumed as before (see §4.2) to be a narrow band normal random process (α rhythm). We note that the derivative of process y(t) is also distributed normally.

The α rhythm spectrum in operator states not strongly differing from the state of rest is concentrated in a narrow band of frequencies and rapidly falls off beyond the limits of this band which results on the one hand from

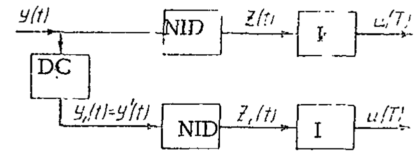


Figure 58

the nature of changes in the spectrum itself (Viner, 1963, 1961; see §4.1), and on the other hand by the frequency properties of the selective П-shaped system. /134
When the degree of emotional stress of the test subject increases, the α rhythm spectrum is "spread" (see §4.1) and at the limit (high emotional stress, spectral peak disappears or is only slightly expressed) its form is approximately determined by the frequency characteristic of the spectral analyzer filter (see Figure 55), which separates the required band of frequencies from the EEG.

In order to trace certain interesting regularities, let us approximate the spectrum of the α rhythm on the basis of the above by a П-shaped curve. The energy bands of the idealized and actual spectra will be assumed equal to $\Delta\omega_y$. In this case, on the basis of (4.2.25), (4.2.73), (4.3.77) and relationship

$$B_{y_1}(\tau) = B_{y'} = -B''_y(\tau)$$

we produce (see Figure 58)

$$B_y(\tau) = \sigma_y^2 R_{y_0}(\tau) \cos \omega_c \tau \tag{4.3.80}$$

$$B_{y_1}(\tau) = -\sigma_y^2 \{[R''_{y_0}(\tau) - \omega_c^2 R_{y_0}(\tau)] \cos \omega_c \tau - 2\omega_c R'_{y_0}(\tau) \sin \omega_c \tau\}. \tag{4.3.81}$$

Let us represent equation (4.3.81) in the form of the product of two rapidly and slowly changing terms [see (4.2.2)]. We note that this representation of the autocorrelation function in the case of a spectrum with rectangular envelope is correct with any ω_c and $\Delta\omega_y$. Consequently,

$$B_{y_1}(\tau) = \sigma_{y_1}^2 R_{y_{10}}(\tau) \cos [\omega_c \tau - \varphi(\tau)], \tag{4.3.82}$$

where $\sigma_{y_1}^2$ is the dispersion of process $y_1(t)$;

$$R_{y_{10}}(\tau) = \frac{\sigma_y^2 \{[\omega_c^2 R_{y_0}(\tau) - R''_{y_0}(\tau)]^2 + [2\omega_c R'_{y_0}(\tau)]^2\}^{1/2}}{\sigma_{y_1}^2} = \frac{\sigma_y^2 [r^2_{y_{10}}(\tau)]^{1/2}}{\sigma_{y_1}^2} \tag{4.3.82'}$$

$$r^2_{y_{10}}(\tau) = [\omega_c^2 R_{y_0}(\tau) - R''_{y_0}(\tau)]^2 + [2\omega_c R'_{y_0}(\tau)]^2;$$

$$\operatorname{tg}\varphi(\tau) = \frac{2\omega_c R'_{y_0}(\tau)}{\omega_c^2 R_{y_0}(\tau) - R''_{y_0}(\tau)}; \tag{4.3.82''}$$

$$R_{y_0}(\tau) = \frac{\sin\frac{\Delta\omega_y\tau}{2}}{\frac{\Delta\omega_y\tau}{2}}; \quad R'_{y_0}(\tau) = \frac{dR_{y_0}(\tau)}{d\tau}; \quad R''_{y_0}(\tau) = \frac{d^2R_{y_0}(\tau)}{d\tau^2}.$$

The mutual correlation function $B_{y_1y}(\tau)$ of processes $y_1(t)$ and $y(t)$ is /135

$$B_{y_1y}(\tau) = -B_{yy_1}(\tau) = B_y(\tau) =$$
$$= \sigma_y^2\{R'_{y_0}(\tau)\cos\omega_c\tau - \omega_c R_{y_0}(\tau)\sin\omega_c\tau\} =$$
$$= \sigma_y\sigma_{y_1}R_{y_{10}y_0}(\tau)\cos[\omega_c\tau + \psi(\tau)], \tag{4.3.83}$$

where

$$R_{y_{10}y_0}(\tau) = \frac{\sigma_y^2\{[R'_{y_0}(\tau)]^2 + [\omega_c R_{y_0}(\tau)]^2\}^{1/2}}{\sigma_y\sigma_{y_1}} = \frac{\sigma_y^2[r^2_{y_{10}y_0}(\tau)]^{1/2}}{\sigma_y\sigma_{y_1}}; \tag{4.3.83'}$$

$$r^2_{y_{10}y_0}(\tau) = \{[R'_{y_0}(\tau)]^2 + [\omega_c R_{y_0}(\tau)]^2\};$$

$$\operatorname{tg}\psi(\tau) = \frac{\omega_c R_{y_0}(\tau)}{R'_{y_0}(\tau)}. \tag{4.3.83''}$$

With a narrow band action (see §4.2), the high-frequency components of the response $z(t)$ of the noninertial nonlinear element have practically no influence on the integrating filter, so that in the following we can analyze only the autocorrelation functions $B_{z_0}(\tau)$ and $B_{z_{10}}(\tau)$ of the low-frequency components at the outputs of the quadratic detectors NID. Assuming in our case a = 1, b = 2, $\nu = 2$ [see (4.2.21), (4.2.27')], we produce

$$B_{z_0}(\tau) = \sigma_y^4 R^2_{y_0}(\tau); \tag{4.3.84}$$

$$B_{z_{10}}(\tau) = \sigma_{y_1}^4 R^2_{y_{10}}(\tau). \tag{4.3.85}$$

The mutual correlation function of processes $z_1(t)$ and $z(t)$ is determined by considering the normal distribution of $y(t)$ and its derivative as follows:

$$B_{z_1,z}(\tau) = \overline{z_1 z_\tau} = \overline{y_1 y_1 y_\tau y_\tau} = 2\,[\overline{y_1 y_\tau}]^2 + \overline{y_1^2}\,\overline{y_\tau^2} = 2B^2_{y_1 y}(\tau) + \sigma^2_{y_1}\sigma^2_y, \tag{4.3.86}$$

where $\overline{z_1(t)} = \sigma^2_{y_1}$, $\overline{z(t)} = \sigma^2_y$ [see (4.2.21) where $\tau \to \infty$]. On the basis of (4.3.83), (4.3.86), the mutual correlation function of the low-frequency components $z_0(t)$ and $z_{10}(t)$ is expressed by the equation

$$B_{z_{10}z}(\tau) = \overline{z_{10} z_{0\tau}} = \sigma^2_{y_1}\sigma^2_y R^2_{y_{10}y_0}(\tau) + \sigma^2_{y_1}\sigma^2_y. \tag{4.3.87}$$

The mathematical expectations of output signals of the integrators $\overline{u(T)}$ and $\overline{u_1(T)}$ over analysis time T are equal to σ^2_y and $\sigma^2_{y_1}$ respectively [see (4.2.24), (4.3.79)]. The dispersion of the derivative of the input action $y_1(t)$ can be found from expression (4.3.82) where $\tau = 0$; however, in this case it is simpler to use the spectral representations for (4.2.25), (4.3.75): /136

$$\sigma^2_{y_1} = \frac{1}{2\pi} \int\limits_{\omega_c - \frac{\Delta\omega_y}{2}}^{\omega_c + \frac{\Delta\omega_y}{2}} 2S_0\omega^2 d\omega = \sigma^2_y \left(\omega_c^2 + \frac{\Delta\omega_y^2}{12}\right), \tag{4.3.88}$$

where $2S_0$ is the spectral density of $y(t)$, constant within the band $\Delta\omega_y$:

$$\sigma^2_y = \frac{1}{2\pi} \int\limits_{\omega_c - \frac{\Delta\omega_y}{2}}^{\omega_c + \frac{\Delta\omega_y}{2}} 2S_0 d\omega = \frac{S_0}{\pi}\,\Delta\omega_y. \tag{4.3.89}$$

The calculations of dispersions of processes $u(T)$ and $u_1(T)$ using precise formula (4.2.28) yield rather cumbersome expressions[1]. In the following we will use approximate equation (4.2.31), which is correct in situations of interest to us $\Delta f_y T \gg 1$. On the basis of (4.2.53), (4.2.54), (4.3.79), (4.3.82'),

$$\rho_h(\tau, T) = \begin{cases} \frac{1}{T^2}(T - |\tau|) & |\tau| < T \\ 0 & |\tau| > T \end{cases}, \tag{4.3.90}$$

therefore (a = 1, b = 2, ν = 2)

$$\sigma_u^2(T) = \sigma_y^4 \frac{2}{T} \int_0^\infty R_{y_0}^2(\tau)\, d\tau; \tag{4.3.91}$$

$$\sigma_{u_1}^2(T) = \sigma_{y_1}^4 \frac{2}{T} \int_0^\infty R_{y_{1_0}}^2(\tau)\, d\tau = \sigma_y^4 \frac{2}{T} \int_0^\infty r_{y_{1_0}}^2(\tau)\, d\tau. \tag{4.3.92}$$

Calculations using formulas (4.3.91), (4.3.92) are performed in Appendix 1 (A.1.2) and Appendix 2 (A.2.5), (A.2.11):

$$\sigma_{u_1}^2(T) = \frac{2\pi\sigma_y^4 \omega_c^4}{\Delta\omega_y T}\left(1 + \frac{\Delta\omega_y^2}{2\omega_c^2} + \frac{\Delta\omega_y^4}{80\omega_c^4}\right). \tag{4.3.93}$$

$$\sigma_u^2(T) = \frac{2\sigma_y^4\pi}{\Delta\omega_y T}; \tag{4.3.94}$$

The analysis of expressions (4.3.93), (4.3.94) shows that the dispersions $\sigma_u^2(T)$ and $\sigma_{u_1}^2(T)$ depend on the quantities $\Delta\omega_y T$ and approach zero with unlimited increase in the latter. /137

Let us represent the mixed second moment $B_{uu_1}(T)$ of signals u(T) and $u_1(T)$ at the outputs of the integrating filters in the form

[1] Determination of the dispersions of processes u(T) and $u_1(T)$ using precise formula (4.2.28) in the case here being analyzed can be performed on the basis of the expressions presented in Appendices 1 and 2 [(A.1.5) and (A.2.20)], where the corresponding integrals are calculated, allowing us to estimate the errors arising when approximate equation (4.2.31) is used. Formulas (A.1.5) and (A.2.20) are correct for narrow band and broad band processes.

$$B_{uu_1}(T) = \overline{u(T)\,u_1(T)} - \overline{u(T)}\;\overline{u_1(T)}.$$

Keeping in mind (4.2.8), (4.2.12), (4.2.15) and (4.2.30), we can write

$$\begin{aligned} B_{uu_1}(T) &= \iint\limits_{-\infty}^{\infty} h(u, T)\, h(v, T)\, \overline{z(T-u)\, z(T-v)}\, du\, dv - \\ &- \iint\limits_{-\infty}^{\infty} \overline{z_1(t)}\; \overline{z(t)}\, h(u, T)\, h(v, T)\, du\, dv = \\ &= \int\limits_{-\infty}^{\infty} \rho_h(\tau, T)\, [B_{z_1z}(\tau) - \overline{z_1(t)}\;\overline{z(t)}]\, d\tau, \end{aligned}$$

(4.3.95)

in which we have performed replacement of variables u - v = τ.

Considering further the narrow band width of input action y(t) and equations (4.2.55), (4.2.30), (4.3.87), (4.3.90), (4.3.83'), we produce

$$B_{uu_1}(T) = \sigma_{y_1}^2 \sigma_y^2 \cdot \frac{2}{T} \int\limits_0^{\infty} R_{y_{10}y_1}^2(\tau)\, d\tau = \sigma_y^4 \frac{2}{T} \int\limits_0^{\infty} r_{y_{10}y_0}^2(\tau)\, d\tau. \tag{4.3.96}$$

Performing integration, we find the final result [see (A.2.13)]:

$$B_{uu_1}(T) = \frac{2\pi\omega_c^2\sigma_y^4}{\Delta\omega_y T}\left[1 + \frac{\Delta\omega_y^2}{12\,\omega_c^2}\right]. \tag{4.3.97}$$

In the following, assuming in our case $\Delta\omega_y \ll \omega_c$, we will approximately consider signal u(T) [or u_1(T)] random

$$u(T) = \frac{1}{T} \int\limits_0^T y^2(t)\, dt$$

distributed normally, with parameters $\overline{u(T)}$, $\sigma_u^2(T)$ (or $\overline{u_1(T)}$, $\sigma_{u_1}^2(T)$) where $\Delta f_y T \gg 1$ (Slepian, 1958).

The distribution function $w_2(y_1,\ldots,y_n)$ of a certain set of transformed random quantities $u_1,\ldots,u_n$ can be generally found from the known joint probability density $w_1(x_1,\ldots,x_n)$ of the initial random quantities $v_1,\ldots,v_n$ and Jacobian of the transform upon transition from variables $x_1,\ldots,x_n$ to variables $y_1,\ldots,y_n$. In many cases, with a large number of variables or with rather /138 complex functional dependences between initial and transformed random quantities, the usage of the direct method represents considerable computational difficulties. Therefore, we will use an approximate method for evaluation of the distribution and its parameters as applicable to the quantity $n_T(0)$, which is random over observation time T and is a function of the random argument $u_1(T)$ and $u(T)$:

$$n_T(0) = \frac{1}{\pi}\left\{\frac{u_1(T)}{u(T)}\right\}^{\frac{1}{2}}. \qquad (4.3.98)$$

Suppose we have the function $v = \phi(u_1,\ldots,u_n)$ of random quantities $u_1,\ldots,u_n$. In this case, when the masses of the distribution probabilities $(u_1,\ldots,u_n)$ are concentrated mainly in a small area around point A $(\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_n)$ -- the common center, we can replace function $\phi(u_1,\ldots,u_n)$ with the linear terms of its expansion into a Taylor series about point A with a certain degree of approximation (Smirnov, Dunin-Barkovskiy, 1965):

$$\varphi(u_1\ldots u_n) \approx \varphi(\bar{u}_1\bar{u}_2\ldots\bar{u}_n) + \sum_{i=1}^{n}\frac{\partial\varphi}{\partial u_i}(u_i-\bar{u}_i). \qquad (4.3.99)$$

In the partial derivative $\partial\phi/\partial u_i$, argument $(u_1,u_2,\ldots,u_n)$ is replaced by $(\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_n)$ respectively:

$$\varphi'_{u_i}(\bar{u}_1\bar{u}_2\ldots\bar{u}_n) = \frac{\partial\varphi}{\partial u_i}.$$

Thus, if the distribution density of arguments $u_1,\ldots,u_n$ is normal, we can assume approximately that the distribution density of the probabilities of function $\phi(u_1,u_2,\ldots,u_n)$, a linear combination of the normally distributed random quantities (4.3.99), is also normal. The parameters of this distribution are determined on the basis of the theorems on the mean value and dispersion of a linear function of random quantities:

$$\bar{v} = \overline{\varphi(u_1 u_2 \dots u_n)} \approx (\bar{u}_1 \bar{u}_2 \dots \bar{u}_n); \quad (4.3.100)$$

$$\sigma_v^2 = \sum_i^n \sum_j^n \frac{\partial\varphi}{\partial u_i} \cdot \frac{\partial\varphi}{\partial u_j} \overline{(u_j - \bar{u}_j)(u_i - \bar{u}_i)}, \quad (4.3.101)$$

where $\overline{(u_i - \bar{u}_i)(u_j - \bar{u}_j)} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j = B_{ij}$ is the mixed second order moment of quantities u_j, u_i, where $B_{ij} = B_{ji}$ [see (4.3.86), (4.3.95)]. For an estimate of the parameters produced, we can use the first three terms of expansion $\phi(u_1, u_2, \dots, u_n)$ into a Taylor series about point A:

$$\varphi(u_1 \dots u_n) \approx \varphi(\bar{u}_1 \bar{u}_2 \dots \bar{u}_n) + \sum_{i=1} \frac{\partial\varphi}{\partial u_i}(u_i - \bar{u}_i) + \\ + \frac{1}{2} \sum_i^n \sum_j^n \frac{\partial^2\varphi}{\partial u_i \partial u_j}(u_i - \bar{u}_i)(u_j - \bar{u}_j).$$

Let us introduce for simplification of our inscription the centered quantities $U_j = u_j - \bar{u}_j$, where (for normally distributed arguments $u_1 \dots u_n$) /139

$$\bar{v} = \overline{\varphi(u_1 u_2 \dots u_n)} = \varphi(\bar{u}_1 \bar{u}_2 \dots \bar{u}_n) + \frac{1}{2} \sum_i^n \sum_j^n \frac{\partial^2\varphi}{\partial u_i \partial u_j} \overline{U_j U_i}; \quad (4.3.102)$$

$$\sigma_v^2 = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial\varphi}{\partial u_i} \cdot \frac{\partial\varphi}{\partial u_j} \overline{U_i U_j} + \frac{1}{4} \sum_i^n \sum_j^n \sum_k^n \sum_l^n \frac{\partial^2\varphi}{\partial u_i \partial u_j} \cdot \frac{\partial^2\varphi}{\partial u_k \partial u_l} \overline{U_i U_j U_k U_l}, \quad (4.3.103)$$

where

$$\overline{U_i U_j U_k} = 0; \quad \overline{U_i U_j U_k U_l} = B_{ij} B_{kl} + B_{jk} B_{il} + B_{jl} B_{ik}.$$

Correspondingly, the relative errors in determination of mathematical expectation $\delta(\bar{v})$ and dispersion $\delta(\sigma_v^2)$ due to linearization of the function $\phi(u_1 \dots u_n)$ can be estimated using the formulas

$$\delta(\bar{v}) = \left| \frac{\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\varphi}{\partial\bar{u}_i\partial\bar{u}_j}B_{ij}}{\varphi(\bar{u}_1\bar{u}_2\ldots\bar{u}_n)+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\varphi}{\partial\bar{u}_i\partial\bar{u}_j}B_{ij}} \right|, \tag{4.3.104}$$

$$\delta(\sigma_v^2) = \left| \frac{\frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\frac{\partial^2\varphi}{\partial u_i\partial u_j}\cdot\frac{\partial^2\varphi}{\partial u_k\partial u_l}(B_{jk}B_{il}+B_{jl}B_{ik}+B_{ij}B_{kl})}{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\varphi}{\partial u_i}\cdot\frac{\partial\varphi}{\partial u_j}B_{ij}+\frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\frac{\partial^2\varphi}{\partial u_i\partial u_j}\times\frac{\partial^2\varphi}{\partial u_k\partial u_l}(B_{ij}B_{kl}+B_{jk}B_{il}+B_{jl}B_{ik})} \right|. \tag{4.3.105}$$

We will assume further that in our case $\Delta f_y T \gg 1$ (dispersions (4.3.93), (4.3.94) are rather small), we can use the expansion of function (4.3.98) into a Taylor series about point A $[u(T), u_1(T)]$, and limit ourselves to linear terms. The errors in calculations of $\bar{v}$ and σ_v^2 thus produced can be evaluated on the basis of equalities (4.3.104), (4.3.105). The mathematical expectation of the quantities $n_T(0)$ is determined from (4.3.98) on the basis of (4.3.88), (4.3.89), (4.3.100) considering inequality $\Delta\omega_y \ll \omega_c$:

$$\overline{n_T(0)} = \frac{1}{\pi}\left\{\omega_c^2 + \frac{\Delta\omega_y^2}{12}\right\}^{\frac{1}{2}} \simeq \frac{\omega_c}{\pi}\left(1 + \frac{\Delta\omega_y^2}{24\omega_c^2}\right). \tag{4.3.106}$$

The dispersion $n_T(0)$ in this case is represented in the form [see (4.3.101), (4.3.97)]: /140

$$\sigma_n^2 = \left\{\frac{\partial[n_T(0)]}{\partial u}\right\}^2\sigma_u^2 + 2\frac{\partial[n_T(0)]}{\partial u}\cdot\frac{\partial[n_T(0)]}{\partial u_1}B_{uu_1} + \left\{\frac{\partial[n_T(0)]}{\partial u_1}\right\}^2\sigma_{u_1}^2; \tag{4.3.107}$$

$$B_{jj} = B_{u_j u_j} = \sigma_{u_j}^2.$$

Let us find the coefficients of expression (4.3.107), determined by the partial derivatives of function $n_T(0)$ of random quantities u, u_1 [see (4.3.98), (4.3.99)]:

$$\left\{\frac{\partial\,[n_T\,(0)]}{\partial u}\right\}^2=\left\{-\frac{\bar{u}_1}{2\pi\bar{u}^2}\left[\frac{\bar{u}_1}{\bar{u}}\right]^{-\frac{1}{2}}\right\}^2=\frac{\bar{u}_1^2}{4\pi^2\bar{u}^4}\left[\frac{\bar{u}_1}{\bar{u}}\right]^{-1}=\frac{\left(\omega_c^2+\frac{\Delta\omega_y^2}{12}\right)}{4\pi^2\sigma_y^4};$$

$$\left\{\frac{\partial\,[n_T\,(0)]}{\partial u_1}\right\}^2=\left\{\frac{1}{2\pi\bar{u}}\left[\frac{\bar{u}_1}{\bar{u}}\right]^{-\frac{1}{2}}\right\}^2=\frac{1}{4\pi^2\bar{u}^2}\left[\frac{\bar{u}_1}{\bar{u}}\right]^{-1}=\frac{1}{4\pi^2\sigma_y^4}\left(\omega_c^2+\frac{\Delta\omega_y^2}{12}\right)^{-1};$$

$$\frac{\partial\,[n_T\,(0)]}{\partial u}\cdot\frac{\partial\,[n_T\,(0)]}{\partial u_1}=-\frac{\bar{u}_1}{4\pi^2\bar{u}^3}\left[\frac{\bar{u}_1}{\bar{u}}\right]^{-1}=-\frac{1}{4\pi^2\sigma_y^4}.$$

Consequently [see (4.3.107)],

$$\begin{aligned}\sigma_n^2 &= \frac{\left(\omega_c^2+\frac{\Delta\omega_y^2}{12}\right)}{4\pi^2\sigma_y^4}\cdot\frac{2\sigma_y^4\pi}{\Delta\omega_y T}-\frac{2}{4\pi^2\sigma_y^4}\cdot\frac{2\pi\omega_c^2\sigma_y^4}{\Delta\omega_y T}\left[1+\frac{\Delta\omega_y^2}{12}\right]+\\ &+\frac{\left(\omega_c^2+\frac{\Delta\omega_y^2}{12}\right)^{-1}}{4\pi^2\sigma_y^4}\cdot\frac{2\pi\sigma_y^4\omega_c^4\left[1+\frac{\Delta\omega_y^2}{2\omega_c^2}+\frac{\Delta\omega_y^4}{80\,\omega_c^4}\right]}{\Delta\omega_y T}=\\ &=\frac{\omega_c^2\left(\frac{\Delta\omega_y^2}{3\omega_c^2}+\frac{\Delta\omega_y^4}{180\,\omega_c^4}\right)}{2\pi\Delta\omega_y T\left(1+\frac{\Delta\omega_y^2}{12\,\omega_c^2}\right)}=\frac{\Delta\omega_y^2\left(\frac{1}{3}+\frac{\Delta\omega_y^2}{180\,\omega_c^2}\right)}{2\pi\Delta\omega_y T\left(1+\frac{\Delta\omega_y^2}{12\,\omega_c^2}\right)}.\end{aligned}\tag{4.3.108}$$

If we ignore the term $\Delta\omega_y^2/180\omega_c^2$ for narrow band processes, formula (4.3.108) can be represented in the form

$$\sigma_n^2=\frac{\omega_c^2\left(\frac{\Delta\omega_y^2}{\omega_c^2}\right)}{6\pi\Delta\omega_y T\left(1+\frac{\Delta\omega_y^2}{12\,\omega_c^2}\right)}=\frac{\Delta\omega_y^2}{6\pi\Delta\omega_y T\left(1+\frac{\Delta\omega_y^2}{12\,\omega_c^2}\right)}.\tag{4.3.109}$$

Thus, in the case at hand, expansion of function $n_T(0)$ of random quantities $u(T)$ and $u_1(T)$ into a Taylor series, the mathematical expectation $n_T(0)$ corresponds with expression (4.3.75) for the mean number of intersections of the zero level per unit time (unbiased estimate; see Chapter 4, Introduction), while dispersion σ_n^2 (4.3.109) decreases in proportion to observation time T (consistent estimate; see Chapter 5, Introduction). The distribution of $n_T(0)$ is assumed approximately normal on the basis of the above. /141

The errors in calculation of mathematical expectation (4.3.104) and dispersion (4.3.105) are calculated in Appendix 3 [see (A.3.4) and (A.3.7)], and are determined by the expressions (estimate of accuracy of linearization method)

$$\delta[\overline{n_T(0)}] \approx \frac{\pi}{12\Delta\omega_y T\frac{\omega_c^2}{\Delta\omega_y^2} - \pi}; \tag{4.3.110}$$

$$\delta(\sigma_n^2) = \frac{\pi}{\frac{4}{3}\Delta\omega_y T + \pi}. \tag{4.3.111}$$

In the general case, the dispersion of the mean number of intersections of the zero level per unit time will depend not only on the energy band, but on the form of the spectrum of the process being analyzed. Investigation of the behavior of the error (4.3.107) with arbitrary spectral functions is complex, and even for concrete situations represents computational difficulties. However, on the basis of several works (Tikhonov, 1966; Klokov, 1964), we can assume that for sufficiently high values of time T, the error in calculation of the mean number of intersections of the zero level by process y(t) with various spectral forms (Klokov, 1964) will not differ significantly, and that in this case the expression (4.3.108) can be accepted as an estimate.

The results produced were used in a special series of experiments -- 20 experiments (ten test subjects, selected as a function of clarity of α rhythm), where the degree of emotional stress was determined either by the skin galvanic reaction or by the introduction of a game situation (see Chapter 2). The difference between the significant and any of the insignificant signals in a series of six stimuli was investigated, where when the former (significant signal) was used the dispersion of the mean number of intersections of the zero level was determined approximately by the pass band of the spectral analyzer filter (separation of α rhythm) with a rather rectangular (see Figure 55) frequency characteristic (limit case), while for the latter the band was estimated as about 1 Hz (time of appearance of each signal T did not vary during the process of the experiment). In comparing the two values $\overline{n_{T,i}(0)}$ and $\overline{n_{T,k}(0)}$, where the i-th situation corresponded to /142
the effect of the significant stimulus, the normal distribution was used for quantity q:

$$q = \frac{D}{\sigma_D},$$

where D is the difference between the i-th and k-th estimates of the mean number of intersections of the zero line by process y(t), each of which was additionally averaged for 6-10 realizations of the signal (see Chapter 2); σ_D^2 is the sum of dispersions corresponding to the i-th and k-th situations.

The processing of the material produced showed that in 13 experiments the significant signal was separated unambiguously: $(\overline{n_{T,i}(0)} > \overline{n_{T,k}(0)}), q > t_\beta$; while in two other experiments, a group of two signals was defined, containing the significant stimulus. In remaining experiments, the reliability of differentiation was low. For the normal distribution, parameter $t_\beta = 1.96$ corresponds to the probability $1 - 2\beta = 0.95$ of determining random quantity q in the interval ±1.96 in testing the zero hypothesis (Van der Waarden, 1960)

$$\overline{n_{T,i}(0)} - \overline{n_{T,k}(0)} = 0.$$

The preliminary results produced, probably, could be considered satisfactory, considering the degree of approximation of the estimates and the game nature of most of the situations investigated , in which the test subjects could vary their attitude toward the incoming signal stimuli within certain limits.

However, the data produced do confirm the fact that for a sufficiently reliable evaluation of the state of the test subject, it is desirable to have a variety of physiological signals.

CHAPTER 5

FREQUENCY OF HEART CONTRACTIONS

ABSTRACT. The auto-correlation functions of cardiac rhythm are analyzed, and it is determined that after a steeply dropping initial sector, this function fluctuates in a complex manner about the abscissa as pulse frequency varies with respiration and emotional state. An electrical analogue for vagus nerve inhibition is presented. Experimental modeling of the influence of emotional stress on cardiac contraction frequency by digital computer is described.

Introduction

The cardiac rhythm is frequently used in practice to estimate the degree of emotional stress of the test subject. The nature of the change of heart contraction intervals T_{R-R} is rather complex even when the operator is at rest. On the one hand, slow variations in the value of T_{R-R} may occur with a period on the order of one minute or more, while on the other hand -- rapid changes in T_{R-R} may occur, resulting primarily due to the influence of respiration on the cardiac contraction frequency (respiratory regulation). /143

Frequently, the operator state is estimated using a sampling mean interval $T_{(R-R),T}$ or the mean frequency of the cardiac rhythm $f_{c,T}$:

$$f_{c,T} = [T_{(R-R),T}]^{-1} = \left[\frac{1}{N}\sum_{i=1}^{N} T_{(R-R),i}\right]^{-1};$$

$$T_{(R-R),T} = \frac{1}{N}\sum_{i=1}^{N} T_{(R-R),i},$$

determined over a certain time sector T, where $T = \sum_{i=1}^{N} T_{(R-R),i}$. The averaging time T is selected on the basis of the experimental conditions and the required accuracy for estimation of $f_{c,T}$ (or $T_{(R-R),T}$), and in many cases is characterized by quantities on the order of one minute and longer. The influence of respiratory regulation on the frequency of heart contractions is smoothed to some extent due to averaging for several respiratory cycles. If we assume in this case that it is possible to use the results of the theory of stable random processes, we can analyze problems of convergence of the estimates $f_{c,T}$ or $T_{(R-R),T}$.

Let us determine the sampling average x_N as the arithmetic mean of N random quantities x_i:

$$x_N = \frac{1}{N}\sum_{i=1}^{N} x_i,$$

where x_i is the value of the sampling function x(t) of the probabilistic process, which is stable in the broad sense, at the i-th moment in time, or the i-th value of the sample of random quantity x. We know that the mathematical expectation of the sampling mean x_N is equal to /144

$$\bar{x}_N = \frac{1}{N}\sum_{i=1}^{N} \bar{x}_i = \bar{x}$$

the mean value of the probabilistic process being studied (or the random quantity). The statistics for which the mathematical expectation is equal to the quantity estimated is called an unbiased estimate; therefore, the sampling mean is an unbiased estimate for the mean value.

On the basis of the equalities presented, the dispersion of the sampling mean $\sigma^2(x_N)$ can be represented in the form

$$\sigma^2(x_N) = \overline{(x_N - \bar{x})(x_N - \bar{x})} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} \overline{[x_i x_j]} - (\bar{x})^2.$$

For our subsequent discussions, we require information on the correlation between the i-th and j-th samples. Let us assume that the readings are correlated rather strongly and that we can consider approximately

$$\overline{x_j x_i} = (\bar{x}^2) = \sigma_x^2 + (\bar{x})^2$$

for all values i,j. In this case

$$\sigma^2(x_N) = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\overline{x_i x_j} - (\bar{x})^2\right] = \sigma_x^2.$$

Consequently, if the samples are strongly correlated, regardless of their number the dispersion of the sampling mean is approximately equal to the dispersion of the probabilistic process being analyzed (or a random quantity). In this case, a single measurement gives an equally good (or bad) estimate of the mean as any number of measurements.

With uncorrelated readings

$$\overline{x_j x_i} = \bar{x}_i \bar{x}_j = (\bar{x})^2, \qquad i \neq j;$$
$$\overline{x_i x_j} = \sigma_x^2 + (\bar{x})^2, \qquad i = j.$$

Therefore

$$\sigma^2(x_N) = \frac{\sigma_x^2}{N}.$$

It follows from this equation that with unlimited increase in the number of measurements N, the dispersion of the sampling mean approaches zero: /145

$$\lim_{N \to \infty} \sigma^2(x_N) = 0.$$

This latter represents consistency of estimate x_N.

In order to make a judgment concerning the degree and nature of correlation couplings between the i-th and j-th samples, we must know the correlation (or normalized correlation) function. As an example, Figure 59 shows the two normalized correlation functions $R_T(n)$ of intervals of heart contractions T_{R-R} (solid line and dotted line), the values of which $[R_T(n)]$ were evaluated using the formulae below (N = 450), from experiments with the operator in the state of rest (eyes open):

$$R_T(n) = \frac{N \sum_{i=1}^{N-n} [T_{(R-R),i} - \bar{T}_{(R-R),T}][T_{(R-R),(i+n)} - \bar{T}_{(R-R),T}]}{(N-n) \sum_{i=1}^{N} [T_{(R-R),i} - \bar{T}_{(R-R),T}]^2}.$$

The value of n characterizes the shift in function $R_T(n)$ in the number of intervals T_{R-R}: $n = 1,2,3...$ and for stable processes $n = i - j$.

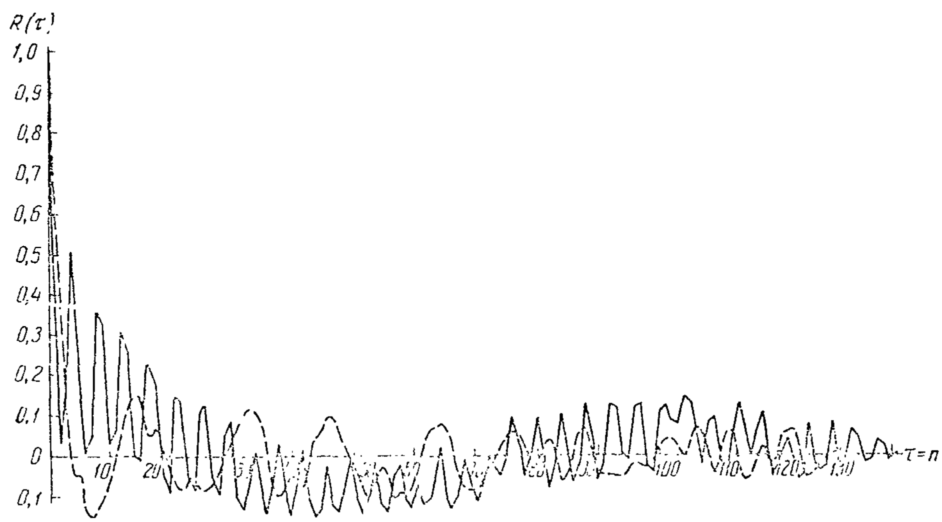


Figure 59

It follows from analysis of Figure 59 that the autocorrelation functions have a rather steeply dropping initial sector and subsequently, attenuating, fluctuate about the abscissa, with rapid oscillations superimposed on slower oscillations. This characterizes the structure of the process investigated, which was mentioned above, although in the case of the function $R_T(n)$, shown /146
on Figure 59 by the dotted curve, we can hardly explain the rapid fluctuations only by the influence of respiratory regulation.

According to the χ^2 criterion, the hypothesis of normal distribution of T_{R-R} for states differing slightly from the rest state, which can be judged from the change in the mean interval $\overline{\Delta T}_{R-R}$ of heart contractions $\overline{\Delta T}_{R-R} \leqslant (0,1-0,15)\overline{T}_{R-R}$, in most cases does not contradict the experimental data.

The nature of changes in the heart rhythm (EKG_2) and respiration (R_2) in the realizations (N = 450-500) for which calculations were performed is shown on Figure 60.

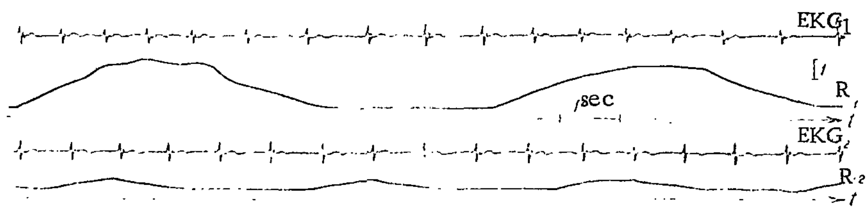


Figure 60

In a number of cases in order to produce supplementary information usually lost with long averaging times, it is desirable to estimate $\overline{T}_{R-R}$ using short time sectors $T \geqslant (3\text{-}10)$ sec. This problem may arise with continuous tracking of the operator state where the operator is under the influence of signals (stimuli) varying in length and nature. The averaging time in this case is determined on the one hand by the minimum time of action of the signal, and on the other hand by the nature of the reaction of the test subject to various stimuli.

Experience shows that one of the principal factors influencing errors in the determination of $\overline{T}_{R-R}$ (or $\overline{f}_c$) for short realizations is the respiratory regulation. Let us analyze the graphs presented on Figure 60. Quiet, shallow breathing (R_2) causes relatively slight deviations in the heart contraction intervals (EKG_2), while an increase in the depth and change in the nature of respiration (R_1) may cause noticeable fluctuations in the value of T_{R-R} (EKG_1), sometimes amounting to one-half of the mean value (Clynes, 1960).

It was shown above that if the process whose values are the interval T_{R-R} can be assumed stable (see Figure 60, EKG_2, R_2), the dispersion of the sampling mean $\sigma^2(T_{(R-R),T})$, evaluated through short time interval T (samples considered correlated, see Figure 59) is approximately equal to the dispersion of the process itself $\sigma^2(T_{R-R})$. If we assume in the general case a /147
rather arbitrary change in respiration with various operator states, as we have already seen, at least the dispersion of heart contraction intervals T_{R-R} will depend on time and will be determined by the nature of respiratory regulation. In this case, the concepts and methods from the theory of stable processes can hardly be applied.

High emotional stress usually causes significant changes in quantity $\overline{T}_{R-R}$ in comparison with the rest state:

$$\overline{\Delta T}_{R-R} \geqslant (0.3 \div 0.5)\, \overline{T}_{R-R}.$$

Here, the interference resulting from the influence of respiration in most cases can hardly have essential significance. There is principal interest in the situation of comparatively slight emotional stress (or other states), for which the errors arising resulting from respiratory regulation in the estimation of $\overline{T}_{R-R}$ for short realizations are rather great and may be comparable to the change in the mean interval of heart contractions $\overline{\Delta T}_{R-R}$ upon transition from one state to another.

Experiments show that the error in measurement of $\overline{T}_{R-R}$ over short time sector T increases with increasing depth and rate of change of respiratory waves (see Figure 60) and reach 10-15% of the mean value in the state of operator's rest, which is comparable to the changes in the quantity $\overline{\Delta T}_{R-R}$ (signal) with slight emotional stress ($\overline{\Delta T}_{R-R} \leqslant (0.1\text{-}0.15)\overline{T}_{R-R}$).

These discussions show that in order to decrease this error it is necessary somehow to compensate for the influence of respiratory regulation on fluctuations in cardiac rhythm. This requires analysis of physiological information concerning the operation of the heartbeat frequency regulation system.

§5.1. Physiological Regulation of Heartbeat Frequency

The complex, multi-loop regulation of the cardiovascular system is under the direct influence of emotional factors and is an autonomic function of the nervous system, involving the participation of both sympathetic and parasympathetic sectors. Like the other functional systems (in this case we have in mind the internal motor system), the cardiovascular system has proprioceptors, informing the brain of the movements and principal physical conditions of its operation. The extension receptors are contained in the carotid sinus (a special nerve junction in the area of the arc of the aorta /148 and the branching of the carotid arteries) and in the aorta, as well as other pressure-sensitive areas of the cardiovascular system. In contrast to the system of skeletal muscles, the cardiovascular system automatically repeats its motor acts, requiring no conscious effort. This rhythmic activity, necessary for continued existence of the organism, is apparently provided by nervous-muscular properties similar to those causing periodic generation of electrical impulses in the neurons and muscle fibers. A small nervous-muscular structure in the heart -- the "master oscillator" -- is the source of the repeated excitation, but the frequency is adapted to the requirements of periodic muscular contractions. The excitation of the human heart occurs approximately once per second and does not correspond to the frequency of neuron activity, which varies between 50 and 100 Hz. The pulses of the "oscillator" propagate toward the atria and further along the central nerve fibers of the heart to the ventricles, causing contraction of the cardiac muscle.

The frequency of heart contractions determined by the "master oscillator," is subjected to regulating nervous influences. Pulses arriving on the fibers of the vagus nerve decrease the frequency of the "master oscillator of the heart"; this influence is looked upon as a parasympathetic process which is achieved through acetylcholin. The influences of the pulses of the sympathetic fibers appear as an acceleration of the work of the heart, which has an important role to play during hard work, emotional stress, etc. Even under normal conditions in the state of rest there is a certain tonus of the vagus nerve, so that the frequency of heart contractions is lower than the frequency of natural oscillations of the "cardiac oscillator," which would be observed without the inhibiting influence of the vagus nerve. A change in this delaying influence of the vagus results from various reflector paths. The pressure in the area of the carotid sinus and receptors of the arch of the aorta, extension of the receptor zones around the right atrium and lungs, as well as in the muscles of the thorax result in the presence of constant reflector influences which directly change the summary influence of the vagus on the work of the heart. The vagus effect may be increased by other reflexes which have no influence under normal conditions, for example the reflex arising with strong pressure on the stomach or pressure on the eye.

As a result of the regulatory influence of normal reflector mechanisms, respiration has a noticeable influence on the frequency of heart contractions. This effect is quite different than the influence of respiration on the heart resulting from the comparatively slow changes in the content of /149
oxygen and carbon dioxide in the blood, transmitted through the chemoreceptors.

The influence of respiration on the work of the heart is achieved through vagus regulation; this can be shown by injecting atropine into a test subject, thus eliminating changes in pulse frequency related to respiration (Clynes, 1960. In the general case, as follows from the above, the frequency of heart contractions is determined simultaneously by influences on the "heart master oscillator" of the parasympathetic (retarding) and sympathetic (accelerating) segments of the human nervous system.

§5.2. Model of Control of Frequency of "Cardiac Master Oscillator"

The physiological information on the work of the cardiovascular system allows us to represent the process of control of the frequency of the "cardiac master oscillator" as the result of influence on its parameters of regulating signals arriving through the sympathetic and parasympathetic nerve fibers. Thus, the effect of acetylcholin (vagus regulation) can be looked upon as a factor having a parametric influence in the circuit, determining the oscillating mode of the "oscillator."

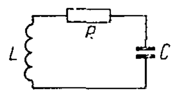


Figure 61

The observations of Otsuka (1958) have shown that the initial action potentials of the "cardiac master oscillator," determining its excitation frequency, can be approximately represented as sine waves. The parametric control of the frequency of a sine-wave oscillator can be described by the Hill equation.

Let us analyze as an example an RLC circuit (Figure 61) in which the resistance, inductance and capacitance change with time:

$$R = R_0 + r(t); \quad L = L_0 + l(t); \quad S = 1/C = S_0 + s(t),$$

where r(t), l(t), s(t) are variable terms, while R_0, L_0, S_0 are constants. Also, we will consider that the constant terms are greater than the corresponding amplitudes of the variable terms:

$$S_0 > |s(t)|; \; L_0 > |l(t)|; \; R_0 > |r(t)|.$$

The differential equation of free oscillations in this circuit will be /150

$$\frac{d^2Q}{dt^2} + \left(R + \frac{dL}{dt}\right)\frac{1}{L} \cdot \frac{dQ}{dt} + \frac{S}{L} Q = 0,$$

if we take as the initial function the charge of condenser Q where u_L = d/dt[L(dQ/dt)] is the voltage across the coil; u_R = dQ/dt(R) is the voltage across the resistor; u_C = Q/C is the voltage across the condenser. We produce further

$$\left(R + \frac{dL}{dt}\right)\frac{1}{L} = A(t); \quad \frac{S}{L} = B(t) \quad \text{и} \quad Q(t) = x(t)\, e^{-\frac{1}{2}\int A(t)dt}.$$

Then, the equation for free oscillations of the condenser charge will take on the form

$$\frac{d^2x}{dt^2} + \left[B(t) - \frac{1}{2} \cdot \frac{dA}{dt} - \frac{1}{4} A^2(t)\right] x = 0.$$

Introducing the symbols

$$B(t) - \frac{1}{2} \cdot \frac{dA}{dt} - \frac{1}{4} A^2(t) = a - u(t),$$

we produce the differential equation

$$\frac{d^2x}{dt^2} + [a - u(t)]\, x = 0, \qquad (5.2.1)$$

which establishes the dependence of the oscillating mode in the circuit on changes in its parameters R, L, C and is called the Hill equation. In the general case, equations like (5.2.1) arise in describing systems whose parameters change under the influence of external excitation. As applicable to the problem here being analyzed concerning the influence of respiration on the frequency of heart contractions, oscillating function x(t) describes the change in the initial action potential of the "cardiac master oscillator," the moments of excitation of which will correspond in this case to maximum values of x(t), determining the breakthrough of the physiological membrane. Parameter a determines the mean cardiac rhythm frequency, while function u(t) characterizes the influence of changes in vagus inhibition resulting from respiratory regulation on the "cardiac oscillator." An equation such as (5.2.1) was used in the work of Clynes (1960) in a study of changes in the interval T_{R-R} arising as a result of respiration in a test subject at rest in the standing position.

In the case $u(t) \equiv 0$, the solution of equation (5.2.1) can be represented in the form of a periodic function /151

$$x = C \cos(\sqrt{a}\, t + \varphi),$$

where C, ϕ are the initial amplitude and phase of the oscillations, the frequency of which f_{c0} is described by the equation

$$f_{c0} = \frac{1}{2\pi} \sqrt{a}. \qquad (5.2.2)$$

We note that as applicable to this parameter, the case $u(t) \equiv 0$ corresponds to constant parameters R_0, L_0, C_0 of the oscillating circuit, which characterize the stable oscillating mode of the oscillator.

It was stated above that for a certain state of the operator, the frequency of heart contractions f_c is determined by the joint influence of the sympathetic S and parasympathetic U segments of the nervous system on the frequency of natural oscillations of the "cardiac master oscillator" f_0

$$f_c = F(U, S, f_0). \tag{5.2.3}$$

We will assume further that the i-th state of the test subject over a rather short observation time is stable or changes little, so that the resulting deviation in cardiac contraction intervals is slight in comparison with its i-th mean value and the dispersion of quantities T_{R-R} produced, as experiments have shown, primarily as a result of respiratory regulation with any operator state (considering the operator states analyzed in this chapter). If we exclude the parasympathetic effect, determined by the influence of respiration $[U_i = U_{i0}]$, the mean value of frequency $f_{c,i}$ during time of analysis T can be represented in the form [see (5.2.2)]

$$F_{i,T}(U_{i0}, S_i; f_0) \simeq \frac{1}{2\pi} \sqrt{a_i} = \text{const.} \tag{5.2.4}$$

In this case, parameter a characterizes the result of the combined action of sympathetic (accelerating) and parasympathetic (retarding) segments of the nervous system on the "cardiac master oscillator."

On the basis of the above, we can use the resulting value of a for an evaluation of the state of rest or weak emotional stress in the test subject, this value being determined from equation (5.2.1), with compensation for the effects of respiratory regulation on the heart frequency.

§5.3. Transfer Characteristic of "Respiration-Vagus Inhibition"

In defining function u(t), the problem arose of studying the transient processes produced when respiration was performed as follows: "inhale -- hold -- exhale -- hold" and determination of the transfer characteristic in the link "respiration -- vagus inhibition," which would allow these transient processes to be reproduced in the solution of equation (5.2.1). /152

Experiments were performed with test subjects in the sitting position with various depths of respiration both in the state of rest and in the state of comparatively weak emotional stress, which was determined from changes in pulse frequency.

Processing of the experimental material for the states investigated showed that the cardiac rhythm with staged respiration after the inhalation (or exhalation) sector shows a phase of increased frequency with a subsequent phase of decreased frequency and further tendency toward return to the initial frequency[1]. The change in intervals of cardiac contraction T_{R-R} during inhalation is generally greater than during exhalation, while in turn the duration of each phase of the cardiac rhythm is increased upon exhalation in comparison with inhalation.

In many cases, the transient processes, beginning with the phase of decrease in quantity T_{R-R}, subsequently have the nature of damping oscillations. This can be determined, in particular, by the voluntary muscular regulation or comparatively great physical stress of the test subject during respiration (for example, if the breath is held for some time t_3 > 15-20 sec either inhaled or exhaled).

Figure 62 illustrates the specific features of transient phenomena in the cardiac rhythm[2]. Each interval, determined by the difference in time $t_j - t_i$ ($t_j > t_i$) is laid out at moment t_j along the ordinate. The graphs of $T_{R-R(1)}$ and $T_{R-R(2)}$ characterize the two-phase transient processes of the cardiac rhythm during inhalation R_1 and exhalation R_2, while the graph of $T_{R-R(3)}$ has the nature of damping oscillations during inhalation R_3. The mean values of intervals T_{R-R}, calculated preliminarily for rather large samples ($T_{(R-R),T} \approx \overline{T}_{R-R}$) are shown on the figure by the dotted lines.

The experiments show that various combinations are possible: the transient process during inhalation has the nature of damping oscillations, while during exhalation it is two-phased, or vice versa, etc. The changes in cardiac rhythm during staged respiration were observed in most cases in the investigated states and depended little on the depth of respiration. /153

Function u(t), which provides good correspondence to actual transient processes (staged respiration) in the situations of interest to us when substituted into equation (5.2.1) (for more detail see §5.4), was produced by transforming analog signal L(t) using a certain rule, representing the change in the area of the thorax during respiration R. The transforming functions for inhalation (dL/dt > 0) and exhalation (dL/dt < 0) were different in the general case. Each of them was composed of a linear chain with subsequent nonlinear weighing of the result as a function of its sign. In operator

[1] The identical nature of the transient processes during inhalation and exhalation can be explained by the influence of the receptors, sensitive to the direction of the perturbation.
[2] In this and subsequent illustrations the R axis shows the dimensionless units, while the mean amplitude R during quiet respiration was on the order of 0.4-0.6 in these experiments.

form, the sequence of these operations, for example for inhalation, can be described as follows: /154

$$
\begin{aligned}
&1.\ v_{in}(p) = L(p) R_{in}(p),\ L' = \frac{dL}{dt} > 0; \\
&2.\ u_{in}(p) = \begin{cases} k_1 v_{in}(p), & v_{in}(t) > 0; \\ k_2 v_{in}(p), & v_{in}(t) < 0, \end{cases}
\end{aligned} \tag{5.3.5}
$$

where $R_{in}(p)$ is the transfer function of the linear chain for inhalation; $u_{in}(p)$ is the result of weighing of L(p) by the inhalation transforming function (change in vagus inhibition); k_1 and k_2 are coefficients ($k_1 > 0$; $k_2 > 0$);

$$v_{in}(p) \doteq v_{in}(t);\ u_{in}(p) \doteq u_{in}(t);\ L(p) \doteq L(t).$$

Similar transformations were performed for exhalation, but only with new values of the weight coefficient with respect to the sign k_3, k_4 and another transfer characteristic of the linear chain -- $R_{ex}(p)$. If u(p) and L(p) are the Carson-Heaviside transformations,

$$\varphi(p) = p\int_0^\infty f(t)\, e^{-pt}\, dt$$

for the respiration process L(t) and the change in vagus inhibition u(t), the transfer characteristics $R_{in}(p)$ and $R_{ex}(p)$ can be written in the form

$$R_{in}(p) = \frac{k' p^2}{p^2 + 2\alpha p + \omega_0^2},\quad \frac{dL}{dt} > 0; \tag{5.3.6}$$

$$R_{ex}(p) = \frac{k'' p^2}{(p^2 + 2\alpha p + \omega_0^2)(p + \gamma)},\quad \frac{dL}{dt} < 0, \tag{5.3.7}$$

where α, γ, ω_0 are constants, determined by the nature of the transient process, while k' and k" are the sensitivity coefficients of the linear circuits.

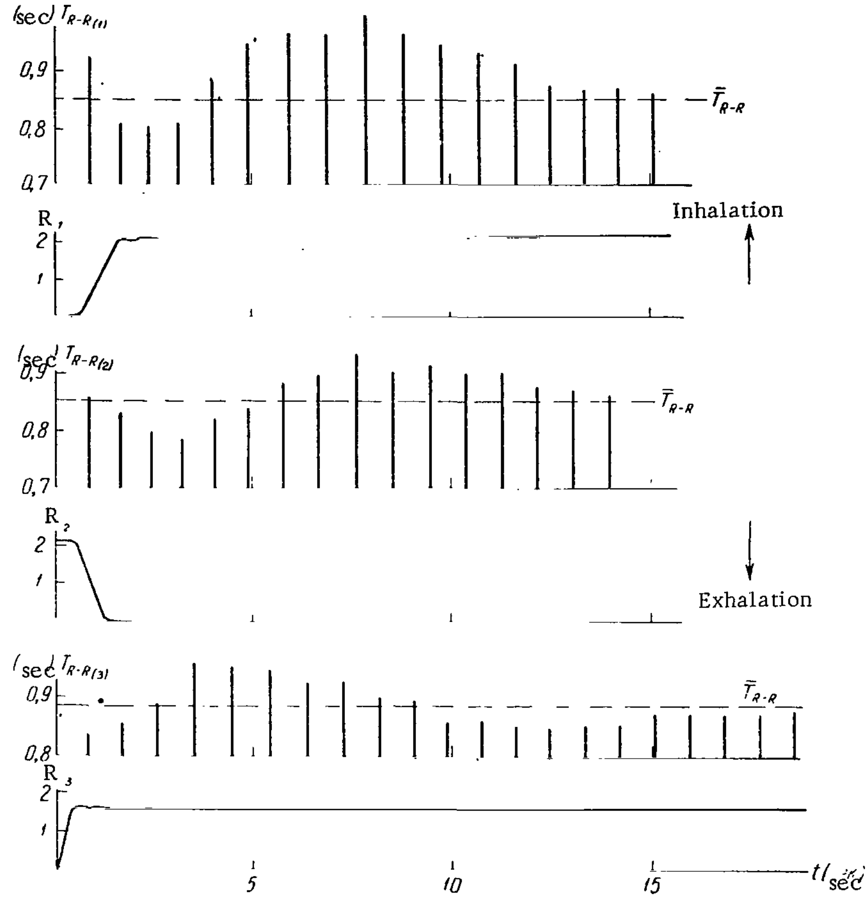


Figure 62

Transfer functions (5.3.6) and (5.3.7) can be produced in principle by successive conjunction of the disconnected RLC circuit and integrating circuit R_1C_1. Output signal v(t) is taken from condenser C_1, while for $L' > 0$, resistor R_1 is connected, and when $L' < 0$ it is shorted. The change in the sign of the transfer characteristic of this circuit should be performed simultaneously with commutation of resistor R_1. When long time constants must be produced, special operational amplifiers with feedback can be used.

As follows from equations (5.3.6) and (5.3.7), the transfer function of the circuit for exhalation differs from the corresponding function for inhalation in the addition of the RC integrating circuit, connected through a decoupling element, which slightly retards the phases of acceleration and deceleration of the cardiac rhythm in comparison with inhalation (see /155

Figure 62). In order to find the transient characteristic h(t), determined by linear transformation circuits L(t), we must solve the equation

$$h(t) = R(p)1. \tag{5.3.8}$$

As a result, we produce for inhalation, for example,

$$\alpha^2 > \omega_0^2;\ h_{\text{in}}(t) = \frac{1}{m-n}\left(ne^{-nt} - me^{-mt}\right), \tag{5.3.9}$$

$$\alpha^2 = \omega_0^2;\ h_{\text{in}}(t) = e^{-\alpha t}(1 - \alpha t); \tag{5.3.10}$$

$$\alpha < \omega_0^2;\ h_{\text{in}}(t) = -\frac{\omega_0}{\omega} e^{-\alpha t} \sin(\omega t - \theta). \tag{5.3.11}$$

Here (-n);(-m) are the roots of the equation $p^2 + 2\alpha p + \omega_0^2 = 0$:

$$\omega = \sqrt{\omega_0^2 - \alpha^2};\ \tan\theta = \frac{\omega}{\alpha}.$$

Analysis of the dependencies (5.3.9), (5.3.10) and (5.3.11) shows that the transient process where $\alpha^2 \geqslant \omega_0^2$ has two phases, while for the values $\alpha^2 < \omega_0^2$ it has the nature of damping oscillations.

We note that expression (5.3.9), determining the reaction of the linear system to a unit change in the input action, corresponds to the reaction of the transforming function to inhalation used in the work of Clynes (1960).

Figure 63 shows a series RLC circuit, the transfer function of which is described by equation (5.3.6), as well as graphs of the transient characteristics defined by formulas (5.3.9), (5.3.10) and (5.3.11).

In this case

$$\alpha = \frac{R}{2L};\ \omega_0^2 = \frac{1}{LC}.$$

The transient functions for inhalation can be found similarly. The coefficients of nonlinear weighing (by sign) of the outputs of the linear circuits -- k_1, k_2 and k_3, k_4 -- defined the relationship between changes in intervals of heart contractions for the phases of acceleration and deceleration during inhalation and exhalation and in the general case (various /156 states of the test subject and types of respiration) may depend on u_{in} and u_{ex}; however, this latter problem will not be investigated here.

Of course, we are not recommending these transient characteristics for all possible cases of the influence of respiration on the pulse frequency. However, most practical results, some of which are presented below, show that the transfer functions of weighting L(t) circuits selected on the basis of experimental modeling by digital computer and theoretical assumptions, allow us to produce rather good correspondence between the solution of equation (5.2.1) in the situations of interest to us (estimation of the value of $\overline{T}_{R-R}$ for comparatively short realizations for the operator states being investigated) and the actual fluctuations in cardiac rhythm due to respiratory regulation.

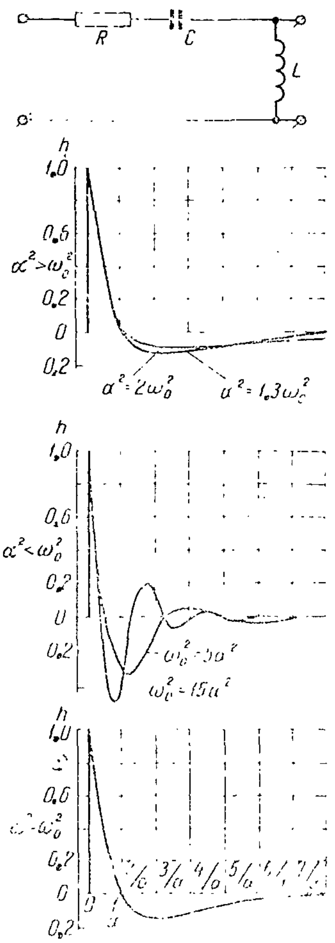


Figure 63

§5.4. Experimental Modeling by Digital Computer

The solution of equation (5.2.1) for the general case of input actions (frequency of changes and value of u(t) comparable respectively to frequency of oscillations of function x(t) and value of parameter a) was performed[1] using the M-20 digital computer. A general block diagram of the investigations is presented on Figure 64. The program for processing called for the formation of function u(t) as a result of weighing of the signal of the change in the area of the thorax during respiration L(t), which could be fixed discretely or in the form of approximating expressions. /157
The function of vagus inhibition u(t) with the parameters of transfer characteristics selected α, ω_0, γ (or n, m, ω) and the coefficients k_1, k_2 and k_3, k_4 was determined on the basis of the expressions (see §5.3):

$$v_{in}(p) = R_{in}(p)L(p), \quad \frac{dL}{dt} > 0$$
$$v_{ex}(p) = R_{ex}(p)L(p), \quad \frac{dL}{dt} < 0 \qquad (5.4.12)$$

[1] The authors express their deep gratitude to Engineer P. A. Lyuoseva for her help in performing this section of our work.

which correspond to the differential equations

$$\frac{d^2 v_{in}(t)}{dt^2} + 2\alpha \frac{dv_{in}(t)}{dt} + \omega_0^2 v_{in}(t) = -k' \frac{d^2 L(t)}{dt^2}, \ \frac{dL}{dt} > 0;$$

$$\frac{d^3 v_{ex}(t)}{dt^3} + (2\alpha + \gamma) \frac{d^2 v_{ex}(t)}{dt^2} + (\omega_0^2 + 2\alpha\gamma) \frac{dv_{ex}(t)}{dt} + \gamma\omega_0^2 v_{ex}(t) =$$
$$= k'' \frac{d^2 L(t)}{dt^2}, \ \frac{dL}{dt} < 0,$$

with subsequent weighing of the resulting values v(t) as a function of the sign:

$$v_{in}(t) = \begin{cases} v_{in}(t) k_1, & v_{in}(t) > 0 \\ v_{in}(t) k_2, & v_{in}(t) < 0 \end{cases} \quad \frac{dL}{dt} > 0;$$
$$v_{ex}(t) = \begin{cases} v_{ex}(t) k_3, & v_{ex}(t) > 0 \\ v_{ex}(t) k_4, & v_{ex}(t) < 0 \end{cases} \quad \frac{dL}{dt} < 0. \qquad (5.4.12')$$

In many cases with zero initial conditions when the transient process from inhalation (exhalation) is practically damped by the beginning of exhalation (inhalation), approximate calculation of function v(t) can be performed using the Duhamel integral

$$v(t) = \int_0^t L'(\tau) h(t - \tau) d\tau,$$

where h(t) is the transient characteristic of the linear circuit during inhalation (exhalation), the stable mode of which (currents, condenser charges) is assumed constant as the transfer function is changed. In order to solve the differential equation (5.2.1), the values of the resulting variable coefficient u(t) were found, and quantity $\hat{a}$ was estimated [see (5.2.2)], characterizing the mean interval of cardiac contractions during observation time T (N readings T_{R-R}):

$$\hat{a} = \frac{4\pi^2}{T^2_{(R-R),T}}; \qquad (5.4.13)$$
$$T_{(R-R),T} = \frac{1}{N} \sum_{i=1}^{N} T_{(R-R),i}.$$

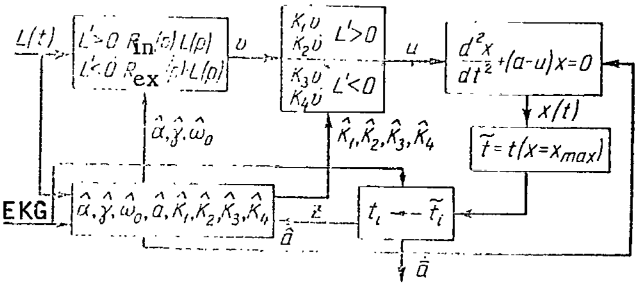


Figure 64

In the comparison section ($\tilde{t}_i \leftarrow t_i$), the sequence of intervals of contractions of the actual heart $T_{(R-R),i}$ were compared to those produced by modeling $\tilde{T}_{(R-R),i}$: /158

$$T_{(R-R),\,i} = t_i - t_{i-1}$$
$$\tilde{T}_{(R-R),\,i} = \tilde{t}_i - \tilde{t}_{i-1}. \qquad (5.4.14)$$

Moments in time t_i corresponded to the sequence of excitation of the cardiac muscle, while moments in time $\tilde{t}_i$ corresponded to startup of the model of the "cardiac master oscillator," determined by the maximum value of function x(t).

In order to compare the quantities T_{R-R} and $\tilde{T}_{R-R}$, we used the least squares method. The feedback loop z from the output of the comparison section (see Figure 64) carried a value of coefficient $\hat{a}$ with a correction so that in the solution of equation (5.2.1) the value of the error $\varepsilon^2(\hat{a})$ was minimized:

$$\varepsilon^2(\hat{a}) = \sum_{i=1}^{N} (\tilde{T}_{(R-R),\,i} - T_{R-R})^2, \qquad (5.4.15)$$

where i is the ordinal number of interval T_{R-R} (or $\tilde{T}_{R-R}$) in the realization of the process being investigated. The parameters α, ω_0, γ, k_1, k_2, k_3 and k_4 were assumed constant.

If the equality

$$\varepsilon^2(\hat{a}) = A = \min, \qquad (5.4.16)$$

is satisfied, the basic error in the estimate $\bar{T}_{R+R}$ arising due to respiratory regulation is compensated to a considerable extent and continues to decrease with decreasing A. In this case, $\hat{a} = \bar{\dot{a}}$, where $\bar{\dot{a}}$ is the mean value of the coefficient, calculated on the machine for a certain state of the test subject using the predetermined realization of the process in correspondence with expression (5.4.13):

$$\dot{a} = 4\pi^2 (\dot{\bar{T}}_{R-R})^{-2}. \tag{5.4.17}$$

With complete exclusion of parasympathetic respiratory effects and unchanged operator state over the observation interval we produce [see (5.2.4)]

$$\dot{a} = \tilde{a} = a.$$

In the process of minimization of error $\varepsilon^2(\hat{a})$ if this was required, a shift of the realization $\tilde{T}_{(R-R),i}$ along the time axis was performed, usually within the limits of one to two intervals T_{R-R}, which could decrease the error due to the comparatively small deviations of parameters of the transfer function /159 α, ω_0, γ and incomplete knowledge of the initial conditions in the solution of equation (5.2.1). The discrete nature of the change in $\hat{a}$ [see (5.4.16)] was determined by two conditions: on the one hand, the required accuracy of calculations, and on the other -- machine time economy, and was (0.001-0.05) $\hat{a}$ (depending on the magnitude of the signal expected $\overline{\Delta T}_{R-R}$). In each concrete case, if required by the experiment, the quantization step a could be clarified.

The estimate of the values of α, ω_0, γ (m, n, ω) and coefficients k_1, k_2, k_3, k_4 was performed, as a rule, with the test subject in the quiet state. We first measured certain parameters of the envelope of the actual transient process of the cardiac rhythm with staged breathing "inhale -- hold" or "exhale -- hold," such as time $\tilde{t}_n$ of intersection of the level of the mean interval $\overline{T}_{R-R}$, the moments of attainment of the minimal and maximal values $\tilde{t}_{min}$, $\tilde{t}_{max}$, the attenuation time $\tilde{t}_{at}$, etc. The envelope here is taken to mean the smooth curve connecting the ends of the intervals T_{R-R}, laid out along the ordinate at moments of excitation of the cardiac muscle t_i [see (5.4.14), Figure 62]. It was assumed that the measured parameters agreed approximately with the same quantities for the function $\phi^2(t) = a - u(t)$. On the basis of the theoretically determined dependence v(t) for the type of transient characteristic selected [see (5.3.9), (5.3.10), (5.3.11)] and the change in the input action with finite front t_f for inhalation (exhalation), which differed from the resulting value of u(t) only by the weight coefficients k_1, k_2, k_3 and k_4 [see (5.4.12)], we calculated in general form moments in time t_n, t_{max}, t_{min}, t_{at}, etc.

$$
\begin{aligned}
t_n &= t_n(\alpha, \omega_0, \gamma, t_f); \\
t_{max} &= t_{max}(\alpha)\, \omega_0, \gamma, t_f); \\
&\cdots\cdots\cdots\cdots \\
t_{at} &= t_a(\alpha, \omega_0, \gamma, t_f).
\end{aligned}
\qquad (5.4.18)
$$

The solution of equation system (5.4.18) for α, ω_0, γ with substitution of the experimental quantities t_n, t_{max}, t_{min}, t_{at}, etc. allowed us to produce the estimates $\hat{\alpha}$, $\hat{\omega}_0$, $\hat{\gamma}$, which were then averaged for several realizations with various depths and rates of staged breathing, approximating trapezoidal curves in these cases.

As an example, let us analyze the sequence of operations of approximate calculations of parameters m and n from realizations of the mean rhythm with breath held inhaled (staged action L(t) with finite front t_f) for the transient function described by equation (5.3.9):

$$
h_{in} = \frac{1}{m-n}(ne^{-nt} - me^{-mt}).
$$

Using the Duhamel integral and keeping in mind /160

$$
\frac{dL}{dt} = c > 0,
$$

we produce:

$$
0 \leqslant t \leqslant t_f, \quad v_{in_1}(t) = \int_0^t c h_{in}(t-\tau)\, d\tau = \frac{c}{m-n}(e^{-mt} - e^{-nt});
$$

$$
t_f < t < \infty, \quad v_{in_2}(t) = \int_0^{t_f} c h_{in}(t-\tau)\, d\tau = \frac{c}{m-n}[(e^{-mt} - e^{-nt}) -
$$

$$
-(e^{-m(t-t_f)} - e^{-n(t-t_f)})] = v_{in_1}(t) - v_{in_1}(t - t_f).
$$

The time of intersection of mean level t_n by the process is found from the relationship

$$
v_{in_1}(t) = v_{in_1}(t - t_f)
$$

and is equal to:

$$t_n = \frac{1}{m-n} \ln \frac{1-e^{mt_f}}{1-e^{nt_f}}. \quad (5.4.19)$$

If the transient function investigated (5.3.9) is two-phased, m > n, and therefore the attenuation $\hat{t}_{at}$ of the process (return of cardiac rhythm to initial level) is determined primarily by parameter n considering inhalation time $\hat{t}_f$.

Assuming further

$$e^{-n(t_{at}-t_f)} = 0,1,$$

let us find the estimate $\hat{n}$:

$$\hat{n} = \frac{2,3}{\hat{t}_{at} - \hat{t}_f}.$$

Substitution of the experimental quantities $\hat{t}_f$, $\hat{n}$ and $\hat{t}_n$ into equation (5.4.19) allows us to produce estimate $\hat{m}$.

Approximate calculation of k_1, k_2, k_3 and k_4 was performed on the basis of formula (5.4.13). In the transient processes "inhale -- hold -- exhale -- hold," we selected time sectors t', where the intervals T_{R-R} changed slowly and determined the time average $T_{(R-R),t'}$. Here, we considered u(t) ≈ u = = const, so that

$$\hat{u} = \hat{a} - \frac{4\pi^2}{T_{(R-R),t'}}, \quad T_{(R-R),t'} > T_{(R-R),T};$$

$$\hat{u} = \frac{4\pi^2}{T_{(R-R),t'}} - \hat{a}, \quad T_{(R-R),t'} < T_{(R-R),T}.$$

In the ratio $\hat{u}/\hat{v} = \hat{k}$, the quantity $\hat{v}$ (over time t', function $\hat{v}(t)$ was assumed equal to $\hat{v}$) was determined by substitution into the theoretically calculated dependence [see (5.4.12)] of the estimates $\hat{\alpha}$, $\hat{\gamma}$, $\hat{\omega}_0$, $\hat{t}_f$. Experiments showed that the approximation for k is satisfactory if the intervals t' are selected in the areas of maximum values of T_{R-R} [N = 3-4] for the phase of retardation of the cardiac rhythm during inhalation and exhalation. On the basis of most /161 experimental data, coefficients k_1, k_2 and k_3, k_4 (5.4.12') can be approximately determined from the expressions (where either member of each pair is known):

$$\frac{\hat{k}_1}{\hat{k}_2} \simeq 1 \div 3{,}5; \quad \frac{\hat{k}_3}{\hat{k}_4} \simeq 2 : 5.$$

Final selection of parameters α, γ, ω_0, k_1, k_2, k_3 and k_4, if it is required for the fixed coincidence of theoretical and experimental transient processes, was performed by solving equation (5.2.1) by machine (see Figure 64). The required correction was achieved in the circuit of feedback z. In this case, the method described above was used first to minimize errors of divergence with respect to certain time points (t_{max}, t_{min}, t_n, etc.) of the calculated and experimentally produced envelopes of the heart contraction intervals (clarification of α, γ, ω_0), then for general minimization of amplitude differences in the quantities T_{R-R} and $\tilde{T}_{R-R}$ (clarification of k_1, k_2, k_3, k_4).

Parameter a (or $\overline{T}_{R-R}$) was considered unchanged in these cases (state of operator rest) and was estimated from a preliminary sample (see Figure 60; EKG_2, R_2) as lasting several minutes. Calculations showed that the accuracy of measurement of $\overline{T}_{R-R}$ where the number of independent readings N = 35-40 is rather high. For example, in one of the worst cases, the confidence interval J_β constructed for $\overline{T}_{R-R}$ and corresponding to a confidence probability $1 - 2\beta = 0.95$, was $J_\beta = (0.835 - 0.012; 0.835 + 0.012)$; $T_{(R-R),T} = 0.835$.

In calculation of J_β, the Student distribution was used (Van der Waarden, 1960; Smirnov, Dunin-Barkovskiy, 1965; Nalimov, 1960).

We analyze below a number of practical results. Figure 65 shows realizations of the envelopes of transient processes for heart contraction intervals T_{R-R}, calculated by machine (dotted line) and produced in experimental situations (solid lines) with the test subject at rest (eyes open) and under slight emotional stress (viewing of low-significance signals) with staged breathing "inhale -- hold -- exhale -- hold." The values of the intervals $T_{(R-R),i}$ are laid out on graphs at moments in time t_i [see (5.4.14)] along the ordinate and the points produced are connected by smooth curves. The plan of the construction is shown in the right upper corner of Figure 65, A. The graphs on Figure 65, A illustrate the two-phased transient process during inhalation and exhalation ($n = 0.3$; $m = 0.53$; $\gamma = 0.65$; $k_1 = 1.4$; $k_2 = 0.7$; $k_3 = 1.0$; $k_4 = 0.6$) while the graphs of Figure 65, B have /162 the nature of attenuating oscillations upon inhalation ($\alpha = 0.2$; $\omega = 0.5$; $k_1 = k_2 = 1$) and are two-phased upon exhalation ($n = 0.25$; $m = 0.5$; $\gamma = 0.6$; $k_3 = 2.8$; $k_4 = 1.2$).

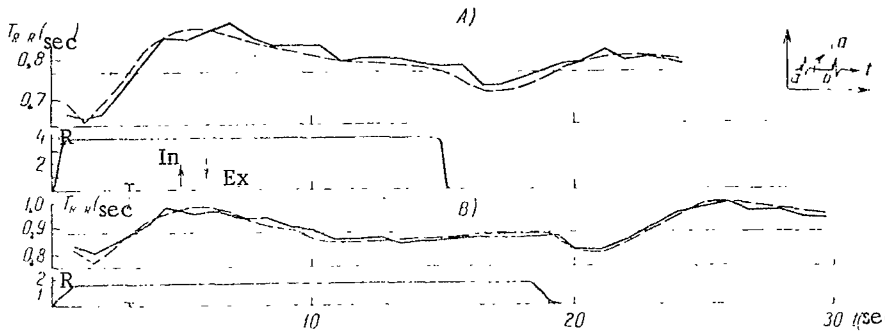


Figure 65

In the first case (see Figure 65, A) as a result of solution of equation (5.2.1) by machine, an estimate was produced of the mean interval of heart contractions $\bar{T}_{R-R} = 0.766$, while in the second case (see Figure 65, B), $\bar{T}_{R-R} = 0.885$. The preliminary measurements of $T_{(R-R),T}$ for time T = 3 min with quiet respiration in the same experiments gave the following results: $T_{(R-R),T} = 0.780$ (operator rest) and $T_{(R-R),T} = 0.892$ (weak emotional stress). We note that for the situation characterized by the graphs of Figure 65, A, it is possible to clarify $T_{(R-R),T}$ if the criterion $\varepsilon^2(\hat{a}) = \min$ is satisfied (the quantization step with respect to $\hat{a}$ was somewhat large). The mean amplitude R with quiet breathing in these experiments was determined from the magnitude of the vertical sector laid out at moment t = 3 sec (see Figure 65).

Figure 66 illustrates the usage of the method with comparatively strong emotional stress (expectation of painful stimulus) for staged, interrupted respiration R, where a rather strong masking effect is produced by the dispersion of quantities $\bar{T}_{R-R}$. In the case of several respiration cycles during the influence of the signal, the phenomenon of averaging due to positive and negative waves of the transient characteristics on inhalation and exhalation is most strongly seen. The abscissa determines the ordinal number N of the intervals T_{R-R} during analysis time T for each of the four realizations shown on the Figure. The actual curves of the transient processes are shown by the solid lines, while the calculated curves are shown by the dotted lines. To the right of each graph we show the corresponding type of respiration R, while at moment in time t = 20 sec we show a vertical sector characterizing the mean amplitude R with quiet respiration. /164

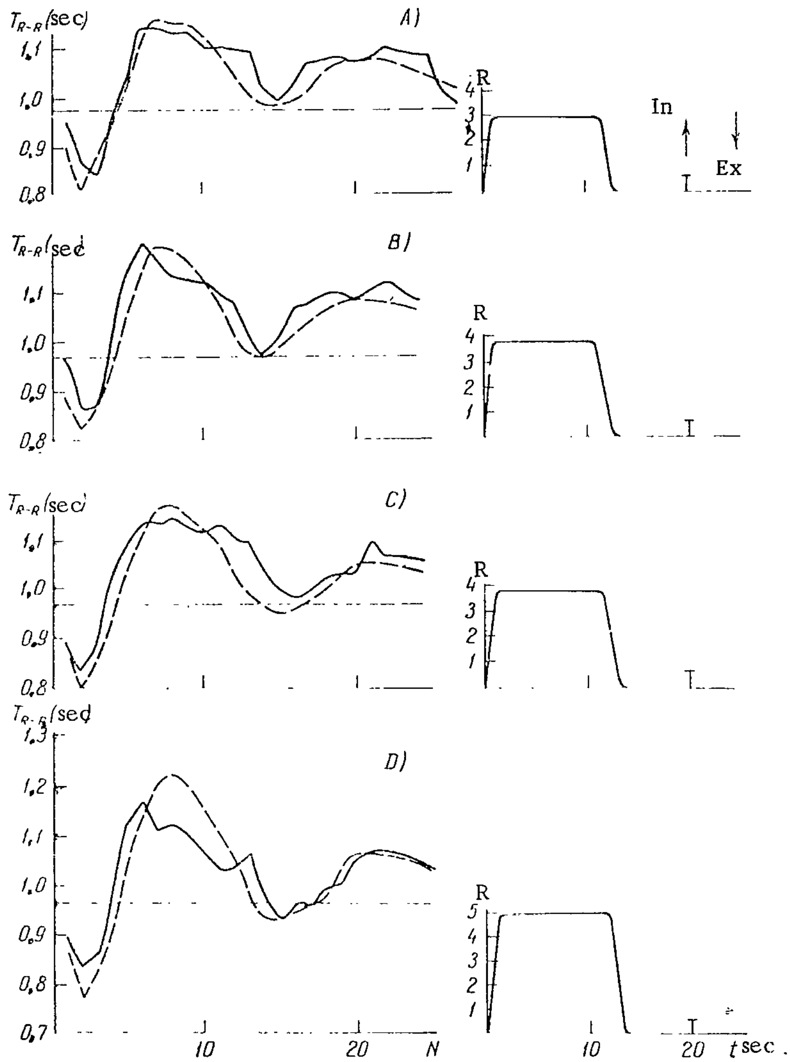


Figure 66

An estimate of the mean value of the parameter being measured, calculated before the beginning of the experiment for realizations lasting $T = 3$ min, amounted to $T_{(R-R),T} = 0.86$.

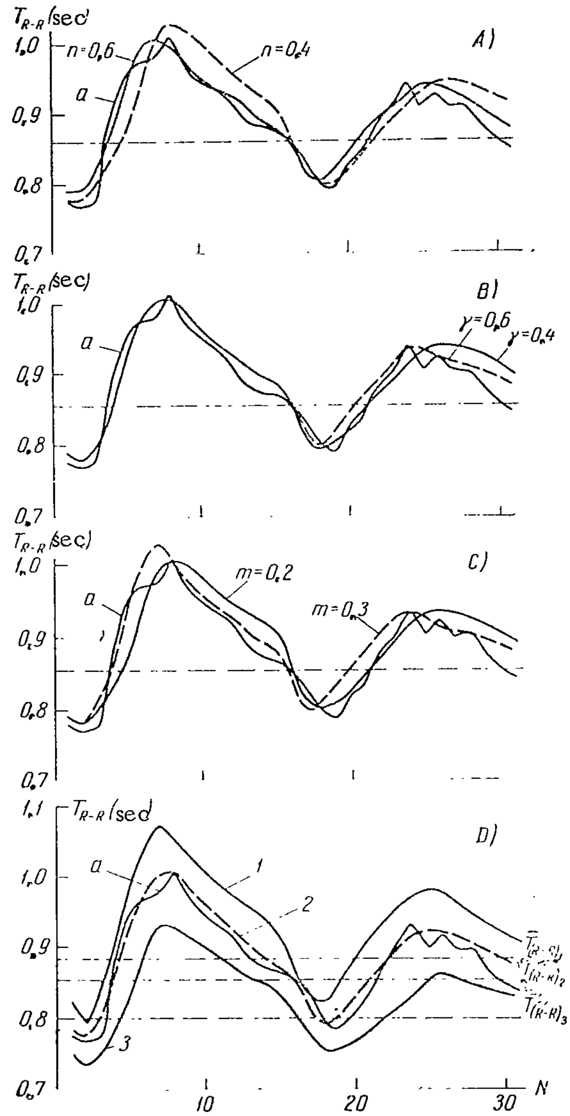
Since in this case the significant signal was rather large (increment of mean value of intervals T_{R-R}), its separation was performed with a large quantization step with respect to $\hat{a}$ (machine estimates $\overline{T}_{R-R} = 0.965$). On the basis of the realization used in these experiments $T_{(R-R),j}$ with quiet respiration (expectation of electrical skin stimulus) with a total duration of $T = 2.8$ min, the time average $T_{(R-R),T} \approx 0.980$ was calculated.

Analysis of the graphs indicates that the estimate $T_{R-R,T}$ performed using five-six intervals T_{R-R} selected at random without consideration of respiratory regulation can give an error reaching 15% of the value of $T_{(R-R),T} = 0.98$, which exceeds the value of the signal $\Delta T_{(R-R),T} = 0.98-$ $-0.86 = 0.12$.

The duration of an experiment in this situation was 30 min. The parameters of the transforming function ($n = 0.22$; $m = 0.43$; $\gamma = 0.65$; $k_1 = 1.4$; $k_2 = 0.6$; $k_3 = 1.0$; $k_4 = 0.15$) were selected in the state of subject rest. At the end of the experiment, a certain divergence was noted between the calculated and actual transient processes, the latter having a tendency to be drawn out with increasing emotional stress, which in the cases of interest to us takes on principal significance.

Figure 67, A, B, C shows the influence of a change in parameters m, n, γ on the nature of the transient processes ($\overline{T}_{R-R} = \text{const}$) with the same type of staged respiration: "inhale -- hold -- exhale -- hold." The actual curve is labeled with the index a. The graphs of Figure 67, D illustrate the stages of fulfillment of criterion $\varepsilon^2(\hat{a}) = \min$ for three values of the estimate $\overline{T}_{(R-R),1}$; $\overline{T}_{(R-R),2}$ and $\overline{T}_{(R-R),3}$. In this case, the constants selected for the transforming function were determined by the values $m = 0.5$; $n = 0.25$; $\gamma = 0.51$; $k_1 = 6.0$; $k_2 = 1.5$; $k_3 = 2.0$; $k_4 = 1.0$.

The dependencies presented on Figure 68 were constructed for weak emotional state of the test subject, near the rest state ($n = 0.3$; $m = 0.5$; $\gamma = 0.65$; $k_1 = 4.3$; $k_2 = 1.5$; $k_3 = 1.7$-; $k_4 = 0.45$).



T_{R-R} (sec)
1,0
0,9
0,8
0,7
n=0,6
n=0,4
a
A)
B)
γ=0,6
γ=0,4
C)
m=0,2
m=0,3
1,1
D)
1
2
3
10
20
30
N

Figure 67

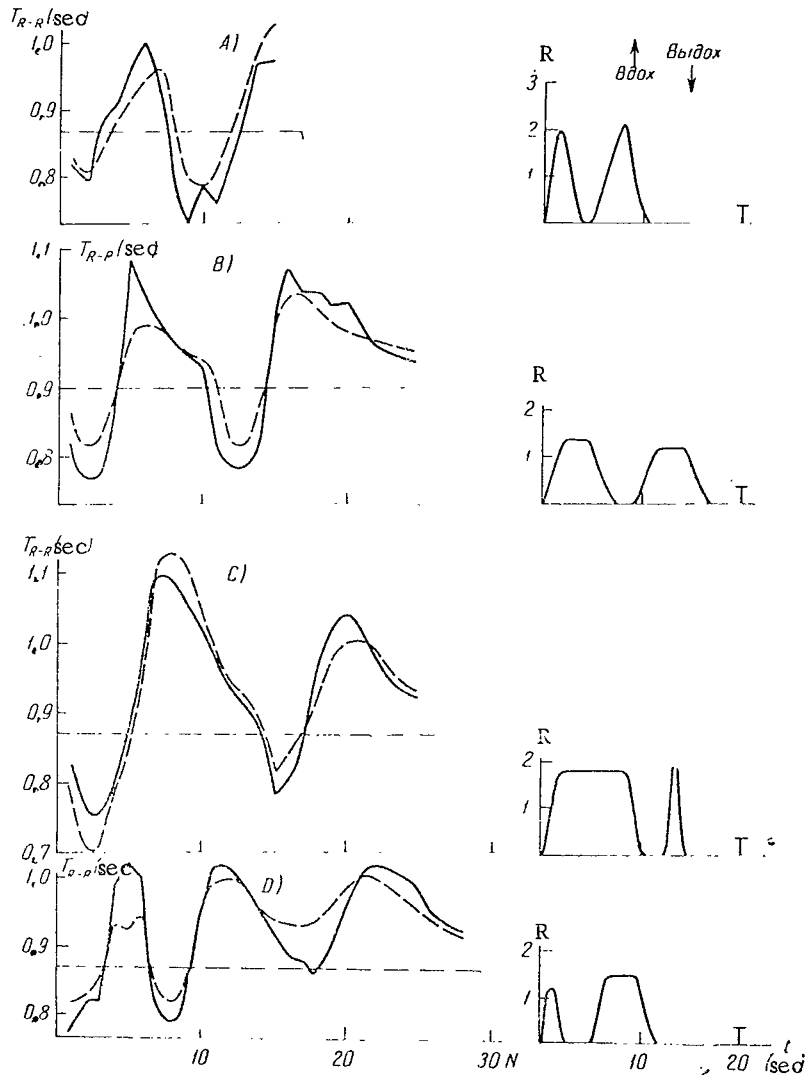


Figure 68

During the time of action of a certain signal ($T \approx$ 15-30 sec), two cycles of inhalation and exhalation could occur with different nature of respiration, as shown on the figure. The mean amplitude R with quiet respiration was characterized, as above, by the vertical sector laid out at moment in time t = 20 sec. The curves calculated on the machine are shown by the dotted lines. The estimates of $T_{(R-R),T}$, calculated using realizations /167
$T_{(R-R),i}$ several minutes long (quiet respiration) fluctuated during the process of the experiment between 0.9 and 0.862. The values of $\overline{T}_{R-R}$ calculated in the model are shown on the figure by the dotted lines parallel to the abscissa. The total duration of the experiment was 60 min. At the end of the experiment (Figure 68), considerable divergence was noted between the actual curves of changes in intervals T_{R-R} and those calculated by machine, which, of course, introduces error into the estimate $\overline{T}_{R-R}$. However, in the case at hand, when the criterion $\varepsilon^2(\hat{a})$ = min is satisfied, the value of this error is rather low, which can be determined from the value of $\overline{T}_{R-R}$.

The results produced allow us to conclude that in the situations analyzed (state of test subject, nature of breathing) the model produces satisfactory results when the mean interval of heart contractions is measured using short realizations $T_{(R-R),i}$. However, with longer experiments, the necessity may arise of adjusting the model by its parameters to the type of transfer characteristics in the "respiration-vagus inhibition" link. It would be desirable to continue the investigations of changes in the transfer characteristics with various test subject states and problems related to combination of the functions of vagus inhibition resulting from inhalation and exhalation.

§5.5. Estimate of Mean Interval of Heart Contractions Using Simplified Method

Let us represent formula (5.2.1) in the form

$$\frac{d^2x}{dt^2} + \varphi^2(t)\,x = 0,$$

where $\phi^2(t) = a - u(t)$ is a variable coefficient.

We will assume that the function $\phi^2(t)$ is characterized by a rather high mean value a, about which relatively small oscillations u(t) occur. On the basis of the VKB method (Khedding, 1965; N. Freman, P. U. Freman, 1967; Kanningkhem, 1962), when the conditions below are fulfilled

$$|\varphi^2(t)| \gg \left|\frac{\varphi''(t)}{2\varphi(t)} - \frac{3}{4}\left[\frac{\varphi'(t)}{\varphi(t)}\right]^2\right|; \qquad (5.5.20)$$
$$\varphi'(t) = \frac{d\varphi}{dt}; \quad \varphi''(t) = \frac{d^2\varphi}{dt^2}$$

the approximate solution of equation (5.2.1) in this case can be written as follows:

$$x = [\varphi(t)]^{-\frac{1}{2}} B \sin\left[\theta + \int_0^t \varphi(t)\,dt\right], \qquad (5.5.21)$$

where $\phi(t)$ changes more slowly than $\sin\left[\theta + \int_0^t \varphi(t)dt\right]$. B, θ are arbitrary constants. /168

Differentiating equation (5.5.21)

$$x' = -\frac{1}{2}[\varphi(t)]^{-\frac{3}{2}} \varphi'(t) B \sin\left[\theta + \int_0^t \varphi(t)\,dt\right] + [\varphi(t)]^{-\frac{1}{2}} \varphi(t) \times$$
$$\times B \cos\left[\theta + \int_0^t \varphi(t)\,dt\right]$$

and setting the first derivative equal to zero, we can find moments in time t_k corresponding to the k-th extreme points of x(t) from the equation

$$\cot\left[\theta + \int_0^t \varphi(t)\,dt\right] = \frac{1}{2}\cdot\frac{\varphi'(t)}{\varphi^2(t)} = -\frac{1}{4}\cdot\frac{u'(t)}{\varphi^3(t)}, \qquad (5.5.22)$$

among which we can use known methods to determine t_i such that the values of the function investigated are maximal. The difference in successive moments in time (for the two neighboring maxima x(t))

$$\tilde{t}_i - \tilde{t}_{i-1} = \tilde{T}_{(R-R),i} \qquad (5.5.23)$$

will characterize the "heart contraction intervals," produced by modeling.

Calculation of intervals (5.5.23) does not in this case require integration of expression (5.2.1); however, solution of the transcendental equation (5.5.22) is rather difficult.

For further simplification of the problem, we will consider that with sufficiently small, slowly changing, continuous component u(t) [$\phi^2(t) = a - u(t)$]

$$\cot\left[\theta + \int_0^t \varphi(t)\,dt\right] \approx 0$$

or

$$u'(t) \ll 4\varphi^3(t). \tag{5.5.24}$$

Under these conditions, moments t_i of attainment of the maximum values of the functions

$$x_1 = [\varphi(t)]^{-\frac{1}{2}} B \sin\left[\theta + \int_0^t \varphi(t)\,dt\right], \quad x_2 = B \sin\left[\theta + \int_0^t \varphi(t)\,dt\right]$$

are approximately determined by the same equation

$$\theta + \int_0^t \varphi(t)\,dt = (4l - 3)\frac{\pi}{2}, \quad l = 1, 2, \ldots$$

Let us analyze the behavior of circular frequency $\tilde{\omega}_t$:

$$\tilde{\omega}_t = \frac{d}{dt}\left[\theta + \int_0^t \varphi(t)\,dt\right] = \varphi(t) = \sqrt{a - u(t)}. \tag{5.5.25}$$

Considering the above concerning the nature of the function $\phi^2(t)$, let us represent equation (5.5.25) approximately in the form /169

$$\tilde{\omega}_t \approx \sqrt{a} - \frac{1}{2\sqrt{a}} u(t) \tag{5.5.26}$$

In order to calculate dependence (5.5.25) or (5.5.26), it is required, as follows from the formulas, only that we know the characteristics of vagus inhibition u(t), the result of weighing of the signal of change in the area of the thorax during respiration L(t).

On the other hand, from an actual electrocardiogram we can determine the sequence of quantities $\omega_{t,i}$:

$$\omega_{t,\,i} = 2\pi \frac{1}{T_{(R-R),\,i}}, \quad (5.5.27)$$

which in this case lie along the ordinate at moments in time t_i^* (center of i-th interval T_{R-R}):

$$t_i^* = \frac{t_i - t_{i-1}}{2},$$

where t_i is the time of the i-th excitation of the cardiac muscle.

Joining for convenience the experimental points of (5.5.27) with a smooth curve (ω_t), we can use the methods described in §5.4 to compare the dependence ω_t produced with function $\tilde{\omega}_t$, determined in the model. As a result, we will find an estimate $\dot{\bar{\omega}}_1$, related to the estimates of the parameters investigated $\dot{\bar{a}}_1$ and $\dot{\bar{T}}_{(R-R),1}$ with the formula

$$\dot{\bar{\omega}}_1 = \sqrt{\dot{\bar{a}}_1} = 2\pi\,(\dot{\bar{T}}_{(R-R),\,1})^{-1}. \quad (5.5.28)$$

As an example, let us analyze one possible simplified block diagram (Figure 69), realizing the method described above in the case of transfer functions of the linear circuit with the form [see (5.3.6), (5.3.7); $\alpha^2 < \omega_0^2$]

$$R_{\text{in}} = -\frac{k'p^2}{(p+n)(p+m)};$$

$$R_{\text{ex}} = \frac{k''p^2}{(p+m)(p+n)(p+\gamma)},$$

corresponding to the transient characteristic

$$h_{in}(t) = \frac{1}{m-n}(ne^{-nt} - me^{-mt});$$

$$h_{ex}(t) = \frac{-ne^{-nt}}{(m-n)(\gamma-n)} + \frac{-me^{-mt}}{(n-m)(\gamma-m)} + \frac{-\gamma e^{-\gamma t}}{(m-\gamma)(n-\gamma)}$$

applicable to staged interrupted inhalation and exhalation ($t_{at} \geqslant$ 10-15 sec) /170 with deep breathing, leading to considerable fluctuation of the intervals of heart contractions.

We will consider the parameters n, m, γ to be selected as a result of preliminary experimentation. Signal L(t) is differentiated in the circuit DC_1 (time constant 1/n) and, depending on the sign of the derivative

$$L'(t) > 0 \text{ -- inhalation; } L'(t) < 0 \text{ -- exhalation}$$

enters the following units: integrator -- IC (time constant $1/\gamma$) and differentiating device DC_2 (time constant 1/m) for exhalation, or circuit IDC_2, which changes the sign of the signal to the opposite sign with subsequent differentiation (time constant 1/m) -- for inhalation. The outputs of the linear circuits are weighted according to the sign in proportion to coefficients k_1, k_2 and k_3, k_4 (nonlinear devices with varying amplification for positive and negative effects). The output function $\tilde{\omega}_t$ of unit C, /171 combining the transformed inhalation and exhalation signals, is fed to comparison circuit ($\omega_t \leftarrow \tilde{\omega}_t$) to be compared with experimental dependence ω_t. Unit C also receives the value of $\hat{a}$ predetermined for the processing interval [see (5.4.13)] (in order to reduce search time), which is adjusted in the process of minimization of error $\varepsilon^2(\hat{a})$ using the feedback loop. Where equation (5.4.16) is satisfied, an estimate of the mean time of measured quantity $\bar{a}_1$ is output, determined in the model and related to the remaining parameters of formula (5.5.28). Figure 70 shows the experimental dependence ω_t (dotted line) and calculated dependence $\tilde{\omega}_t$ (solid line) for rather deep respiration R with varying nature. The graphs of ω_t and $\tilde{\omega}_t$ are constructed with respect to quantities $\dot{\bar{\omega}} = 7.3$, determined by machine solution of equation (5.2.1) for the same input actions and time of existence of the signal T $\approx$ 22 sec. Analysis of these curves (ω_t, $\tilde{\omega}_t$) shows that in the case at hand when the criterion $\varepsilon^2(\hat{a}) = \min$ is satisfied, a simplified method yields a slightly low estimate ($\dot{\bar{\omega}}_1 \approx 7.23$) in comparison with the quantity calculated by machine. This error generally results from the degree of approximation of solution (5.5.21) for comparatively long and rapidly changing input actions.

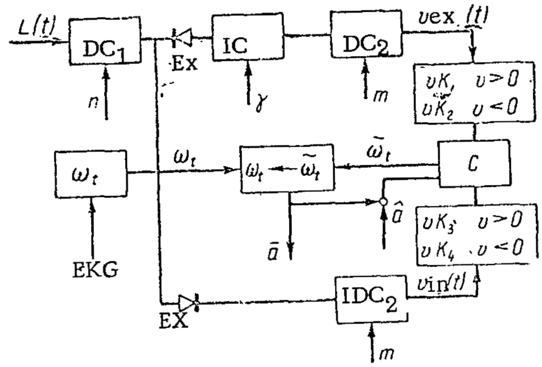


Figure 69

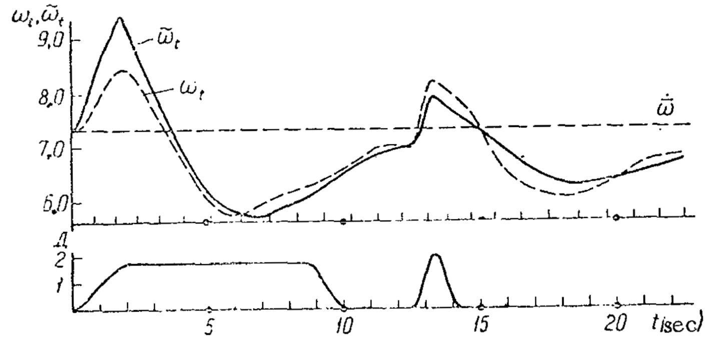


Figure 70

CHAPTER 6

THE SPEECH SIGNAL

ABSTRACT. Experiments indicate that the changes in the spectral conditions of a speech signal are among the most reliable for determination of the emotional state of a human operator. Experiments preformed included modeling a human speech under the influence of emotion by actors, which confirm the stable changes in frequency composition, etc. with emotional coloring.

Introduction

We know that in addition to the symptoms perceived directly by an observer (blushing or paleness of the face, nature of expression, etc.) emotions are accompanied by changes which can be detected by special methods of investigation (some of which have been described in preceding sections) of various physiological processes: EEG, SGR, EKG, changes in blood pressure, respiration, skin temperature, chemical composition of the blood, saliva, urine, etc. /172

The recording of these changes requires contact pickups, which is not always convenient for an actively working operator. Many vegetative indicators used for practical purposes (for example pulse, respiration, arterial pressure) do not allow at the present time reliable differentiation of positive and negative emotions and, furthermore, change sharply during physical loading, thus ceasing to become reliable criteria for the degree of emotional stress. The solution of practical problems requires, on the one hand, a more precise analysis of signals already known, and on the other hand -- a search for new signals which can have independent significance or can be processed together with other signal parameters in order to increase the reliability of detection.

With this in mind, it seems quite interesting to analyze the objective parameters of the intonations of speech. We know that the intonation coloration of speech is a reliable means of communications. Recording of a speech signal does not require contact pickups, requires no additional telemetry channels, since the recordings of conversations using existing communications channels can be used.

Practical experience and certain special work in this area (Artem'yev, 1961; Vitt, 1965; Zhinkin, 1958; Friedhoff, 1962; Hoffe, 1956-1957;

Lieberman, Michaels, 1962) indicate that objective indices to be used to draw conclusions concerning the emotional state of the speaker can be found in the intonation characteristics of speech.

We analyze below some problems of the utilization of the results of spectral analysis of a speech signal to evaluate the degree of emotional stress of an operator[1].

Before going over to a description of the data produced, we would like to discuss certain concepts from the theory of speech formation and methods of spectral processing of a speech signal, which will be required for our further presentation. /173

§6.1. Model of Speech Channel

The speech signal, as we know, represents the reaction of the resonance (filtering) system of the speech path to the effects of one or more generators of audible oscillations. In other words, the sound oscillations created in surrounding space can be represented as functions dependent on the parameters of the corresponding sources of sound and on the parameters of the complex systems of resonators, including those such as the loading resistance of the mouth (radiator). Resonators are formed by the cavities of the mouth and pharynx, and in many cases by the nasal cavity.

We could suggest a large number of models of speech formation satisfying these requirements. However, in our further analysis it is desirable to select those which imitate rather well the acoustical picture of actual speech signals and which are also convenient from the point of view of their mathematical analysis. In this sense, it is expedient to use electronic models of the speech forming paths, which are comparatively simply realized technically and allow us to use apparatus from the theory of electrical circuits for their investigation. Figure 71 a shows an electronic model of the speech path (without considering the nasal cavity) which is frequently used at the present time (Sapozhkov, 1963; Fant, 1964). In the case at hand, the larynx is the source of oscillations E, which has a certain internal impedance Z (impedance of the cavities before the source). The vocal cords, closing and opening the larynx, modulate the flow of air and create a sequence of pulses (see Figure 71 b) which are near saw-toothed in form (strictly speaking, the vocal cords not only modulate the flow of air, but also vibrate, creating additional oscillations imposed on the basic impulses). The frequency spectrum $\Phi_i(f)$ produced with this sound pressure contains a large number of harmonic components (Figure 71 c), the amplitude of which decreases as their number increases at about 12 db per octave. It should be noted that in the general case the sequence of pulses created by

[1] The authors express their deep gratitude for cooperation in this section of the work to comrades V. A. Popov, L. S. Khachatur'yanets and A. G. Tishchenko.

the vocal cords is not periodic in the mathematical sense, which is explained by the finite duration of the sound and the unstable repetition frequency of the pulses themselves, which may depend on loudness, type of vocal material, emotional content (Voloshin, 1966; Vitt, 1965), etc. In connection with this, representation of spectrum $\Phi_i(f)$ as a linear harmonic spectrum is an idealization.

The repetition frequency of pulses from the oscillation source E characterizes the type of voice speaking (bass, baritone, tenor, alto, contralto, soprano) and is called the base tone frequency F_{bt}. For Russian speech, the mean value of F_{bt} lies between 97 and 200 Hz for masculine voices and 200-320 Hz for feminine voices. /174

The sequence of pulses with repetition frequency T_i excites quadrupole M (input: 1-1, output: 2-2), representing the speech path from the source of sound to the aperture of the mouth (see Figure 71 a). If we represent the frequency and phase characteristics of the quadrupole by $C_M(f)$ and $\alpha_M(f)$ respectively, its output will carry oscillations with frequency spectrum $\Phi(f)$ (see Figure 71 e):

$$\Phi(f) = \Phi_i(f)\,C_M(f)$$

and phase spectrum $\alpha(f)$:

$$\alpha(f) = \alpha_i(f) + \alpha_M(f),$$

where

$$S(f) = \Phi(f)\,e^{j\alpha(f)};$$
$$S_i(f) = \Phi_i(f)\,e^{j\alpha_i(f)},$$

where $S_i(f)$ and $S(f)$ are the complex spectra.

One possible realization of the signal at the output of the quadrupole is shown on Figure 71 g.

The spectral interpretation presented above is correct if the characteristics of the resonant speech path system are independent of the characteristics of the source of oscillations E, which is an idealizing assumption.

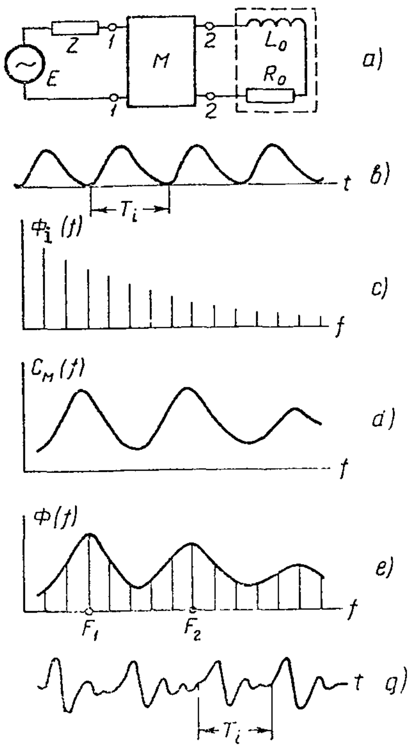


Figure 71

The frequency areas of the maxima observed on the spectral picture of speech sounds are called formant zones or simply formants. The location of the spectral peaks corresponds approximately to the resonances of the speech path. It can be shown that the inequality of frequencies of spectral maxima of the output signal (Figure 71 e) and the resonance frequencies of the spectral characteristic of the quadrupole M (see Figure 71 d) can be explained principally by the decrease in spectrum $S_i(f)$ with increasing frequency. However, this difference is not great, and in practice it is /175
frequently ignored. The formant zones are represented in increasing order F1, F2, F3, ... and their frequencies as F_1, F_2, F_3, ... respectively. Figure 71 e shows the first two formants. Since the formants are considerably stronger than the other components, it is they which act primarily on the ear of the listener, forming the sound of each individual speech sound. Thus, it seems that one of the principal characteristics of the sound spectrum of speech is the number of formants and their placement in the frequency area. Where there are no clearly expressed formant zones in the spectrum of the signal, we can use the method of spectral moments for analysis:

$$M_0 = \sum_n P_n; \quad M_2 = \sum_n f_n^2 P_n^2 = f_{II}^2 M_0; \qquad (6.1.1)$$

$$M_1 = \sum_n f_n P_n = f_I M_0; \quad \Delta f = \frac{M_2}{M_0} - \left(\frac{M_1}{M_0}\right)^2 = f_{II}^2 - f_I^2,$$

where P_n is the intensity of the n-th band of the spectrum; f_n is the mean frequency of the n-th spectral band; Δf characterizes the width of the spectrum; M_0 characterizes the intensity of the entire spectrum; M_1 characterizes the mean frequency of the spectrum, called the centroid of the spectrum (or the weighted mean frequency); M_2 characterizes the mean square frequency of the spectrum.

On the basis of formulas (6.1.1), the spectrum can be characterized by the intensity M_0, mean frequency f_I and width Δf. These moments can be used for spectra with clearly expressed formant areas as well. However, in this case, they should be determined for each of the formant zones separately.

The nulls of the output signal spectrum are sometimes called anti-formants. The form of discrete spectral distribution analyzed above (Figure 71 e) corresponds principally to the picture which obtains during the pronunciation of vowel sounds, which have clear signal periodicity. Many consonant sounds are aperiodic, so that their frequency spectra are either completely solid or contain solid frequency spectrum sectors (Pokrovskiy, 1962).

When voiceless, as well as many voiced consonants are pronounced, the noise source of sound oscillations in many cases is turbulent, located in one of the constrictions of the vocal path. It may be either "continuous" (sounds pronounced over some period of time), or pulsed (in the case of the so-called plosive sounds).

Continuous speech formation is determined primarily by two factors: the source of oscillations and the filter (Fant, 1964). Upon transition from /176 phoneme to phoneme[1], modulation of the oscillations from the variable sound source (consisting of voice, noise, voice and noise together, with variable rise and fall of oscillations, etc.) by the slowly changing filter function of the speech path occurs.

Thus, the transition from a long nasal to a long, sonorous, throaty sound is characterized by a change in the filter function; the transition from a long dental fricative to a vowel sound is characterized by a change both in the source of oscillations and in the filter function of the speech path.

Thus, the continuous speech signal, carrying semantic information, is characterized by changes in the parameters of the oscillation source and the filter function of the speech path, and can be represented on the basis of analysis of speech formation in the general case as an unstable random process (Voloshin, 1966).

§6.2. Problems of Spectral Analysis of Speech Signals

It was shown above that the acoustical picture of speech sounds (based on the model of speech formation which we have analyzed) is characterized by the source of oscillations and the filter system of the speech path. For example, spectrum $S(\omega)$ of the absolutely integrable output signal $f(t)$

[1] Standardized speech sounds are referred to as phonemes in speech signal transmission technology. The Russian language includes 40-41 phonemes, explained by correspondence of many consonants to two sounds: hard and soft, while almost half the vowels have dual sounds (y plus vowel).

$$S(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \tag{6.2.2}$$

is primarily determined by the transfer characteristic of quadrupole M (Figure 71 a). A monotonous decrease at about 6 db/oct of spectral function $S_i(\omega)$ of the sound source (under the influence of the radiator) can be considered in the model by introduction of a correcting coefficient (increasing the amplification of the signal by 6 db/oct with increasing frequency). The spectral density of the energy can be found from the expression

$$\Phi^2(\omega) = S(\omega) S^*(\omega),$$

where $S^*(\omega)$ is complexly conjugate with $S(\omega)$.

In order to judge the properties of a speech signal at a given moment in time, we must go over to the instantaneous spectrum

$$S_T(\omega, t) = \int_{t-T}^{t} f(\tau) e^{-j\omega\tau} d\tau, \tag{6.2.3}$$

where $S_T(\omega, t)$ is defined as the spectrum of the sector of the process of duration T, immediately preceding the given moment t. The constant integration interval in this case moves along the time axis, and its location is unchanged relative to instantaneous time t. With a more general definition of the instantaneous spectrum in the integrand of (6.2.3), we introduce the sliding (related to the instantaneous moment of observation) weight function (Kharkevich, 1957) /177

$$S_p(\omega, t) = \int_{-\infty}^{t} p(\tau - t) f(\tau) e^{-j\omega t} d\tau. \tag{6.2.4}$$

Expression (6.2.3) follows from equation (6.2.4), if

$$p(x) = \gamma(x + T) - \gamma(x),$$

where

$$\gamma(x) = \begin{cases} 0 \text{ when } x < 0; \\ 1 \text{ when } x > 0. \end{cases}$$

Transform (6.2.4), which can be rather simply achieved using a system of frequency filters, is frequently used for analysis of speech signals. Here, weight function p(x) is selected in the form (Fano, 1950)

$$p(x) = Ke^{\alpha x}\gamma(-x);$$

$$\gamma(-x) = \begin{cases} 1 \text{ when } x < 0; \\ 0 \text{ when } x > 0 \end{cases}$$

(K is a constant coefficient), which reflects the actual result of spectral analysis using actual filters with time constant $1/\alpha$. However, the usage of this method for speech signals encounters certain difficulties. On the one hand, extended sounds have varying spectral structure and their analysis requires filters with great resolving capacity with respect to frequency, while on the other hand the plosive sounds are characterized primarily by time dependences, which requires a filter with low resolving capacity. The compromise solution should be selected on the basis of the concrete conditions at hand.

It was noted above that in the general case the continuous speech signal is an unstable, random process. Going over to the instantaneous power spectrum, we can write (Levin, 1966)

$$G(\omega, t) = \int_{-\infty}^{\infty} B(t, \tau)\, e^{-j\omega\tau}\, d\tau,$$

where $B(t, \tau)$ is the correlation function of the random signal.

At the present time, with the various methods used for processing speech /178
processes, the calculation of correlation functions is performed over intervals (usually several tens of milliseconds) in which the signal is looked upon as a quasi-stable process, while $R(\tau)$ is looked upon as an even function of τ. In the general case for unstable signals this is untrue, and therefore the correlators used have some defects in their filter analyzers. The usage of digital computers for stricter calculation of $R(t, \tau)$ using a large quantity of statistical material is economically unsuitable in comparison with the filter method, and no method is yet known for using function $R(t, \tau)$ as calculated from a set of realizations (Voloshin, 1966).

In recent years, due to the intensive development of computer equipment, we have seen ever broader introduction of digital computers into investigations on the analysis and recognition of speech signals. The usage of digital computers allows the operation of the known analyzing devices to be modeled, allows processing of the signal very close to the mathematical model to be achieved, and in many cases allows a significant acceleration and reduction in price of search work, for example in spectral analysis, where frequent changes in filter parameters are required (selection of filter with optimal parameters) or where it is necessary to use filters with frequency-time characteristics which cannot be achieved in actual systems with losses, etc.

The principal method of numerical spectral analysis at the present time is the calculation of the Fourier integral [see (6.2.2)].

Function f(t) is determined by readings at discrete moments in time, calculation function $S(\omega)$ is also produced discrete. In order to convert analog quantities into digital quantities and back, special devices called "analog-digital" and "digital-analog" converters have been developed (see Chapter 2). During time quantization of a signal, it is assumed that its spectrum is limited by frequency f_0. In this case, on the basis of Kotel'nikov's theorem (1933), the quantization frequency can be determined from the formula

$$f_Q = 2f_t.$$

Integral (6.2.2), considering these conditions, can be represented in the following form (Kharkevich, 1957)

$$S(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt = \Delta t \sum_{k=-\infty}^{k=\infty} f(k\Delta t)\, e^{-j\omega k\Delta t}, \tag{6.2.5}$$

where

$$-\omega_t \leqslant \omega \leqslant \omega_t; \quad \Delta t = \frac{1}{2f_t}.$$

The calculation of a sum with an infinite number of terms is /179
impossible; therefore, as a result of limitation of the integration limits (or addition limits) errors arise, detailed analysis of which has been presented in the literature (Voloshin, 1964).

In conclusion, we must note that the productivity of modern computers in many cases is insufficient, which hinders, for example, the production of results of spectral analysis of speech signals in real time.

§6.3. Results of Experimental Modeling of Human Emotions

We noted above (Chapter 2) that investigation of the intonation characteristics of speech in the area of spectral parameters (other characteristics are not being analyzed at the present time) and the selection of experimental material, it was decided to use the principle of modeling of human emotions by acting with subsequent testing (clarification) of the speech intonation estimates under actual emotional situations.

The processing of the main material (300 pronunciations of words) was performed using a spectral analyzer of the Danish firm "Bruel Kjaer" with a linear channel L having a pass band of 20-40,000 Hz and a set of one-third octave filters covering the pass band 40-40,000 Hz. We present below the filter numbers N and the corresponding mean tuning frequencies f (in Hz).

N . . .	1	2	3	4	5	6	7	8	9
f . . .	40	50	63	80	100	125	160	200	250
N . . .	10	11	12	13	14	15	16		
f . . .	315	400	500	630	800	1000	1250		
N . . .	17	18	19	20	21	22	23	24	25
f . . .	1600	2000	2500	3150	4000	5000	6300	8000	10000
N . . .	26	27	28	29	30	31			
f . . .	12500	16000	20000	25000	31500	40000			

A magnetic tape recording carrying the words was formed into a loop, during each revolution of which the output, proportional to the signal power in the pass band of the filter in question, was recorded. The results of the analysis were recorded on paper using a special spectral analyzer stripchart recorder with a dynamic range of 50 (or 75) db. The measurement error did not exceed 0.3 (or 0.5) db. The area of investigation was limited to frequency f = 1250 Hz, since the pass band of the filters of the analyzer being used was too broad above this frequency ($\Delta f > 300$ Hz). Narrower band width filters are needed in order to study the spectral characteristics of speech signals (Sopozhkov, 1963).

During the process of the experiments, we analyzed the behavior of /180
spectral peaks A_1, A_2 (determined from the output values of the corresponding filters) in the frequency areas of the base tone 100-315 Hz and the position of the first formant 315-1250 Hz.

With increasing emotional stress, regardless of the nature of the emotion itself, we generally observe an increase in middle frequencies f_I and in most cases in the energy of the spectral maxima (in relation to the energy of the process at the output of linear filter L). We note that processing of the experimental material showed that there was less information to be gained from the energy characteristic of peak A_1 (or A_2) than the deflections in the corresponding middle frequency from its value during quiet speech. Since the behavior of spectral peaks A_1 and A_2 was largely identical in our experiments, we present below the results concerning primarily the investigation of output signals of the spectral analyzer in the frequency band 315-1250 Hz.

As an estimate of the degree of emotional stress of the test subject, we use the quantity M:

$$M = \frac{\sum_{i=1}^{U} f_i P_i}{\sum_{i=1}^{U} P_i} \ln C \sum_{l=1}^{W} \frac{P_l}{P}, \quad U \geqslant W, \tag{6.3.6}$$

where P_l, P_i, P are the powers of the processes at the outputs of the l-th, i-th and linear L filters respectively, during the time of action of the input signal; f_i is the frequency to which the i-th filter is tuned; C is the proportionality coefficient.

This quantity allowed us to determine the position of the mean frequency f_I within the limits of the pass band of the k-th filter $|U > 1|$:

$$f_I = \frac{\sum_{i=1}^{U} f_i P_i}{\sum_{i=1}^{U} P_i} \tag{6.3.7}$$

and to consider the influence of x changes in the energy characteristics of the spectral maximum, determined primarily for the situations of formant structure of vowel sounds analyzed (particularly plosive sounds):

$$x = \ln C \sum_{l=1}^{W} \frac{P_l}{P}. \tag{6.3.8}$$

The proportionality coefficient C, characterizing the weight of the sum following the logarithm [see (6.3.6), (6.3.8)], on the basis of experimental data, was selected as 10^3 (area of analysis 315-1250 Hz). /181 On the basis of preliminary experimental processing (see also Shearme, 1959), the group of filters U was selected at a level (2-4) db below the filter with the maximum mean power $P_{k,max}$ at the output. The value of W in these experiments was found from the conditions

$$W = U \text{ when } U = 1;\ 2;$$
$$W = 2 \text{ when } U > 2$$

and the l-th filters (or filter) from set U was determined, such that the value of the centroid f_I [see (6.1.1)] was located between their adjustment frequencies (or within the adjustment frequency where W = U = 1). The sum $\sum_l P_l$ estimated the minimum energy (mean power) of the spectral maximum considering its expected width and the characteristics of the filter system of the spectral analyzer, which had slight overlap in the frequency range investigated (in the overwhelming majority of cases $U \leqslant 4$).

The increase in quantity M corresponded to an increase in degree of emotional stress in 80% of all experiments performed with actors. We note that when there were two separate spectral peaks in the frequency band being analyzed (with a gap of -(2-4) db relative to the greater of the two) the significant peak is that for which estimate f_I is greater. The unsuccessful experimental results, in addition to the low resolution of the analysis, are explained partially by the experience of emotionally colored states during recording of the background level, i.e., when the actors imitated quiet situations, which was indicated by electrocardiographic recordings. Naturally, this led to disappearance of characteristics detected in comparative analysis. Figures 72 and 73 show the spectra (output signals in spectral analyzer channels) of the expressions "Ya Almaz" [I am Almaz] and "Ponyal" [Roger] (pronounced by two actors), allowing us to obtain some idea of the changes in the values of x, f_I [see (6.3.7), (6.3.8)] in various emotional situations:

Figure 72 shows -- A quiet, B triumph, C fear; on Figure 73 -- A is quiet, B is joy, C is alarm.

Analysis of the figures indicates a considerable increase in the pulse frequency (above the norm) under these conditions of emotional tension. Recording of spectra on paper was performed in logarithmic scale using the stripchart recorder described above. In order to investigate the behavior of the speech signal envelopes in several frequency bands and clarify the nature of changes in the spectral structure, considering the coarseness of the resolution of the spectral analyzer with the one-third octave filters, some of the calculations (90 pronunciations of words) were performed by type M-20 digital /184
digital computer (see §6.2) at the Institute of Mathematics, Siberian Affiliate, Academy of Sciences USSR[1].

As an example, Figure 74 shows the envelopes calculated by machine for instantaneous spectra $\Phi_1(f)$, $\Phi_2(f)$ [during observation time (0-T) and (T-2T) respectively] and current spectra $\Phi_{1-2}(t)$, $\Phi(f)$ [observation time in the first case (0-2T) and in the second case (0-T_c)] for the word "Ponyal" in the frequency area being investigated when spoken by an actor under certain conditions (see Chapter 2) for the state of rest (see Figure 74 A) and joyous excitation (see Figure 74 B). The frequency step of the spectral readings was 25 Hz. The analysis interval T = 100 msec was determined by the length of the voiced (vowel) sounds, T_c from the duration of realization of the word (coordinate origin at point t = 0). It follows from analysis of the graphs of $\Phi(f)$ that during emotional tension the spectral areas with various degrees of energy concentration (200-350, 375-675, 875-1050 Hz) were expanded (particularly 375-675 Hz) and shifted toward the higher frequencies relative to the corresponding zones for the state of rest (150-225, /185
325-450, 475-625 Hz). However, in any of the situations analyzed, the resulting spectrum $\Phi(f)$ is a function of the entire realization investigated [see $\Phi_1(f)$, $\Phi_2(f)$, $\Phi_{1-2}(f)$], the width and frequency placement of the area (second) with greatest concentration of energy in both cases depend on the nature of the formant structure of the spectrum of $\Phi_1(f)$ in the frequency zone being analyzed, which is determined basically by the quasi-stable sector of the plosive vowel sound, since the duration of the plosive "P" sound is short in comparison to the

[1] The authors express their deep gratitude to the colleagues of the institute N. G. Zagoruyko, G. Ya. Voloshin, B. M. Kurilov, V. V. Vlasov, V. K. Lozovskiy, for their cooperation and help in performing this part of our work.

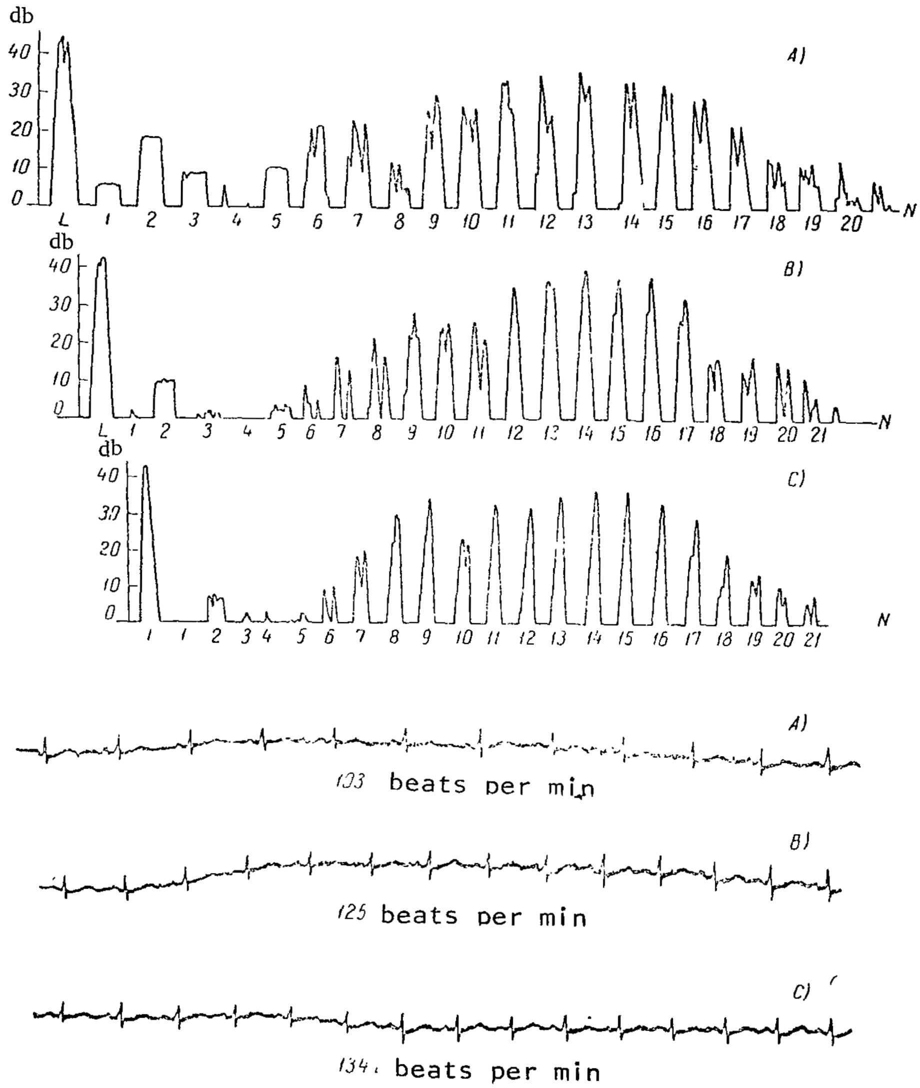
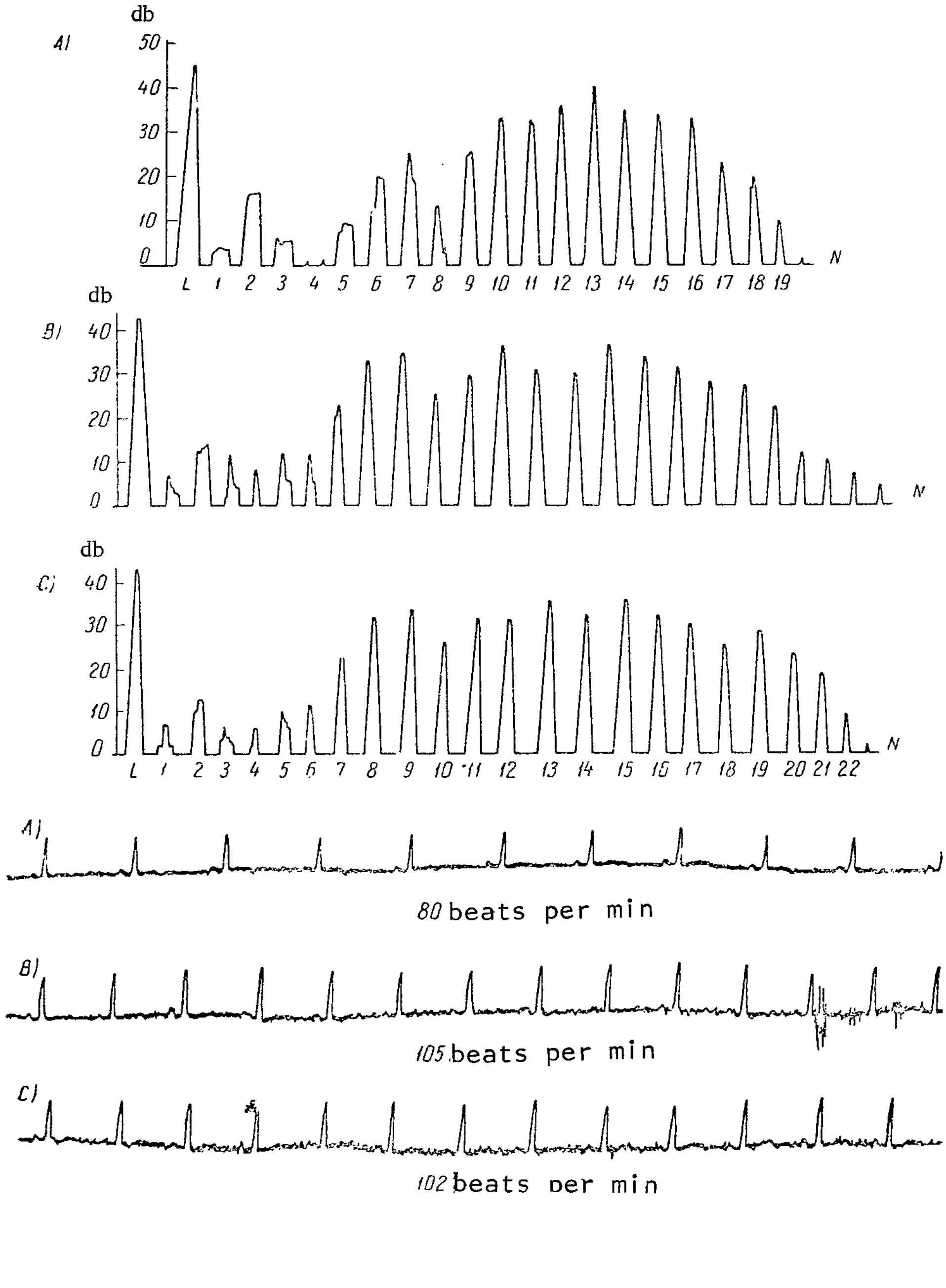

db
40
30
20
10
0
A)
L 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
N
B)
L 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
C)
1 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
A)
103 beats per min
B)
125 beats per min
C)
134 beats per min

Figure 72



db
A)
50
40
30
20
10
0
N
L 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
db
B)
40
30
20
10
0
N
db
C)
40
30
20
10
0
N
L 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
A)
80 beats per min
B)
105 beats per min
C)
102 beats per min

Figure 7.3

observation interval T[1]. In this sense, composition of graphs of $\Phi_1(f)$ (see Figure 74 A and 74 B) allows us to perform differentiation of situations according to the degree of emotional stress. These specifics were also observed in experimental modeling of negative emotions. Analysis of the changes in the spectral composition of vowel sounds for various emotional states was not performed separately.

The preliminary results of machine processing also showed that characteristics related to the usage of the envelopes of the speech signal in certain frequency areas might be quite informative (see Chapter 2). In the case at hand, spectral resolution of the process was achieved using five slightly overlapping filters, the pass bands of which were determined using the formula

$$\Delta f_N = 2^{N-1}(225 - 450) \text{ Hz}$$

where N is the ordinal number of the filter ($1 \leqslant N \leqslant 5$).

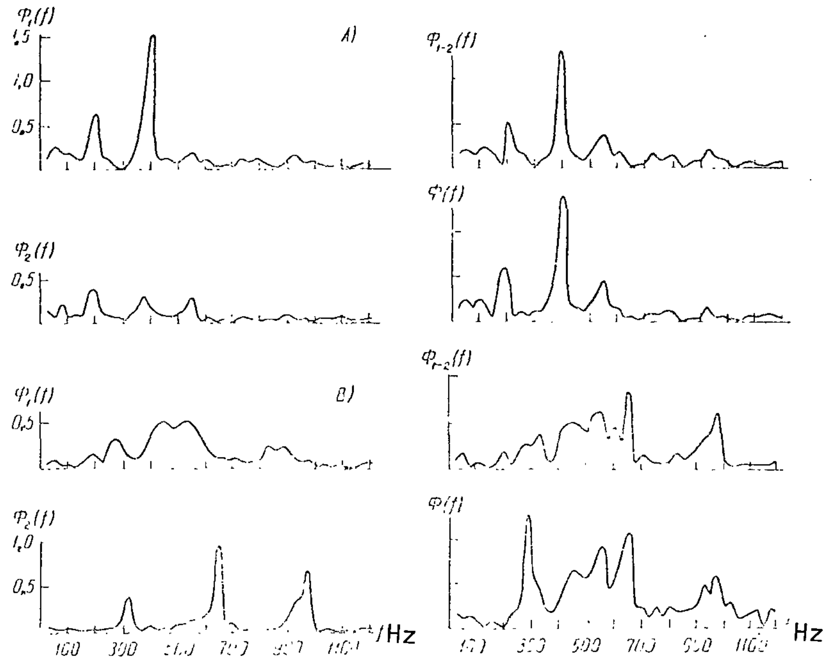


Figure 74

[1] These discussions are true for any word with a similar spectral composition.

After the corresponding nonlinear transforms, the envelopes of signal $u_N(t)$ were separated in each of the channels by an integrating filter with a cutoff frequency f_{co} = 25 Hz, which in turn determined the quantization interval $T_q = [2f_{co}]^{-1}$ for input of data to the digital machine.

In order to estimate the degree and nature of emotional stress, we use the quantities λ, μ, η:

$$\mu = \begin{cases} \frac{\alpha_1}{\alpha_i} & \text{if } \alpha_1 < \alpha_5; \\ \frac{\alpha_1}{\alpha_5} & \text{if } \alpha_5 < \alpha_1; \end{cases} \qquad \eta = \begin{cases} \frac{\alpha_5}{\alpha_2} & \text{if } \alpha_2 < \alpha_3; \\ \frac{\alpha_5}{\alpha_3} & \text{if } \alpha_3 < \alpha_2; \end{cases}$$

$$\lambda = \frac{\alpha_1}{\alpha_2} + \frac{\alpha_1}{\alpha_3} + \frac{\alpha_2}{\alpha_3}; \quad \alpha_i = \int_0^{T_c} u_i(t)\,dt; \; i = 1 \ldots 5. \qquad (6.3.9)$$

An increase in emotional stress corresponded to a decrease in the sum λ beneath its value in the state of rest, while subsequent differentiation of the nature of the emotion was performed on the basis of comparison of the quantities μ and η. Where $\mu > \eta$, negative emotions were recorded, while where $\mu < \eta$ -- positive emotions were noted. /186

These rules were used to estimate the states of test subjects in experiments on experimental modeling of human emotions (see Chapter 2) by actors (ten speakers). Conditions were imitated causing the emotions of "fear" or "joy" (pronouncing the words "ponyal," "khorosho" [good]). Proper identification of emotionally colored states was performed in 37 of 40 cases, while in two situations (of those separated) the state of "joy" was confused with the state of "fear" and in two cases the state of "fear" was confused with the state of "joy." The nature of the changes in the speech signal envelopes at the outputs of the five-channel analyzer (word "khorosho") is shown on Figure 75 (A -- "quiet," B -- "joy," C -- "fear").

In conclusion it should be noted that in order to estimate the reliability of the spectral characteristics of speech intonations described in this section, further testing and clarification of the data produced using broad statistical material, including investigations under actual conditions, are desirable. In the next section, we will analyze problems of the practical utilization of certain results.

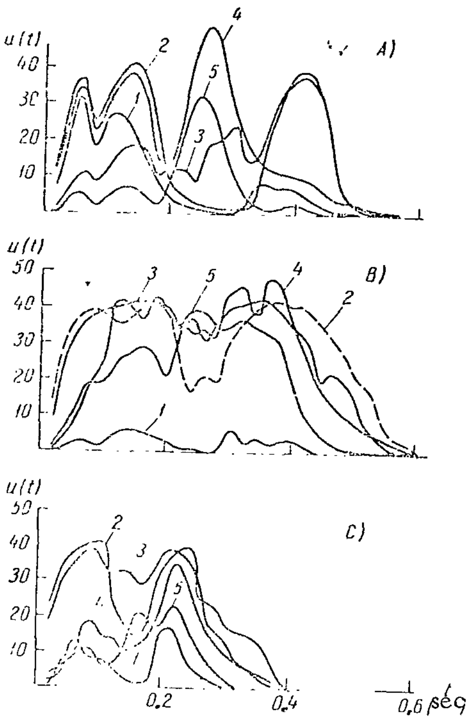


Figure 75

§6.4. Practical Application of Results of Spectral Evaluation of Speech Intonation Characteristics

The method outlined above was used in combination with other electrophysiological indicators for a qualitative evaluation of the degree of emotional stress of cosmonaut A. A. Leonov during /187
his flight into space (the flight of the *Voskhod 2* spacecraft) and training in the heat and pressure chamber. The results of resolution of his speech signal into a spectrum using the band pass spectral analyzer were investigated.

Let us analyze the errors related to application of the method in the presence of additive, stable broad band noise, since the flight recordings of the conversations of the cosmonauts contain noise. Based on spectral resolution of fluctuations in pauses between words, the noise spectrum can be approximately assumed even through the band width of the speech signal analyzed.

Processing of the experimental material on the band pass spectral analyzer (see §6.3) showed that an important portion of the information concerning the degree of emotional stress of the test subject when this method is used is included in changes in the weighted mean frequency f_I; therefore, we analyze below the approximate estimate

$$f_I = \frac{\sum_{i=1}^{U} f_i P_i}{\sum_{i=1}^{U} P_i}, \tag{6.4.10}$$

which has less dispersion than the quantity M [see (6.3.6)], resulting from fluctuation noise, and allows the final expressions to be simplified.

It follows from our analysis of formula (6.4.9) that f_I is a function of the variables $P_1,\ldots,P_U$ and, consequently, the calculation errors in f_I depend on the measurement errors of powers $P_1,\ldots,P_U$ at the outputs of the corresponding spectral analyzer filters, a block diagram of the k-th and m-th channels of which is presented on Figure 76 (m = k + 1; k - 1). The input action $\gamma(t)$ is an additive mixture of the speech signal s(t) and the fluctuation noise n(t)

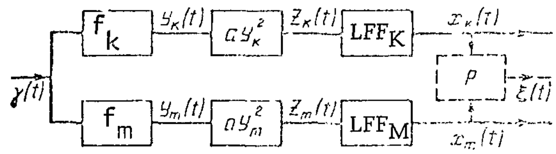


Figure 76

$$\gamma(t) = s(t) + n(t). \tag{6.4.11}$$

Using the results of experiments and the method of investigation of the speech signal, it can be assumed that one of the determining factors in the evaluation of the degree of emotional stress of the operator in this case is the change in the spectrum of vowel sounds, particularly plosive vowels. Considering the fact that the period of oscillations of the base tone ($T_{bt} \leqslant 10$ msec, see §6.1) is much less than the duration T of the quasi-stable sector of the voiced sound (in /188

particular a plosive vowel), we will represent the speech signal at the output of the spectral analyzer in an idealized form (Sopozhkov, 1963; Pokrovskiy, 1962):

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{l} A_n \cos(n\omega_{bt} t - \varphi_n), \tag{6.4.12}$$

where A_n, ϕ_n are the amplitude and phase of the n-th harmonic; ω_{bt} is the angular frequency of the base tone.

Expression (6.4.12) can be written in combined form:

$$s(t) = \frac{1}{2} \sum_{n=-l}^{l} A(j\omega_{bt} n) e^{j\omega_{bt} nt} = \frac{1}{2} \sum_{n=-l}^{l} \dot{A}_n e^{j\omega_{bt} nt}, \tag{6.4.12'}$$

where

$$A(j\omega_{bt}n) = \dot{A}_n = A_a e^{-j\varphi_n} = a_n - jb_n;$$
$$A[j\omega_{bt}(-n)] = \dot{A}_{-n} = A_n e^{j\varphi_n} = a_n + jb_n;$$
$$\dot{A}_n = \frac{2}{T_{bt}} \int\limits_{-\frac{T_{bt}}{2}}^{\frac{T_{bt}}{2}} s(t)\, e^{-j\omega_{bt} nt}\, dt.$$

We note that the relative error in calculation of the correlation function of signal (6.4.12) due to the finite analysis time $T \to 0$ with increasing quantity T and for the observation interval $T \approx$ 100-200 msec ($T_{bt} \leqslant$ 10 msec) does not exceed a few percent (Voloshin, 1964).

Process $y_k(t)[y_m(t)]$, produced as a result of passage of input action $\gamma(t)$ through the filter system of the analyzer $\Phi_k(\Phi_m)$, is squared $ay_k^2(t) = z_k(t)[ay_m^2(t) = z_m(t)]$ and further averaged over time T by the low-frequency filter $LFF_k(LFF_m)$, the pulse transient characteristic h(t) of which is assumed even (see also §4.2, 4.3)

$$h(t) = 1/T. \tag{6.4.13}$$

The output signal $x_k(t)[x_m(t)]$ is evaluated in this case by the mean (over the integration time t = T) power of the process in the k-th (m-th) spectral analyzer channel. The operation of comparison of estimates $x_i(t)$ in the assigned spectral range is represented in Figure 76 arbitrarily by unit P (the block diagram illustrates the case of only two channels for simplicity: k and m). The comparison signal with threshold $\xi(t)$, characterizing the change in state of the test subject, determines the set of values $x_i(t)$ which is then used for calculation of the weighted mean frequency (see §6.3). /189

Since the pass band of resonant system Φ_k (or Φ_m) is much narrower than the frequency band of the input noise n(t), which has an even spectrum, the distribution of noise at the outputs of the corresponding filters of the analyzer will be considered approximately normal.

Considering the above, let us write the expression for the correlation function of process $z_k(t)$ at the output of the nonlinear element of the k-th channel (signal and noise assumed independent):

$$B_{z_k}(t, t-\tau) = \overline{[n_k(t) + s_k(t)]^2 [n_k(t-\tau) + s_k(t-\tau)]^2 a^2} =$$
$$= \overline{n_k^2(t) n_k^2(t-\tau)} + \overline{2n_k^2(t) n_k(t-\tau) s_k(t-\tau)} + \overline{n_k^2(t) s_k^2(t-\tau)} +$$
$$+ \overline{2n_k(t) n_k^2(t-\tau) s_k(t)} + \overline{4n_k(t) n_k(t-\tau) s_k(t) s_k(t-\tau)} +$$
$$+ \overline{2n_k(t) s_k(t) s_k^2(t-\tau)} + \overline{s_k^2(t) n_k^2(t-\tau)} + \overline{2n_k(t-\tau) s_k^2(t) s_k(t-\tau)} +$$
$$+ \overline{s_k^2(t) s_k^2(t-\tau)}.$$

Here, for simplicity, we assume a = 1. Keeping in mind the equality for normally distributed quantities

$$\overline{x_1 x_2 x_3 x_4} = \overline{x_1 x_2} \cdot \overline{x_3 x_4} + \overline{x_1 x_3} \cdot \overline{x_2 x_4} + \overline{x_1 x_4} \cdot \overline{x_2 x_3}; \quad \overline{x_1 x_2 x_3} = 0; \quad \overline{x_i} = 0;$$

we introduce the representation

$$B_x(\tau) = \overline{x(t) x(t-\tau)}; \quad \overline{x^2(t)} = \sigma_x^2,$$

and obtain

$$B_{z_k}(t, t-\tau) = \sigma_{n_k}^4 + 2B_{n_k}^2(\tau) + \sigma_{n_k}^2 [s_k^2(t-\tau) + s_k^2(t)] +$$
$$+ 4B_{n_k}(\tau) s_k(t) s_k(t-\tau) + s_k^2(t) s_k^2(t-\tau), \tag{6.4.14}$$

and, after time-averaging (wavy overline)

$$B_{z_k}^*(\tau) = \widetilde{B_{z_k}(t, t-\tau)} = \widetilde{\sigma_{n_k}^4} + \widetilde{2B_{n_k}^2(\tau)} + \widetilde{\sigma_{n_k}^2 [s_k^2(t-\tau) + s_k(t)]} +$$
$$+ \widetilde{4B_{n_k}(\tau) s_k(t) s_k(t-\tau)} + \widetilde{s_k^2(t) s_k^2(t-\tau)}$$

we find

$$B_{z_k}^*(\tau) = \sigma_{n_k}^4 + 2\sigma_n^2 P_{s_k} + B_{s_k^2}^*(\tau) + 4B_{n_k}(\tau) B_{s_k}^*(\tau) + 2B_{n_k}^2(\tau), \tag{6.4.15}$$

where /190

$$\widetilde{s_k^2(t)\,s_k^2(t-\tau)} = B^*_{s_k^2}(\tau);\quad \widetilde{s_k(t)\,s_k(t-\tau)} = B^*_{s_k}(\tau);\quad \widetilde{s_k^2} = P_{s_k}.$$

In the general case, the time correlation function of the deterministic signal $s_k(t)$, acting where $-\infty < t < \infty$, is determined by the equality

$$B^*_{s_k}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} s_k(t)\, s_k(t-\tau)\, d\tau,$$

which, for example, for a deterministic periodic process with period t_p can be written in the form

$$B^*_{s_k}(\tau) = \frac{1}{T_p} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} s_k(t)\, s_k(t-\tau)\, dt.$$

The time-averaged value of mathematical expectation $m^*_{z_k}$ of process $z_k(t)$ at the output of the k-th squaring device is determined from the expression (a = 1):

$$m_{z_k} = a^2\, \widetilde{[s_k(t) + n_k(t)]^2} = \sigma^2_{n_k} + P_{s_k}. \qquad (6.4.16)$$

Expression (6.4.15), characterizing the energy spectrum of the reaction of the nonlinear element (4.2.25) to the input action $y_k(t)$, allows us, using the known parameters of the integrating filter LFF_k, to perform an estimate of the spectrum at its output (4.2.2). The first two terms of formula (6.4.15) determine the constant component (sum of square of dispersions of noise n(t) and mean power of beat between components of random and deterministic input action frequencies), the third component corresponds to the discrete spectrum (harmonics of beats of components of deterministic portion of process), while the fourth and fifth terms define the continuous portion of the spectrum (result of beating between harmonics of deterministic and random components and components of random portion of reaction $y_k(t)$ of filter Φ_k respectively).

Problems of the calculation of the correlation functions of signal $s_k(t)$ and noise $n_k(t)$ in expression (6.4.15) with fixed input action $\gamma(t)$, parameters of the filtering system of the analyzer Φ_k and squaring device of the k-th channel (see Figure 76) are presented in detail in the literature (Levin, 1966; Tikhonov, 1966).

Calculation of the sums of various combinations of components in the time correlation function $B^*_{s^2}(\tau)$ [see (6.4.15)] in the case of deterministic periodic signal with period T_0 is presented in Appendix 4. We note that process $s_k(t)$ at the output of the k-th filter of the analyzer is represented in the form [see (6.4.12')] /191

$$s_k(t) = \frac{1}{2} \sum_{n=-l}^{l} A(jn\omega_{bt}) H_k(jn\omega_{bt}) e^{jn\omega_{bt} t} = \frac{1}{2} \sum_{n=-l}^{l} \dot{C}_{k,n} e^{jn\omega_{bt} t}, \tag{6.4.17}$$

where

$$\dot{C}_{k,n} = C_{k,n} e^{-j\psi_{k,n}} = C(jn\omega_{bt}) = A(jn\omega_{bt}) H_k(jn\omega_{bt}),$$

where $H_k(jn\omega_{bt})$ is the transfer function of the k-th filter.

For real functions of time, equality (6.4.17) is equivalent to the expression

$$s_k(t) = \frac{C_0}{2} + \sum_{n=1}^{l} C_{k,n} \cos(n\omega_{bt} t - \psi_{k,n}). \tag{6.4.18}$$

Analogous discussions can be performed in estimating the mutual energy spectrum of responses $z_k(t)$ and $z_m(t)$ of the nonlinear elements in the k-th and m-th channels, which are characterized in the general case by the time-averaged mutual correlation functions $B^*_{z_k z_m}(\tau)$ and $B^*_{z_m z_k}(\tau)$ (Levin, 1966):

$$\begin{aligned} B^*_{z_k z_m}(\tau) &= \overline{[n_k(t) + s_k(t)]^2 [n_m(t-\tau) + s_m(t-\tau)]^2} = \\ &= \sigma^2_{n_k}\sigma^2_{n_m} + 2B^2_{n_k n_m}(\tau) + \sigma^2_{n_k} P_{s_m} + 4B_{n_k n_m}(\tau) B^*_{s_k s_m}(\tau) + \\ &\quad + \sigma^2_{n_m} P_{s_k} + B^*_{s^2_k s^2_m}(\tau), \end{aligned} \tag{6.4.19}$$

where

$$B^{*}_{s_k^2 s_m^2}(\tau) = \overline{s_k^2(t)\, s_m^2(t-\tau)}; \quad B_{n_k n_m}(\tau) = \overline{n_k(t)\, n_m(t-\tau)}; \quad a = 1.$$

The mathematical expectation $m_{x_k}(T)$, the dispersion $\sigma^2_{x_k}(T)$ and the second mixed moment $B_{x_k x_m}(T)$ correspond to the outputs of integrating filters LFF_k; LFF_m of the analyzer at moment in time $t = T$ are determined on the basis of formulas (4.2.8), (4.2.13) and (4.3.95):

$$m_{x_k}(T) = \overline{x_k(T)} \int_{-\infty}^{\infty} h(T-t,\, T)\, \overline{z_k(t)}\, dt; \tag{6.4.20}$$

$$\sigma^2_{x_k}(T) = \iint_{-\infty}^{\infty} h(T-t_1,\, T)\, h(T-t_2,\, T)\, [\overline{z_k(t_1)\, z_k(t_2)} - \overline{z_k(t_1)}\; \overline{z_k(t_2)}] \times \times dt_1 dt_2; \tag{6.4.21}$$

$$B_{x_k x_m}(T) = \iint_{-\infty}^{\infty} h(T-t_1,\, T)\, h(T-t_2,\, T)\, [\overline{z_k(t_1)\, z_m(t_2)} - - \overline{z_k(t_1)}\; \overline{z_m(t_2)}]\, dt_1 dt_2. \tag{6.4.22}$$ /192

Process $z_k(t)$ is unstable; therefore, mathematical expectation $\overline{z_k(t)} = m_{z_k}(t)$ cannot be removed from the integral sign (6.4.20).

In many cases, in order to compress the dynamic range, the operation of integration of process $z_k(t)$ is followed by logarithmic transform $\zeta_k(t)$:

$$\zeta_k(t) = \log x_k(t).$$

Since in the case at hand, strict solution of the problem of finding the required characteristics of function $\zeta_k(t = T)$ and the second mixed moment $B_{\zeta_k \zeta_m}(T)$ requires rather complex calculations, assuming at the output of the integrator $m_{x_k}(T) \gg \sigma_{x_k}(T)$, we can use the approximate expressions

$$x_k(T) = \overline{x_k(T)} + x_k(T) - \overline{x_k(T)} = \overline{x_k(T)}\left[1 + \frac{x_k(T) - \overline{x_k(T)}}{\overline{x_k(T)}}\right];$$
$$\zeta_k(T) = \log \overline{x_k(T)} + \log\left[1 + \frac{x_k(T) - \overline{x_k(T)}}{\overline{x_k(T)}}\right] \simeq \log \overline{x_k(T)} + \tag{6.4.23}$$
$$+ \frac{x_k(T) - \overline{x_k(T)}}{\overline{x_k(T)}},$$

allowing us to simplify the required calculations and produce rather satisfactory practical results (Smirnov, Dunin-Barkovskiy, 1965).

On the basis of (6.4.23), the dispersion $\sigma^2_{\zeta_k}$ (T) and mathematical expectation m_{s_k} (T) of the logarithmic function ζ_k(T) are equal to

$$\overline{\zeta_k(T)} = \log \overline{x_k(T)}; \tag{6.4.24}$$

$$\sigma^2_{\zeta_k}(T) = \frac{\sigma^2_{x_k}(T)}{[\overline{x_k(T)}]^2}, \tag{6.4.25}$$

and the nature of its distribution is determined by the distribution of the centered random quantity α_k(T):

$$\alpha_k(T) = \frac{x_k(T) - \overline{x_k(T)}}{\overline{x_k(T)}}. \tag{6.4.26}$$

The second mixed moment $B_{\zeta_k\zeta_m}$ (T) after logarithmic transformation of the actions x_k(T), x_m(T) is easily expressed through the corresponding characteristics (6.4.20), (6.4.22) of the outputs of the integrators [see also (6.4.23), (6.4.24)]: /193

$$B_{\zeta_k\zeta_m}(T) = \overline{[\zeta_k(T) - \overline{\zeta_k(T)}][\zeta_m(T) - \overline{\zeta_m(T)}]} =$$
$$= \frac{\overline{[x_k(T) - \overline{x_k(T)}][x_m(T) - \overline{x_m(T)}]}}{\overline{x_k(T)}\cdot\overline{x_m(T)}} = \frac{B_{x_k x_m}(T)}{\overline{x_k(T)}\cdot\overline{x_m(T)}}. \tag{6.4.27}$$

Let us analyze further the problems related to errors in measurement of the weighted mean frequency f_I due to fluctuation noise. Assuming in our case (integration time T much greater than correlation interval of noise τ_{cor}) the masses of the probability distribution $(x_1,\ldots,x_U)$ are concentrated primarily in a small area about point A $(\overline{x}_1, \ldots, \overline{x}_U)$, we can replace with some approximation the function

$$f_I(T) = \frac{\sum_{i=1}^{U} x_i(T) f_i}{\sum_{i=1}^{U} x_i(T)} \tag{6.4.28}$$

by the linear terms of its expansion into a Taylor series about the point A $[\overline{x_1(T)} \ldots \overline{x_U(T)}]$ (see §4.3)

$$f_I(T) \approx \frac{\sum_{i=1}^{U} \overline{x_i(T)} f_i}{\sum_{i=1}^{U} \overline{x_i(T)}} + \sum_{j=1}^{U} \frac{\partial f_I(T)}{\partial x_j(T)} [x_j(T) - \overline{x_j(T)}]. \tag{6.4.29}$$

The mathematical expectation $m_{f_I}(T)$ and dispersion $\sigma^2_{f_I}(T)$ of centroid $f_I(T)$ can be found on the basis of the expressions

$$m_{f_I}(T) = \overline{f_I(T)} = \frac{\sum_{i=1}^{U} \overline{x_i(T)} f_i}{\sum_{i=1}^{U} \overline{x_i(T)}}; \tag{6.4.30}$$

$$\begin{aligned} \sigma^2_{f_I}(T) &= \sum_{i=1}^{U} \sum_{j=1}^{U} \frac{\partial f_I(T)}{\partial x_i(T)} \cdot \frac{\partial f_I(T)}{\partial x_j(T)} \overline{[x_i(T) - \overline{x_i(T)}][x_j(T) - \overline{x_j(T)}]} = \\ &= \sum_{i=1}^{U} \sum_{j=1}^{U} \left[\frac{f_i - \overline{f_I(T)}}{\sum_{l=1}^{U} \overline{x_l(T)}} \right] \left[\frac{f_j - \overline{f_I(T)}}{\sum_{k=1}^{U} \overline{x_k(T)}} \right] B_{x_i x_j}(T), \end{aligned} \tag{6.4.31}$$

where [see (6.4.28)] /194

$$\frac{\partial f_I(T)}{\partial x_i(T)} = \frac{f_i \sum_{i=1}^{U} \overline{x_i(T)} - \sum_{i=1}^{U} \overline{x_i(T)}\, f_i}{\sum_{i=1}^{U} \sum_{j=1}^{U} \overline{x_i(T)}\, \overline{x_j(T)}} = \frac{f_i - \overline{f_I(T)}}{\sum_{i=1}^{U} \overline{x_i(T)}}; \quad B_{x_i x_j}(T) = \sigma^2_{x_i}(T),\ i = j.$$

Thus, formulas (6.4.30), (6.4.31) allow us to calculate, under the conditions outlined above, the mathematical expectation and the dispersion of the weighted mean frequency f_I on the basis of the characteristics (6.4.20), (6.4.22) (output values $x_i(T)$ of integrating filters $1ff_i$). We note that with normally distributed arguments $x_1,\ldots,x_U$ (Slepian, 1958), it can be considered that the distribution density of the probability of function (6.4.29), a linear combination of random variables $x_1,\ldots,x_U$, is also normal. Since in the practical problem which we are analyzing the frequency characteristics of the spectral analyzer filters are approximately assumed weakly overlapping (and rectangular), we produce [see (6.4.22)]

$$B_{x_k x_m}(T) = 0; \qquad B_{z_k z_m}(t, t-\tau) = 0,$$

and formula (6.4.31) is written in the form

$$\sigma^2_{f_I}(T) = \sum_{i=1}^{U} \left[\frac{f_i - \overline{f_I(T)}}{\sum_{i=1}^{U} \overline{x_i(T)}} \right]^2 \sigma^2_{x_i}(T). \qquad (6.4.32)$$

Expression (6.4.21) can be represented as follows:

$$\sigma^2_{x_k}(T) = \iint_{-\infty}^{\infty} h(T-t_1, T)\, h(T-t_2, T)\, [B_{z_k}(t_1, t_2) - \overline{z_k(t_1)}\, \overline{z_k(t_2)}]\, dt_1 dt_2 = \iint_{-\infty}^{\infty} h(T-t_1, T)\, h(T-t_2, T)\, [2B^2_{n_k}(t_1-t_2) + 4B_{n_k}(t_1-t_2)\, s_k(t_1)\, s_k(t_2)]\, dt_1 dt_2, \qquad (6.4.33)$$

where

[see (6.4.14), where $t = t_1$; $\tau = t_1 - t_2$; $a = 1$]

$$B_{z_k}(t_1, t_2) - \overline{z_k(t_1)}\,\overline{z_k(t_2)} = \overline{[n_k(t_1) + s_k(t_1)]^2 [n_k(t_2) + s_k(t_2)]^2} -$$
$$- \overline{[n_k(t_1) + s_k(t_1)]^2}\,\overline{[n_k(t_2) + s_k(t_2)]^2} = \sigma^4_{n_k} + 2B^2_{n_k}(t_1 - t_2) +$$
$$+ \sigma^2_{n_k}[s_k^2(t_1) + s_k^2(t_2)] + 4B_{n_k}(t_1 - t_2)\, s_k(t_1)\, s_k(t_2) + s_k^2(t_1)\, s_k^2(t_2) -$$
$$- \sigma^4_{n_k} - \sigma^2_{n_k}[s_k^2(t_1) + s_k^2(t_2)] - s_k^2(t_1)\, s_k^2(t_2) = 2B^2_{n_k}(t_1 - t_2) +$$
$$+ 4B_{n_k}(t_1 - t_2)\, s_k(t_1)\, s_k(t_2).$$

Considering equation (6.4.18) for signal $s_k(t)$ in the k-th channel (the constant component in the frequency band investigated is equal to zero), and keeping in mind the general expression for the correlation function of noise $B_{n_k}(t_1 - t_2)$ [see (4.3.80)] /195

$$B_{n_k}(t_1 - t_2) = B_{n_k 0}(t_1 - t_2) \cos \omega_k (t_1 - t_2)$$

at the output of the narrow band filter with frequency characteristics symmetrical relative to the resonant frequency ω_k, we produce, when the following inequality is fulfilled

$$\Delta F_i = \frac{0{,}5}{T} \ll \omega_{bt}$$

where ΔF_i is the pass band of the integrating filter (4.2.28), meaning separation by the integrator of a band of frequencies adjacent to $\omega = 0$ [see (6.4.15); §4.2], an approximate relationship for the dispersion $\sigma^2_{x_k}(T)$ (the upper estimate we produce is $\omega_k = n\omega_{bt}$; $1 \leqslant n \leqslant l$)

$$\sigma^2_{x_k}(T) \approx \iint\limits_{-\infty}^{T} h(T - t_1, T)\, h(T - t_2, T)\, [B^2_{n_k 0}(t_1 - t_2) +$$
$$+ B_{n_k 0}(t_1 - t_2)\, C^2_{k,n}]\, dt_1 dt_2, \qquad (6.4.34)$$

where the integrand uses the video terms of the expansion

$$4B_{n_k 0}(t_1-t_2)\cos\omega_k(t_1-t_2)C_{k,n}\cos(n\omega_{0.T}t_1-\psi_{k,n})C_{k,n}\cos\times$$
$$\times(n\omega_{0.T}t_2-\psi_{k,n}) = 4B_{n_k 0}(t_1-t_2)\cos\omega_k(t_1-t_2)\times$$
$$\times\frac{C^2_{k,n}}{2}[\cos\omega_k(t_1-t_2)+|\cos[\bar{\omega}_k(t_1+t_2)-2\psi_{k,n}]=$$
$$=B_{n_k 0}(t_1-t_2)C^2_{k,n}[1+\cos 2\omega_k(t_1-t_2)+\cos[2\omega_k t_1-\psi_{k,n}]+\cos[2\omega_k t_2-\psi_{k,n}]=$$
$$2B^2_{n_k}(t_1-t_2)=B^2_{n_k 0}(t_1-t_2)[1+\cos 2\omega_k(t_1-t_2)].$$

The other components, arising due to beating of the signal and noise, are not analyzed, since their contribution to the output effect of the integrating filter in this case is insignificant. Performing in (6.4.34) replacement of variables $\tau = t_1 - t_2$; $t_2 = t$ and considering that $\Delta F_i \ll \Delta f_k$, where Δf_k is the pass band of the k-th filter, we produce [see (4.2.15); (4.2.30); (4.2.55); (4.3.90)]

$$\sigma^2_{x_k}(T)\approx\iint_{-\infty}^{\infty}h(T-t,T)\,h(T-t-\tau,T)[B^2_{n_k 0}(\tau)+$$
$$+C^2_{k,n}B_{n_k 0}(\tau)]\,dt\,d\tau\simeq\int_{-\infty}^{\infty}\rho_h(\tau,T)[B^2_{n_k 0}(\tau)+C^2_{k,n}B_{n_k 0}(\tau)]\,d\tau\approx$$
$$\approx\frac{2}{T}\int_0^{\infty}[B^2_{n_k 0}(\tau)+C^2_{k,n}B_{n_k 0}(\tau)]\,d\tau. \qquad (6.4.35)$$

On the basis of similar considerations, formula (6.4.20) is represented in the form /196

$$\overline{x_k(T)}=\int_{-\infty}^{\infty}h(T-t,T)\overline{\left[n_k(t)+\sum_{n=1}^{l}C_{k,n}\cos(n\omega_{0.T}t-\psi_{k,n})\right]^2}\,dt\approx$$
$$\approx\sigma^2_{n_k}+\sum_{n=1}^{l}\frac{C^2_{k,n}}{2}. \qquad (6.4.36)$$

we note that the limits of summation of signal components are determined for the given input process s(t) by the transfer characteristic of the k-th filter $H_k(jn\omega_{bt})$ [see (6.4.17)]. From this point of view, the sum

$$\sum_{n=1}^{l}\frac{C^2_{k,n}}{2}$$

characterizes the power of the signal P_{s_k} at the output of the k-th channel of the spectral analyzer.

The mathematical expectation of quantity $f_I(T)$

$$\overline{f_I(T)} = \frac{\sum_{i=1}^{U} f_i \overline{x_i(T)}}{\sum_{i=1}^{U} \overline{x_i(T)}} = \frac{\sum_{i=1}^{U} f_i (\sigma^2_{n_i} + P_{si})}{\sum_{i=1}^{U} (\sigma^2_{n_i} + P_{si})} = \frac{\sum_{i=1}^{U} f_i P_{si}\left(\frac{\sigma^2_{n_i}}{P_{si}} + 1\right)}{\sum_{i=1}^{U} P_{si}\left(\frac{\sigma^2_{n_i}}{P_{si}} + 1\right)} \qquad (6.4.37)$$

where

$1 + \frac{\sigma^2_{n_i}}{P_{si}} = A = \text{const}$ is equal to [see (6.4.10), $P_i = P_{s_i}$]

$$\overline{f_I(T)} = \frac{\sum_{i=1}^{U} f_i P_{si}}{\sum_{i=1}^{U} P_{si}} = f_I. \qquad (6.4.37')$$

In the case of different signal-noise ratios in the filters of the analyzer

$$\frac{\sigma^2_{n_i}}{P_{s_i}} \lessgtr \frac{\sigma^2_{n_k}}{P_{s_k}}$$

equality (6.4.37') is fulfilled approximately, more accurately, the stronger the inequality

$$\frac{\sigma^2_{n_i}}{P_{si}} < 1.$$

Substituting the value of the correlation function $B_{n_k0}(\tau)$ of low-frequency /197
fluctuations with spectral density $2N_0$ even in the band $\Delta\omega_k$ into integral (6.4.35) [see (4.2.25)]

$$B_{n_k0}(\tau) = \frac{1}{2\pi}\int_{-\frac{\Delta\omega_k}{2}}^{\frac{\Delta\omega_k}{2}} 2N_0 \cos\omega\tau\, d\omega = \sigma^2_{n_k}\frac{\sin\frac{\Delta\omega_k\tau}{2}}{\frac{\Delta\omega_k\tau}{2}};$$

$$\sigma^2_{n_k} = \frac{N_0\Delta\omega_k}{\pi},$$

we produce

$$\sigma^2_{x_k}(T) \simeq \frac{2}{T}\int_0^\infty \left[\sigma^4_{n_k}\frac{\sin^2\left(\frac{\Delta\omega_k\tau}{2}\right)}{\left(\frac{\Delta\omega_k\tau}{2}\right)^2} + C^2_{k,n}\sigma^2_{n_k}\frac{\sin\frac{\Delta\omega_k\tau}{2}}{\frac{\Delta\omega_k\tau}{2}}\right]d\tau =$$

$$= \frac{2\pi\sigma^4_{n_k}}{T\Delta\omega_k}\left(1 + 2\frac{C^2_{k,n}}{2\sigma^2_{n_k}}\right) \tag{6.4.38}$$

Let us transform expression (6.4.32), considering (6.4.38) and (6.4.36):

$$\sigma^2_{f_I}(T) = \sum_{i=1}^{U}\left[\frac{f_i \cdot \overline{f_I(T)}}{\sum_{l=1}^{U}\overline{x_l(T)}}\right]^2\sigma^2_{x_i}(T) =$$

$$= \frac{2\pi[\overline{f_I(T)}]^2}{T}\sum_{i=1}^{U}\left[\frac{\frac{f_i}{\overline{f_I(T)}} - 1}{\sum_{l=1}^{U}\sigma^2_{n_l}(1 + P_{s_l}/\sigma^2_{n_l})}\right]^2\left(1 + 2\frac{C^2_{i,n}}{2\sigma^2_{n_i}}\right)\frac{\sigma^4_{n_i}}{\Delta\omega_i}. \tag{6.4.39}$$

where, as was already noted, $C_{i,n}/2$ is the output harmonic of the signal of the i-th filter, the number n of which depends on the value of the resonant frequency of the i-th channel of the analyzer ω_i and is determined from the condition $n\omega_{bt} = \omega_i$. Analysis of formula (6.4.39) indicates that with otherwise unchanged conditions, dispersion $\sigma^2_{f_I}(T)$ decreases with increasing ratio of the signal-noise value $P_{s_l}/\sigma^2_{n_l}$ at the outputs of the integrators (1ff).

Assuming further to produce a top estimate within the limits of the investigated U-group of filters

$$\min\left\{\frac{P_{sl}}{\sigma^2_{n_l}}\right\} = B = \text{const}, \quad \max\left|\frac{f_i}{f_1(T)} - 1\right| = D = \text{const};$$

$$\min\{\Delta\omega_i\} = E = \text{const}, \quad \frac{C^2_{i,n}}{2\sigma^2_{n_i}} = \frac{P_{si}}{\sigma^2_{n_i}}, \tag{6.4.40}$$

we write equation (6.4.39) in the form /198

$$\sigma^2_{f_1}(T) = \frac{2\pi[f_1(T)]^2 D^2(1+2B)}{TE(1+B)^2} \cdot \frac{\sum\limits_{i=1}^{U} \sigma^4_{n_i}}{\sum\limits_{l=1}^{U}\sum\limits_{m=1}^{U} \sigma^2_{n_l}\sigma^2_{n_m}}. \tag{6.4.41}$$

If the energy bands Δf_i of the filters Φ_i of the analyzer are expanded as the resonant frequencies f_i are increased (for example, third-octave filters) in proportion to a certain coefficient $d > 1$ ($\Delta f_i d = \Delta f_{i+1}$) and the input noise spectrum $n(t)$ is even, the power of fluctuations $\sigma^2_{n_i}$ can be represented in the form

$$\sigma^2_{n_i} d = \sigma^2_{n_{i+1}}, \quad 1 \leqslant i \leqslant U,$$

In this case, using the formula for the sum s_n of n terms of the geometrically increasing progression $d_1, d_2, \ldots, d_n$

$$s_n = \frac{d_1(q^n - 1)}{q - 1}, \quad d_n = d_1 q^{n-1},$$

we find [see (6.4.41)]

$$\frac{\sum\limits_{i=1}^{U} \sigma^4_{n_i}}{\sum\limits_{l=1}^{U}\sum\limits_{m=1}^{U} \sigma^2_{n_l}\sigma^2_{n_m}} = \frac{\sum\limits_{i=1}^{U} d^{2(i-1)}}{\sum\limits_{l=1}^{U}\sum\limits_{m=1}^{U} d^{l+m-2}} = \frac{1 + \dfrac{d^2[d^{2(U-1)} - 1]}{d^2 - 1}}{\left[1 + \dfrac{d(d^{U-1} - 1)}{d - 1}\right]^2} = \frac{(d-1)(d^U+1)}{(d+1)(d^U-1)} \tag{6.4.42}$$

and, consequently,

$$\sigma^2_{f_I}(T) = [\overline{f_I(T)}]^2 \frac{2\pi D^2 (1+2B)(d-1)(d^U+1)}{TE(1+B)^2(d+1)(d^U-1)}, \quad U > 1. \tag{6.4.43}$$

Selecting, on the basis of the conditions of the experiment [see §6.3; 6.4] B = 3; E = 690 1/sec; d = 1.24; T = 200•10^{-3} sec, we produce an upper estimate for the mean square values of the quantity $f_I(T)$:

$$\begin{aligned} \sigma_{f_I}(T) &\simeq 0.017\overline{f_I(T)}, \quad D = 0.17, \quad U = 2; \\ \sigma_{f_I}(T) &\simeq 0.025\overline{f_I(T)}, \quad D = 0.3, \quad U = 3; \\ \sigma_{f_I}(T) &\simeq 0.04\overline{f_I(T)}, \quad D = 0.57, \quad U = 4. \end{aligned} \tag{6.4.43'}$$

Formula (6.4.43) is not correct for determination of the formant area of a speech signal within the limits of the i-th filter (U = 1), since in this case $f_I(T) = f_i$ [see (6.4.30)], and the coefficients of the expansion into a Taylor series (6.4.29) for function $f_I(T)$ become equal to zero ($f_i - \overline{f_I(T)} = 0$). In this case, we can use, for example, the assumption of even distribution of values of the centroid within the limits of the pass band of the filter $\Delta\omega_i$, and consider /199

$$\sigma^2_{f_I} = \frac{\Delta\omega_i^2}{48\pi^2} = \frac{\Delta f_i^2}{12}.$$

Measuring the power of noise at the outputs of the integrating filters in the pauses between words (which was possible in the experiments performed), we can improve the estimate of the weighted mean frequency f_I to some extent.

Representing the output effect of the k-th integrator in the absence of a speech signal as $x'_k(T)$, we can represent equation (6.4.28) in the form

$$f_I^*(T) = \frac{\sum_{i=1}^{U} [x_i(T) - x_i'(T)] f_i}{\sum_{i=1}^{U} [x_i(T) - x_i'(T)]} = \frac{\sum_{i=1}^{U} x_i^*(T) f_i}{\sum_{i=1}^{U} x_i^*(T)}$$

On the basis of the formulas [(6.4.29), (6.4.30), (6.4.32), (6.4.36)], the mathematical expectation and dispersion of the quantity $f_I^*(T)$ are determined by the expressions

$$\overline{f_I^*(T)} = \frac{\sum_{i=1}^{U} P_{s_i} f_i}{\sum_{i=1}^{U} P_{s_i}} = f_I;$$

$$\sigma^2_{f_I^*}(T) = \sum_{i=1}^{U} \left[\frac{f_i - \overline{f_I^*(T)}}{\sum_{k=1}^{U} \overline{x_k^*(T)}} \right]^2 \sigma^2_{x_i^*}(T), \tag{6.4.44}$$

where

$$\overline{X_k^*(T)} = \overline{X_k(T)} - \overline{X_k'(T)} = P_{s_k}$$

Analysis of equations (6.4.44) indicates that the mathematical expectation of the estimate of the weighted mean frequency with noise $\overline{f_I^*(T)}$ is equal to the value of centroid f_I without fluctuation noise and is independent of the signal/noise ratio [see (6.4.37); (6.4.37')], i.e. this estimate is unbiased. Keeping in mind the possibility of rather long (T'), the measurement of noise power in pauses between words ($T' \gg \tau_{cor}$; $\sigma^2_x \gg \sigma^2_{x'_i}$, where $P_{s_i}/\sigma^2_{n_i} \to 0$), we write, on the basis of the theorem of dispersion of a linear combination of random independent quantities, /200

$$\sigma^2_{x_i^*}(T) = \sigma^2_{x_i}(T) + \sigma^2_{x'_i}(T) \approx \sigma^2_{x_i}(T), \tag{6.4.45}$$

from which, considering the factors presented above in concluding formulas (6.4.43), we produce [see (6.4.44)]

$$\sigma^2_{f_I^*}(T) = [\overline{f_I^*(T)}]^2 \frac{2\pi D^{*2}(1+2B)(d-1)(d^U+1)}{TEB^2(d+1)(d^U-1)}, \tag{6.4.46}$$

where

$$\sum_{i=1}^{U} P_{s_i} = \sum_{i=1}^{U} \frac{P_{s_i}}{\sigma^2_{n_i}} \sigma^2_{n_i} = B \sum_{i=1}^{U} \sigma^2_{n_i}.$$

$$D^* = \left| \frac{f_i}{\overline{f_I^*(T)}} - 1 \right| \tag{6.4.47}$$

In order to select the output values $x_k(T)$ (or $x_k^*(T)$) of the group of filters U according to some threshold criterion, required in calculating the weighted mean frequency (see §6.3), we used an approximate method for calculating the dispersion of the mixture $\sigma^2_{x_k}(T)$ in the analyzer channels, since the value of the signal/noise ratio included in equation (6.4.38) is generally unknown. From our analysis, estimating the relative error $\delta_{x_k}(T)$ in expression [see (6.4.36), (6.4.38)]

$$\delta_{x_k}(T) = \frac{\sigma_{x_k}(T)}{x_k(T)} \approx \sqrt{\frac{1}{\Delta f_k T}} \cdot \sqrt{\frac{1+2B}{(1+B)^2}}; \qquad (6.4.48)$$

$$\min\left\{\frac{P_{s_k}}{\sigma^2_{n_k}}\right\} = B,$$

it follows that $\delta_{x_k}(T)$ changes monotonously within the range

$$0 \leqslant \delta_{x_k}(T) \leqslant \frac{1}{\sqrt{\Delta f_k T}}.$$

with simultaneous change in B within the limits ($\Delta f_k T$ -- fixed), $\infty \geqslant B \geqslant 0$.

Measuring further the power of the mixture of signal and noise at the output of the linear filter (in the frequency band ΔF = 5 kHz), we produce for the relative error $\delta(T)$ [see (6.4.48)] /201

$$\frac{\sigma(T)}{x(T)} = \delta(T) \ll 1 \text{ when } B \to 0; \; T = 200 \cdot 10^{-3} \text{ sec.},$$

therefore in the first approximation we can consider a certain measure $x(T)$ equal to the mathematical expectation of the process at the output of the integrator

$$x(T) \approx \overline{x(T)} = P_s + \sigma^2_n,$$

where P_s and σ^2_n are the powers of signal and noise at the input of the spectral analyzer. Similar discussions remain correct also for measurement over time $T' > T$ of the power of noise passing through a linear (or any other) filter of the analyzer:

$$x'(T) \approx \overline{x'(T)} = \sigma_n^2.$$

We note that in comparing the calculated value of quantity $f_I(T)$ or $f_I^*(T)$ with its mathematical expectation, we can also consider certain facts concerning the smallness of the relative error of measurement (6.4.43'). On the basis of experiments performed earlier (with no noise, see §6.3) for the words analyzed, we found that the power of the signal in the j-th filter (in this case the spectral analyzer channel with maximum output within the limits of the frequency band investigated) is characterized by inequality $P_{s_j} \leqslant 0.5\, P_s$, so that, keeping in mind the considerations presented above, we produce

$$P_{s_j} \leqslant \frac{x(T) - x'(T)}{2} = \frac{x^*(T)}{2}. \tag{6.4.49}$$

Thus, assuming $\sum_{i=1}^{U} P_{s_i} = U\frac{P_s}{2}$ and knowing the measured values of $\sigma^2_{n_1} \ldots \sigma^2_{n_j} \ldots \sigma^2_{n_U}$, we can produce maximum estimates of $\sigma^2_{x_1} \ldots \sigma^2_{x_j} \ldots \sigma^2_{x_U}$ [see (6.4.38)] with known parameters $\Delta f_1 \ldots \Delta f_j \ldots \Delta f_U$ and observation time T. Since the random quantities $x_i(T)$ and $x'_i(T)$ have normal distribution in the first approximation, their difference $x_i^*(T) = x_i(T) - x'_i(T)$, like any linear combination of such quantities, is also distributed normally with dispersion $\sigma^2_{x_i}(T)$ [see (6.4.45)]. Further, we analyzed the ratio $q_{\alpha,j}$ for each pair of channels with numbers

$$j,\ \alpha = (j-n), \ldots, (j-1), (j+1), \ldots, (j+p);$$

$$q_{\alpha,j} = \frac{x^*_\alpha(T) - bx^*_j(T)}{\sigma_{\alpha,j}}$$

where /202

$$\sigma_{\alpha,j} = \sqrt{\sigma^2_{x_\alpha}(T) + b^2\sigma^2_{x_j}(T)};$$

b is the threshold coefficient, corresponding in the case at hand to a decrease by 2-4 db of the reading of the j-th channel, $b \geqslant 0.4$; $b^2\sigma^2_{x_j}(T)$ is the dispersion of random quantity $bx^*_j(T)$.

In testing the zero (threshold) hypothesis, if $\overline{x^*_\alpha(T)} = \overline{bx^*_j(T)}$, the modulus of the normally distributed ratio $q_{\alpha,j}$ with probability $1 - 2\beta$ does not exceed the boundary t_β (in our experiments, $t_\beta = 2.58$; $2\beta = 0.01$); $t_\beta \geqslant |q_{\alpha,j}|$. In the case $|q_{\alpha,j}| > t_\beta$, we assume, as we know, the hypothesis

$$\overline{x^*_\alpha(T)} > \overline{bx^*_j(T)}, \quad q_{\alpha,j} > 0;$$
$$\overline{x^*_\alpha(T)} < \overline{bx^*_j(T)}, \quad q_{\alpha,j} < 0,$$

and the zero hypothesis is negated; therefore, quantity U, estimating the number of channels of the analyzer, for which the hypothesis (that the threshold is exceeded) is correct

$$\overline{x^*_\alpha(T)} > \overline{bx^*_j(T)}$$

is determined by the equation

$$U = (j + p) - (j - n) + 1 = p + n + 1. \tag{6.4.50}$$

In conclusion, before going over to the results of our experiments, let us analyze certain problems of estimation of the precision of the method of linearization in calculating the dispersion and mathematical expectation. Retaining for this the first three terms of the expansion of function $f_I(T)$ [see (6.4.28)] into a Taylor series about the point A $[\overline{x_1(T)} \ldots \overline{x_U(T)}]$

$$f_I(T) = \frac{\sum\limits_{i=1}^{U} \overline{x_i(T)}\, f_i}{\sum\limits_{i=1}^{U} \overline{x_i(T)}} + \sum_{i=1}^{U} \frac{\partial f_I(T)}{\partial x_i(T)} \mathring{x}_i(T) +$$

$$+ \frac{1}{2} \sum_{i=1}^{U} \frac{\partial^2 f_I(T)}{\partial x_i^2(T)} \mathring{x}_i^2(T) + \sum_{i<j} \frac{\partial^2 f_I(T)}{\partial x_i(T)\, \partial x_j(T)} \mathring{x}_i(T)\, \mathring{x}_j(T),$$

$$\mathring{x}_i(T) = x_i(T) - \overline{x_i(T)},$$

we produce the independent, normally distributed quantities $x_i(T)$ /203

$$\overline{f_I(T)} = \frac{\sum_{i=1}^{U} \overline{x_i(T)}\, f_i}{\sum_{i=1}^{U} \overline{x_i(T)}} + \frac{1}{2} \sum_{i=1}^{U} \frac{\partial^2 f_I(T)}{\partial x_i^2(T)}\, \sigma^2_{x_i}(T); \tag{6.4.51}$$

$$\begin{aligned} \sigma^2_{f_I}(T) = \sum_{i=1}^{U} \left[\frac{\partial f_I(T)}{\partial x_i(T)}\right]^2 \sigma^2_{x_i}(T) + \frac{1}{2} \sum_{i=1}^{U} \left[\frac{\partial^2 f_I(T)}{\partial x_i^2(T)}\right]^2 \sigma^4_{x_i}(T) + \\ + \sum_{i<j} \left[\frac{\partial^2 f_I(T)}{\partial x_i(T)\, \partial x_j(T)}\right]^2 \sigma^2_{x_i}(T)\, \sigma^2_{x_j}(T), \end{aligned} \tag{6.4.52}$$

where, as before, after calculation of the derivatives of the function $\phi(x_1,x_2,\ldots,x_N)$, the variables should be replaced by their mathematical expectations

$$\frac{\partial \varphi}{\partial x_i} = \varphi'(x_1, \ldots, x_N).$$

Let us determine the coefficients included in expressions [(6.4.51), (6.4.52)]

$$\frac{\partial^2 f_I(T)}{\partial x_i^2(T)} = \frac{\partial}{\partial x_i(T)} \left[\frac{f_i \sum_{i=1}^{U} x_i(T) - \sum_{i=1}^{U} x_i(T)\, f_i}{\sum_{i=1}^{U} \sum_{j=1}^{U} x_i(T)\, x_j(T)} \right] =$$

$$= -\frac{2 \sum_{i=1}^{U} x_i(T) \left[f_i \sum_{i=1}^{U} x_i(T) - \sum_{i=1}^{U} x_i(T)\, f_i \right]}{\sum_{i=1}^{U} \sum_{j=1}^{U} \sum_{k=1}^{U} \sum_{l=1}^{U} x_i(T)\, x_j(T)\, x_k(T)\, x_l(T)};$$

$$\frac{\partial^2 f_I(T)}{\partial x_i^2(T)} = -2 \frac{f_i - \overline{f_I(T)}}{\sum_{i=1}^{U} \sum_{j=1}^{U} \overline{x_i(T)}\, \overline{x_j(T)}};$$

$$\frac{\partial^2 f_I(T)}{\partial x_p(T)\, \partial x_k(T)} = \frac{\partial}{\partial x_k(T)} \left[\frac{f_p \sum_{i=1}^{U} x_i(T) - \sum_{i=1}^{U} x_i(T)\, f_i}{\sum_{i=1}^{U} \sum_{j=1}^{U} x_i(T)\, x_j(T)} \right] = \tag{6.4.53}$$

$$= \frac{(f_p - f_k) \sum_{i=1}^{U} \sum_{j=1}^{U} x_i(T)\, x_j(T) - 2 \sum_{i=1}^{U} x_i(T) \left[f_p \sum_{i=1}^{U} x_i(T) - \sum_{i=1}^{U} x_i(T)\, f_i \right]}{\sum_{i=1}^{U} \sum_{j=1}^{U} \sum_{k=1}^{U} \sum_{l=1}^{U} x_i(T)\, x_j(T)\, x_k(T)\, x_l(T)};$$

$$\frac{\partial^2 f_I(T)}{\partial x_p(T)\,\partial x_k(T)} = -\frac{f_p + f_k - 2\overline{f'_I(T)}}{\sum\limits_{i=1}^{U}\sum\limits_{j=1}^{U}\overline{x_i(T)}\,\overline{x_j(T)}}; \tag{6.4.54}$$

$$\overline{f'_I(T)} = \frac{\sum\limits_{i=1}^{U}\overline{x_i(T)}\,f_i}{\sum\limits_{i=1}^{U}\overline{x_i(T)}}. \tag{6.4.55}$$

Substituting the values of the coefficients which we have found (6.4.53), (6.4.54), into equations (6.4.51) and (6.4.52) considering (6.4.32), we produce

$$\overline{f_I(T)} = \frac{\sum\limits_{i=1}^{U}\overline{x_i(T)}\,f_i}{\sum\limits_{i=1}^{U}\overline{x_i(T)}} - \sum_{i=1}^{U}\left[\frac{f_i - \overline{f'_I(T)}}{\sum\limits_{i=1}^{U}\sum\limits_{j=1}^{U}\overline{x_i(T)}\,\overline{x_j(T)}}\right]\sigma^2_{x_i}(T); \tag{6.4.56}$$

$$\sigma^2_{f_I}(T) = \sum_{i=1}^{U}\left[\frac{f_i - \overline{f'_I(T)}}{\sum\limits_{l=1}^{U}\overline{x_l(T)}}\right]^2\sigma^2_{x_i}(T) + 2\sum_{i=1}^{U}\left[\frac{f_i - \overline{f'_I(T)}}{\sum\limits_{l=1}^{U}\sum\limits_{k=1}^{U}\overline{x_l(T)}\,\overline{x_k(T)}}\right]^2\sigma^4_{x_i}(T) +$$

$$+\sum_{i<j}\left[\frac{f_i + f_j - 2\overline{f'_I(T)}}{\sum\limits_{k=1}^{U}\sum\limits_{l=1}^{U}\overline{x_k(T)}\,\overline{x_l(T)}}\right]^2\sigma^2_{x_i}(T)\,\sigma^2_{x_j}(T). \tag{6.4.57}$$

Let us use the method for production of boundary formula (6.4.43) to estimate expressions (6.4.56), (6.4.57), providing the maximum estimate [see (6.4.38), (6.4.40), (6.4.42)]. We note that the second term in equation (6.4.56) may be equal to zero, negative or positive, which is determined by the relationship between the resulting sums for dispersions $\sigma^2_{x_i}$ (T) and the tuning frequencies of the corresponding analyzer channels f_i, located to the left and right of quantity $\overline{f'_I(T)}$; therefore,

$$\overline{f_I(T)} = \overline{f'_I(T)}\left[1 \pm \frac{2\pi D(1+2B)(d-1)(d^U+1)}{TE(1+B^2)(d+1)(d^U-1)}\right], \quad U > 1, \tag{6.4.58}$$

where $\overline{f'_I(T)}$ characterizes the mathematical expectation of function $f_I(T)$ as it is expanded into a Taylor series, retaining the first two terms of (6.4.30). Let us analyze separately the second and third components of expression (6.4.57). Assuming

$$H = \frac{2\pi(1+2B)}{TE(1+B)^2},$$

we write /205

$$
\begin{aligned}
&2\sum_{i=1}^{U}\left[\frac{f_i-\overline{f'_I(T)}}{\sum\limits_{l=1}^{U}\sum\limits_{k=1}^{U}\overline{x_l(T)}\,\overline{x_k(T)}}\right]^2\sigma^4_{x_i}(T)= \\
&= 2\,[\overline{f'_I(T)}]^2 D^2H^2\,\frac{\sigma^8_{n_1}\sum\limits_{i=1}^{U}d^{4(i-1)}}{\sigma^8_{n_1}\left[\sum\limits_{l=1}^{U}\sum\limits_{k=1}^{U}d^{l-1}d^{k-1}\right]^2}= \\
&= 2\,[\overline{f'_I(T)}]^2 D^2H^2\,\frac{1+\dfrac{d^4\,[d^{4(U-1)}-1]}{d^4-1}}{\left[1+\dfrac{d\,[d^{U-1}-1]}{d-1}\right]^4}= \\
&= 2\,[\overline{f'_I(T)}]^2\,\frac{D^2H^2\,(d^{4U}-1)\,(d-1)^4}{(d^4-1)\,(d^U-1)^4};
\end{aligned}
\qquad (6.4.59)
$$

$$
\begin{aligned}
&\sum_{i<j}\left[\frac{f_i+f_j-2\overline{f'_I(T)}}{\sum\limits_{l=1}^{U}\sum\limits_{k=1}^{U}\overline{x_l(T)}\,\overline{x_k(T)}}\right]^2\sigma^2_{x_i}(T)\,\sigma^2_{x_j}(T)= \\
&= 4\,[\overline{f'_I(T)}]\,D^2H^2\,\frac{\sigma^8_{n_1}\sum\limits_{i<j}d^{2(i-1)}d^{2(j-1)}}{\sigma^8_{n_1}\left[\sum\limits_{l=1}^{U}\sum\limits_{k=1}^{U}d^{l-1}d^{k-1}\right]^2}=
\end{aligned}
$$

$$= 4\,\overline{[f'_I(T)]^2}\, D^2 H^2 \frac{\sum\limits_{i=1}^{U}\sum\limits_{j=1}^{U} d^{2(i-1)} d^{2(j-1)} - \sum\limits_{i=1}^{U} d^{4(i-1)}}{2\left[\sum\limits_{l=1}^{U}\sum\limits_{k=1}^{U} d^{l-1} d^{k-1}\right]^2} =$$

$$= 4\,\overline{[f'_I(T)]^2}\, H^2 D^2 \frac{\left[1+\frac{d^2[d^{2(U-1)}-1]}{d^2-1}\right]^2 - \left[1+\frac{d^4[d^{4(U-1)}-1]}{d^4-1}\right]}{2\left[1+\frac{d[d^{U-1}-1]}{d-1}\right]^4} = \qquad (6.4.60)$$

$$= 4\,\overline{[f'_I(T)]^2}\, H^2 D^2 \frac{(d^{2U}-1)(d^{2U}-d^2)(d-1)^4}{(d^2+1)(d^U-1)^4(d^2-1)^2}.$$

Considering (6.4.38), (6.4.59), (6.4.60), let us represent (6.4.57) for a maximum estimate in the form /206

$$\sigma^2_{f_I}(T) = [\sigma'_{f_I}(T)]^2 \left\{1 + \frac{4\pi(1+2B)}{TE(1+B)^2}\left[\frac{(d^{4U}-1)(d-1)^4(d+1)(d^U-1)}{(d^4-1)(d^U-1)^4(d-1)(d^U+1)} + \right.\right.$$
$$\left.\left. + 2\frac{(d^{2U}-1)(d^{2U}-d^2)(d-1)^4(d+1)(d^U-1)}{(d^4-1)(d^U-1)^4(d-1)(d^U+1)(d^2-1)}\right]\right\},$$

and finally produce

$$\sigma^2_{f_I}(T) = [\sigma'_{f_I}(T)]^2 \left\{1 + \frac{4\pi(1+2B)(d-1)}{TE(1+B)^2(d^U-1)^2(d^2+1)(d+1)} \times \right.$$
$$\left. \times [(d^{2U}+1)(d^2-1)+2(d^{2U}-d^2)]\right\},$$

where $[\sigma'_{f_I}(T)]^2$ characterizes the dispersion of function $f_I(T)$ when expanded into a Taylor series retaining the first two terms [see (6.4.43)].

Calculations using formulas (6.4.58), (6.4.61) allow us to perform an estimate of the accuracy of the method of linearization when calculating the mathematical expectation and dispersion of the function $f_I(T)$:

$$U = 2 \quad \overline{f_I(T)} = \overline{f'_I(T)}[1 \pm 0.0017], \quad \sigma^2_{f_I}(T) = 1.02\,[\sigma'_{f_I}(T)]^2;$$
$$U = 4 \quad \overline{f_I(T)} = \overline{f'_I(T)}[1 \pm 0.003], \quad \sigma^2_{f_I}(T) = 1.014\,[\sigma'_{f_I}(T)]^2.$$

The results presented above (see also §6.3) were used together with the electrophysiological indicators for a practical estimate of the degree of emotional stress of cosmonaut A. A. Leonov as he performed his space walk. Figure 77 presents the spectra (output values of analyzer channel integrators) of the phrase investigated, "Ya Almaz" [I am Almaz], pronounced by

A. A. Leonov during the exit stage (the cosmonaut was located on the rim of the hatch in the pressure chamber) during the flight (Figure 77 A) and during preliminary training in the temperature and pressure chamber TPC (Figure 77 B). The outputs of the spectral analyzer integrators were recorded on paper using a logarithmic stripchart recorder manufactured by the "Bruel Kjaer" firm. The spectrum of the signal, as can be seen from Figure 77 (see also Figure 72) is characterized by rather clear formant maxima, located in the area of frequency analysis (see §6.3).

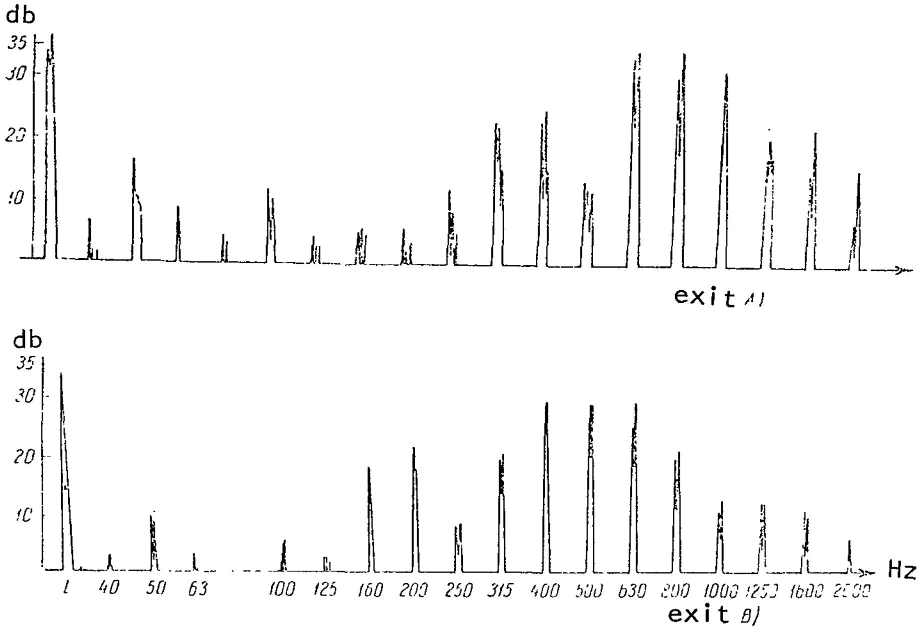


Figure 77

The graphs of change in ratio ε_k

$$\varepsilon_k = \frac{\{f_I(T)\}_k}{\max\{f_I(T)\}},$$

where $\{f_I(T)\}_k$; $\max\{f_I(T)\}$ are the value of the function $f_I(T)$ at the k-th observation interval and its maximum value during the time of the flight and /207 training in the TPC respectively, and the frequency of cardiac contractions $f_{c,k}$ (number of beats per minute) during the stages in the flight and training in the TPC: 1 is preparation for the launch, 2 is 1 min before launch, 3 is the active flight sector, 4 is preparation for the space walk, 5 is exit (on the rim of the hatch of the airlock AL), 6 is the space walk, 7 is return (AL hatch closed) are shown on Figure 78.

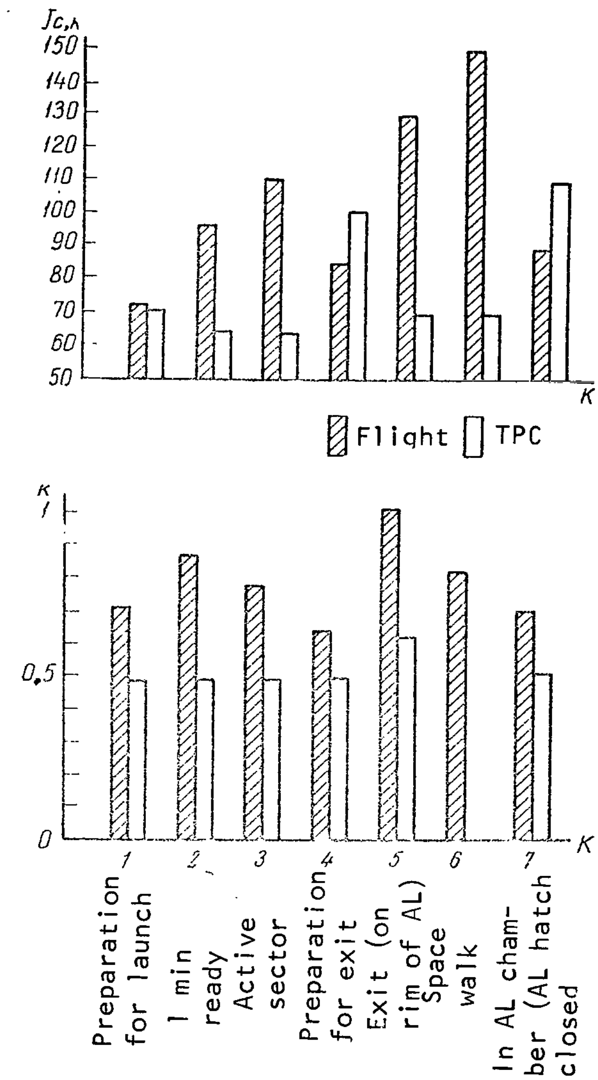


Figure 78

On the basis of the experimental results of modeling of emotions by actors, and analysis of the characteristics $\varepsilon_k; f_{c,k}$, we can draw the following conclusions. Under the TPC conditions, the tests are performed against a background of comparatively stable emotional state with an increase in emotions only at the moment the hatch of the airlock is opened ("exit"), indicated by the behavior of the relative quantity ε_k. In contrast to the TPC, the level of emotional stress during flight is higher, with sharp fluctuations at various observation intervals. The maximum values of ε_k correspond to the stages "1 min before exit" and "exit" ($\varepsilon_k = 1$; $\max\{f_I(T)\} \approx \max\{\overline{f_I(T)}\} \approx$ ≈ 700 Hz), higher stress being noted in the second case. The decrease of ε_k during the stages "active sector" and "preparation to exit" in comparison with the "1 min ready" stage might indicate a decrease in the degree of emotional stress, which, however, still remains higher than in the TPC.

As the launch approaches, the /208
pulse frequency increases continually. One minute before the launch, the pulse exceeds the corresponding value in the TPC by 20-30 beats per minute. This is explained by the great emotional background ("prelaunch state") in the real situation. During the "active sector" stage, the pulse frequency reaches 110 beats per minute, indicating the additional effect of acceleration against the background of comparatively high emotional stress (see change in ε_k). At the same time, in the TPC the pulse did not exceed the normal values. During
the stage of "preparation for exit" the pulse frequency decreases in flight to /209

85 beats per minute, while in the TPC it reaches 100 beats per minute. This results from the considerable physical loading under conditions of terrestrial gravitation. The work of Leonov in the TPC during the preparation to exit and return through the airlock was estimated as difficult, with a working time density of 30%. It is during these stages that we note a sharp increase in the pulse frequency and the number of respiratory cycles per minute. In flight, the physical effort in these stages was comparatively low due to the compensation of the force of gravity. During the "exit" stage, and the "space walk" stage in flight, we see the greatest increase in pulse frequency (130-150 beats per minute). In the TPC during these stages, the pulse frequency was normal.

Analysis of the graph of $f_{c,k}$ shows that both physical and emotional stress cause an increase in the pulse frequency. As yet, it is not possible to differentiate the reason and nature of stress from the data of vegetative reactions (pulse, respiration, etc.). Combined investigation of the characteristics ε_k and $f_{c,k}$ will allow us to approach the solution of this problem. Comparison of the graphs on Figure 78 can lead us to the conclusion that the work of A. A. Leonov in the TPC was characterized primarily by a smooth emotional background, while the increase in the number of heart contractions was caused by physical loading. In actual flight, the emotional stress predominates, indicated by the behavior of $f_{c,k}$ and ε_k against the background of comparatively low physical effort.

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APPENDICES

Appendix 1

Calculation of the expressions /210

$$\Delta_y(T) = \left| 1 - \sqrt{T} \left\{ \frac{\int\limits_0^\infty \left(\frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \right)^2 d\tau}{\int\limits_0^T (T - \tau) \left(\frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \right)^2 d\tau} \right\}^{\frac{1}{2}} \right|; \tag{A.1.1}$$

$$1.\ \int\limits_0^\infty \left(\frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \right)^2 d\tau = \frac{2}{\Delta\omega_y} \int\limits_0^\infty \frac{\sin^2 x}{x^2}\, dx = \frac{\pi}{\Delta\omega_y};$$

$$2.\ \int\limits_0^T (T - \tau) \left(\frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \right)^2 d\tau = T \int\limits_0^T \left(\frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \right)^2 d\tau - \int\limits_0^T \tau \left(\frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \right) d\tau$$

$$3.\ T \int\limits_0^T \left(\frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \right)^2 d\tau = \frac{2T}{\Delta\omega_y} \int\limits_0^{\frac{\Delta\omega_y T}{2}} \frac{\sin^2 x}{x^2}\, dx. \tag{A.1.2}$$

We assume that

$$\int\limits_m^M \varphi(x)\, dx = \psi(M) - \psi(m),$$

then

$$\int_0^M \varphi(x)\,dx = \psi(M) - \lim_{m\to 0}\psi(m).$$

Consequently /211

$$4.\ \int_m^M \frac{\sin^2 x}{x^2}\,dx = \int_m^M \frac{1-\cos 2x}{2x^2}\,dx = -\frac{1}{2}\left[\frac{1}{x}\right]_m^M - \int_{2m}^{2M}\frac{\cos v}{v^2}\,dv = \frac{1}{2m} -$$

$$-\frac{1}{2M} - \left[-\frac{\cos v}{v} - \mathrm{Si}(v)\right]_{2m}^{2M} = \frac{1}{2m} - \frac{1}{2M} + \frac{\cos 2M}{2M} + \mathrm{Si}(2M) - \frac{\cos 2m}{2m} -$$

$$-\mathrm{Si}(2m) = \mathrm{Si}(2M) - \frac{1-\cos 2M}{2M} + \frac{1-\cos 2m}{2m} - \mathrm{Si}(2m);$$

$$5.\ \int_0^M \frac{\sin^2 x}{x^2}\,dx = \mathrm{Si}(2M) - \frac{\sin^2 M}{M} + \lim_{2m\to 0}\frac{1-\cos 2m}{2m} - \lim_{2m\to 0}\mathrm{Si}(2m).$$

On the basis of the L'Hopital rule

$$\lim_{2m\to 0}\frac{1-\cos 2m}{2m} = \lim_{2m\to 0}\frac{2\sin 2m}{2} = 0.$$

Consequently,

$$6.\ \frac{2T}{\Delta\omega_y}\int_0^{\frac{\Delta\omega_y T}{2}}\frac{\sin^2 x}{x^2}\,dx = \frac{2T}{\Delta\omega_y}\left[\mathrm{Si}(\Delta\omega_y T) - \frac{2\sin^2\left(\frac{\Delta\omega_y T}{2}\right)}{\Delta\omega_y T}\right]$$

$\mathrm{Si}(x) = \int_0^x \frac{\sin v}{v}\,dv$ — is the integral sine (A.1.3)

$$7.\ \int_0^T \tau\left(\frac{\sin\frac{\Delta\omega_y\tau}{2}}{\frac{\Delta\omega_y\tau}{2}}\right)^2 d\tau = \frac{2}{\Delta\omega_y}\int_0^T \frac{\sin^2\left(\frac{\Delta\omega_y\tau}{2}\right)}{\frac{\Delta\omega_y\tau}{2}}\,d\tau = \frac{4}{\Delta\omega_y^2}\int_0^{\frac{\Delta\omega_y T}{2}}\frac{\sin^2 x}{x}\,dx =$$

$$= \frac{4}{\Delta\omega_y^2}\int_0^{\frac{\Delta\omega_y T}{2}}\frac{1-\cos 2x}{2x}\,dx = \frac{2}{\Delta\omega_y^2}\int_0^{\Delta\omega_y T}\frac{1-\cos v}{v}\,dv =$$

$= \frac{2}{\Delta\omega_y^2}\left[C_0 + \ln(\Delta\omega_y T) - \mathrm{Ci}(\Delta\omega_y T)\right]$ — (See Yanke, Emde, 1949)

$C_0 = 0{,}5772156$ — is Euler's Constant

$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos v}{v}\,dv$ is the integral cosine

$$\int_0^T \tau \left(\frac{\sin \frac{\Delta\omega_y \tau}{2}}{\frac{\Delta\omega_y \tau}{2}} \right)^2 d\tau = \frac{2}{\Delta\omega_y^2} [C_0 + \ln (\Delta\omega_y T) - \mathrm{Ci}\,(\Delta\omega_y T)]. \qquad (A.1.4)$$ /212

Considering (A.1.2), (A.1.3) and (A.1.4), we can write (A.1.1) in the form

$$\Delta_u (T) =$$

$$= \left| 1 - \sqrt{T} \left\{ \frac{\frac{\pi}{\Delta\omega_y}}{\frac{2T}{\Delta\omega_y} \left[\mathrm{Si}\,(\Delta\omega_y T) - \frac{2 \sin^2 \left(\frac{\Delta\omega_y T}{2} \right)}{\Delta\omega_y T} \right] - \frac{2}{\Delta\omega_y^2} [C_0 + \ln (\Delta\omega_y T) - \mathrm{Ci}\,(\Delta\omega_y T)]} \right\}^{\frac{1}{2}} \right| =$$

$$= \left| 1 - \left\{ \frac{\frac{\pi}{\Delta\omega_y T}}{\frac{2}{\Delta\omega_y T} \left[\mathrm{Si}\,(\Delta\omega_y T) - \frac{2 \sin^2 \left(\frac{\Delta\omega_y T}{2} \right)}{\Delta\omega_y T} \right] - \frac{2}{\Delta\omega_y^2 T^2} [C_0 + \ln (\Delta\omega_y T) - \mathrm{Ci}\,(\Delta\omega_y T)]} \right\}^{\frac{1}{2}} \right| =$$

$$= \left| 1 - \left\{ \frac{\pi \Delta\omega_y T}{2[\Delta\omega_y T\, \mathrm{Si}\,(\Delta\omega_y T) + \cos \Delta\omega_y T - (1 + C_0) - \ln (\Delta\omega_y T) + \mathrm{Ci}\,(\Delta\omega_y T)]} \right\}^{\frac{1}{2}} \right|; \qquad (A.1.5)$$

$$8.\ \int_0^T (T - \tau) \frac{\sin^2 \left(\frac{\Delta\omega_y \tau}{2} \right)}{\left(\frac{\Delta\omega_y \tau}{2} \right)^2} d\tau = \frac{2T^2}{\Delta\omega_y T} \left[\mathrm{Si}\,(\Delta\omega_y T) - \frac{2 \sin^2 \left(\frac{\Delta\omega_y T}{2} \right)}{\Delta\omega_y T} \right] -$$

$$- \frac{2T^2}{\Delta\omega_y T^2} [C_0 + \ln (\Delta\omega_y T) - \mathrm{Ci}\,(\Delta\omega_y T)]. \qquad (A.1.6)$$

Appendix 2

Calculation of the integral (A.2.1)

$$A = \frac{2}{T^2} \int_0^T (T - \tau)\, r_{y_{10}}^2 (\tau)\, d\tau$$

and the related expressions

$$1.\ r^2_{y_{10}}(\tau)=[\omega_c^2 R_{y_0}(\tau)-R''_{y_0}(\tau)]^2+[2\omega_c R'_{y_0}(\tau)]^2 \text{ [see (4.3.82'')]};$$

$$2.\ R'_{y_0}(\tau)=\left(\frac{\sin\frac{\Delta\omega_y\tau}{2}}{\frac{\Delta\omega_y\tau}{2}}\right)'=\frac{\tau\frac{\Delta^2}{4}\cos\frac{\tau\Delta}{2}-\frac{\Delta}{2}\sin\frac{\tau\Delta}{2}}{\left(\frac{\tau\Delta}{2}\right)^2},\ \Delta=\Delta\omega_y;$$

$$3.\ R''_{y_0}(\tau)=\frac{-\tau^2\frac{\Delta^2}{4}\sin\frac{\tau\Delta}{2}-2\tau\frac{\Delta}{2}\cos\frac{\tau\Delta}{2}+2\sin\frac{\tau\Delta}{2}}{\tau^3\frac{\Delta}{2}};$$

$$4.\ \omega_c^2 R_{y_0}(\tau)-R''_{y_0}(\tau)=\omega_c^2\frac{\sin\frac{\tau\Delta}{2}}{\frac{\tau\Delta}{2}}-\frac{-\tau^2\frac{\Delta^2}{4}\sin\frac{\tau\Delta}{2}-2\tau\frac{\Delta}{2}\cos\frac{\tau\Delta}{2}+2\sin\frac{\tau\Delta}{2}}{\tau^3\frac{\Delta}{2}}=\frac{\tau\Delta\cos\frac{\tau\Delta}{2}-\sin\frac{\tau\Delta}{2}\left(2-\omega^2\tau^2-\frac{\tau^2\Delta^2}{4}\right)}{\tau^3\frac{\Delta}{2}};$$

$$5.\ [\omega_c^2 R_{y_0}(\tau)-R''_{y_0}(\tau)]^2=$$

$$=\frac{\tau^2\Delta^2\cos^2\frac{\tau\Delta}{2}-2\tau\Delta\sin\frac{\tau\Delta}{2}\cos\frac{\tau\Delta}{2}\left(2-\omega_c^2\tau^2-\frac{\tau^2\Delta^2}{4}\right)+\sin^2\frac{\tau\Delta}{2}\left(2-\omega_c^2\tau^2-\frac{\tau^2\Delta^2}{4}\right)^2}{\tau^6\frac{\Delta^2}{4}}=$$

$$=\frac{\sin^2\frac{\Delta\tau}{2}\left(4+\omega_c^4\tau^4+\frac{\tau^4\Delta^4}{16}-4\omega_c^2\tau^2-4\frac{\tau^2\Delta^2}{4}+2\frac{\omega_c^2\tau^4\Delta^2}{4}\right)-\tau\Delta\sin 2\frac{\tau\Delta}{2}\left(2-\omega_c^2\tau^2-\frac{\tau^2\Delta^2}{4}\right)}{\tau^6\frac{\Delta^2}{4}}+$$

$$+\frac{\tau^2\Delta^2\cos^2\frac{\tau\Delta}{2}}{\tau^6\frac{\Delta^2}{4}}=\sin^2\frac{\tau\Delta}{2}\left(\frac{16}{\tau^6\Delta^2}+\frac{4\omega_c^4}{\tau^2\Delta^2}+\frac{\Delta^2}{4\tau^2}-\frac{16\omega_c^2}{\tau^4\Delta^2}-\frac{4}{\tau^4}+\frac{2\omega_c^2}{\tau^2}\right)-$$

$$-\sin 2\frac{\tau\Delta}{2}\left(\frac{8}{\tau^5\Delta}-\frac{4\omega_c^2}{\tau^3\Delta}-\frac{\Delta}{\tau^3}\right)+\frac{4}{\tau^4}\cos^2\frac{\tau\Delta}{2};$$

$$6.\ 4\omega_c^2[R'_{y_0}(\tau)]^2=4\omega_c^2\frac{\frac{\tau^2\Delta^4}{16}\cos^2\frac{\tau\Delta}{2}-2\frac{\tau\Delta^3}{8}\sin\frac{\tau\Delta}{2}\cos\frac{\tau\Delta}{2}+\frac{\Delta^2}{4}\sin^2\frac{\tau\Delta}{2}}{\frac{\tau^4\Delta^4}{16}}=$$

$$=\frac{4\omega_c^2}{\tau^2}\cos^2\frac{\tau\Delta}{2}-8\frac{\omega_c^2}{\tau^3\Delta}\sin 2\frac{\tau\Delta}{2}+\frac{16\omega_c^2}{\tau^4\Delta^2}\sin^2\frac{\tau\Delta}{2}; \tag{A.2.2}$$

$$7.\ [\omega_c^2 R_{y_0}(\tau)-R''_{y_0}(\tau)]^2+[2\omega_c R'_{y_0}(\tau)]^2=\sin^2\frac{\tau\Delta}{2}\left(\frac{16}{\tau^6\Delta^2}+\frac{4\omega_c^4}{\tau^2\Delta^2}+\frac{\Delta^2}{4\tau^2}-\right.$$

$$\left.-\frac{4}{\tau^4}+\frac{2\omega_c^2}{\tau^2}\right)-\sin 2\frac{\Delta\tau}{2}\left(\frac{8}{\tau^5\Delta}+\frac{4\omega_c^2}{\tau^3\Delta}-\frac{\Delta}{\tau^3}\right)+\cos^2\frac{\tau\Delta}{2}\left(\frac{4}{\tau^4}+\frac{4\omega_c^2}{\tau^2}\right)=$$

$$=\sin^2\frac{\tau\Delta}{2}\left(\frac{\frac{\Delta^4}{4}}{\frac{\tau^6\Delta^6}{64}}+\frac{\omega_c^4}{\frac{\tau^2\Delta^2}{4}}+\frac{\frac{\Delta^4}{16}}{\frac{\tau^2\Delta^2}{4}}-\frac{\frac{\Delta^4}{4}}{\frac{\tau^4\Delta^4}{16}}+\frac{\frac{\omega_c^2\Delta^2}{2}}{\frac{\tau^2\Delta^2}{4}}\right)-$$

$$-\sin 2\frac{\Delta\tau}{2}\left[\frac{\frac{\Delta^4}{4}}{\frac{\tau^5\Delta^5}{32}}+\frac{\frac{\Delta^2}{8}(4\omega_c^2-\Delta^2)}{\frac{\tau^3\Delta^3}{8}}\right]+\cos^2\frac{\tau\Delta}{2}\left(\frac{\frac{\Delta^4}{4}}{\frac{\tau^4\Delta^4}{16}}+\frac{\Delta^2\omega_c^2}{\frac{\tau^2\Delta^2}{4}}\right).$$

Assuming $\tau\Delta/2 = x$, we can write the last equation as follows: /214

$$8.\ r^2_{y_{10}}(x) = \sin^2 x\left(\frac{\Delta^4}{4x^6} + \frac{\omega_c^4}{x^2} + \frac{\Delta^4}{16x^2} - \frac{\Delta^4}{4x^4} + \frac{\omega_c^2\Delta^2}{2x^2}\right) - $$
$$- \sin 2x\left[\frac{\Delta^4}{4x^5} + \frac{\Delta^2(4\omega_c^2 - \Delta^2)}{8x^3}\right] + \cos^2 x\left(\frac{\Delta^4}{4x^4} + \frac{\Delta^2\omega_c^2}{x^2}\right) \qquad \text{(A.2.3)}$$
$$9.\ x r^2_{y_{10}}(x) = \sin^2 x\left(\frac{\Delta^4}{4x^5} + \frac{\omega_c^4}{x} + \frac{\Delta^4}{16x} - \frac{\Delta^4}{4x^3} + \frac{\omega_c^2\Delta^2}{2x}\right) - $$
$$- \sin 2x\left[\frac{\Delta^4}{4x^4} + \frac{\Delta^2(4\omega_c^2 - \Delta^2)}{8x^2}\right] + \cos^2 x\left(\frac{\Delta^4}{4x^3} + \frac{\Delta^2\omega_c^2}{x}\right). \qquad \text{(A.2.3')}$$

On the basis of (A.2.2), we have [see (A.2.3)]

$$10.\ [R'_{y_0}(x)]^2 + [\omega_c R_{y_0}(x)]^2 = \frac{\Delta^2}{4x^2}\cos^2 x - \frac{\Delta^2}{4x^3}\sin 2x + $$
$$+ \frac{\Delta^2}{4x^4}\sin^2 x + \omega_c^2\,\frac{\sin^2 x}{x^2}. \qquad \text{(A.2.4)}$$

The expression (A.2.1) can be written in the form

$$11.\ \frac{2}{T^2}\int_0^T (T-\tau)\, r^2_{y_{10}}(\tau)\, d\tau = \frac{2}{T}\left\{\frac{2}{\Delta}\int_0^{T\frac{\Delta}{2}} r^2_{y_{10}}(x)\, dx\right\} - $$
$$- \frac{2}{T^2}\left\{\frac{4}{\Delta^2}\int_0^{T\frac{\Delta}{2}} x r^2_{y_{10}}(x)\, dx\right\}, \qquad \text{(A.2.5)}$$

where the integrands are determined by the equations (A.2.3) and (A.2.3'). The integrals (A.2.5), considering the relationship

$$12.\ \int_m^M \frac{\sin^2 x}{x^n}\, dx = \frac{1}{2}\int_m^M \frac{1-\cos 2x}{x^n}\, dx = \frac{1}{2}\int_m^M \frac{dx}{x^n} - 2^{n-2}\int_{2m}^{2M} \frac{\cos v}{v^n}\, dv; \qquad \text{(A.2.6)}$$
$$13.\ \int_m^M \frac{\cos^2 x}{x^n}\, dx = \frac{1}{2}\int_m^M \frac{1+\cos 2x}{x^n}\, dx = \frac{1}{2}\int_m^M \frac{dx}{x^n} + 2^{n-2}\int_{2m}^{2M} \frac{\cos v}{v^n}\, dv;$$

$$14.\ \int_m^M \frac{\sin 2x}{x^n}\,dx = 2^{n-1}\int_{2m}^{2M} \frac{\sin v}{v^n}\,dv,$$

are reduced to tabular integrals (Dvayt, 1948) with subsequent limit transfer as $m \to 0$

$$15.\ \frac{2}{\Delta}\int_m^M r^2_{g_{1_0}}(x)\,dx = \frac{2}{\Delta}\left\{\int_m^M \left[\omega_c^4 + \frac{\Delta^4}{16} + \frac{\omega_c^2\Delta^2}{2}\right]\frac{\sin^2 x}{x^2}\,dx + \right.$$

$$+\int_m^M \frac{\Delta^4}{4}\cdot\frac{\sin^2 x}{x^6}\,dx - \int_m^M \frac{\Delta^4}{4}\,\frac{\sin^2 x}{x^4}\,dx - \int_m^M \frac{\Delta^4}{4}\,\frac{\sin 2x}{x^5}\,dx -$$

$$\left. - \int_m^M \frac{\Delta^2(4\omega_c^2 - \Delta^2)}{8}\cdot\frac{\sin 2x}{x^3} + \int_m^M \frac{\Delta^4}{4}\,\frac{\cos^2 x}{x^4}\,dx + \int_m^M \frac{\Delta^2\omega_c^2\cos^2 x}{x^2}\,dx\right\}. \qquad \text{(A.2.7)}$$ /215

The first term of this sum is calculated similarly to (A.1.3)

$$16.\ B(\Delta, T) = \int_0^{T\frac{\Delta}{2}} \left[\omega_c^4 + \frac{\Delta^4}{16} + \frac{\omega_c^2\Delta^2}{2}\right]\frac{\sin^2 x}{x^2}\,dx = \left(\omega_c^4 + \frac{\Delta^4}{16} + \frac{\omega_c^2\Delta^2}{2}\right)\times$$

$$\times\left[\mathrm{Si}(\Delta T) - \frac{2\sin^2\frac{\Delta T}{2}}{\Delta T}\right].$$

Representing the result of integration within the limits m,M of the first term of expression (A.2.7) as $B_1(\Delta, T)$ and keeping in mind (A.2.6), we produce

$$17.\ \frac{2}{\Delta}\int_{m}^{M} r_{y_{10}}^{2}(v)\,dv = \frac{2}{\Delta}\Bigg\{\Bigg\{B_1(\Delta, T) + \frac{\Delta^4}{4}\Bigg\{\frac{1}{10}\left[\frac{1}{x^5}\right]_M^m - 16\Bigg[-\frac{\cos v}{5v^5} +$$
$$+\frac{\sin v}{20v^4} + \frac{\cos v}{60v^3} - \frac{\sin v}{120v^2} - \frac{\cos v}{120v} - \frac{\mathrm{Si}\,(v)}{120}\Bigg]_{2m}^{2M}\Bigg\} -$$
$$-\frac{\Delta^4}{4}\Bigg\{\frac{1}{6}\left[\frac{1}{x^3}\right]_M^m - 4\Bigg[-\frac{\cos v}{3v^3} + \frac{\sin v}{6v^2} + \frac{\cos v}{6v} + \frac{\mathrm{Si}\,(v)}{6}\Bigg]_{2m}^{2M}\Bigg\} -$$
$$-\frac{\Delta^4}{4}\Bigg\{16\Bigg[-\frac{\sin v}{4v^4} - \frac{\cos v}{12v^3} + \frac{\sin v}{24v^2} + \frac{\cos v}{24v} + \frac{\mathrm{Si}\,(v)}{24}\Bigg]_{2m}^{2M}\Bigg\} -$$
$$-\frac{\Delta^2(4\omega_c^2 - \Delta^2)}{8}\Bigg\{4\Bigg[-\frac{\sin v}{2v^2} - \frac{\cos v}{2v} - \frac{\mathrm{Si}\,(v)}{2}\Bigg]_{2m}^{M}\Bigg\} + \frac{\Delta^4}{4}\Bigg\{\frac{1}{6}\left[\frac{1}{x^3}\right]_M^m +$$
$$+4\Bigg[-\frac{\cos v}{3v^3} + \frac{\sin v}{6v^2} + \frac{\cos v}{6v} + \frac{\mathrm{Si}\,(v)}{6}\Bigg]_{2m}^{2M}\Bigg\} + \Delta^2\omega_c^2\Bigg\{\frac{1}{2}\left[\frac{1}{x}\right]_M^m +$$
$$+\Bigg[-\frac{\cos v}{v} - \mathrm{Si}\,(v)\Bigg]_{2m}^{2M}\Bigg\}\Bigg\}\Bigg\}. \qquad \text{(A.2.8)}$$

Grouping the terms and performing certain contractions, we represent (A.2.7) in the form

$$18.\ \frac{2}{\Delta}\int_{m}^{M} r_{y_{10}}^{2}(x)\,dx = \frac{2}{\Delta}\Bigg\{B_1(\Delta, T) + \frac{\Delta^4}{40}\left[\frac{1}{x^5}\right]_M^m + \frac{\Delta^2\omega_c^2}{2}\left[\frac{1}{x}\right]_M^m +$$
$$+\Bigg[\frac{4\Delta^4}{5v^5}\cos v + \frac{4\Delta^4}{5v^4}\sin v - \frac{2\Delta^4}{5v^3}\cos v + \left(\omega_c^2\Delta^2 - \frac{\Delta^4}{20}\right)\frac{\sin v}{v^2} -$$
$$-\frac{\Delta^4}{20v}\cos v - \frac{\Delta^4}{20}\mathrm{Si}\,(v)\Bigg]_{2m}^{2M}\Bigg\}. \qquad \text{(A.2.9)}$$

Assuming further the value of the integral for the upper limit M equal to $\phi(M) + B(\Delta, T)$, we write /216

$$19.\ \frac{2}{\Delta}\int_{0}^{M} r_{y_{10}}^{2}(x)\,dx = \frac{2}{\Delta}\Bigg\{B(\Delta, T) + \varphi(M) + \lim_{m\to 0}\Bigg[\frac{\Delta^4}{40m^5} + \frac{\Delta^2\omega_c^2}{2m} -$$
$$-\frac{\Delta^4\cos 2m}{40m^5} - \frac{2\Delta^4\sin 2m}{40m^4} + \frac{2\Delta^4\cos 2m}{40m^3} -$$
$$-\left(\omega_c^2\Delta^2 - \frac{\Delta^4}{20}\right)\frac{\sin 2m}{4m^2} + \frac{\Delta^4\cos 2m}{40m} + \frac{\Delta^4\,\mathrm{Si}\,(2m)}{20}\Bigg] =$$
$$= \frac{2}{\Delta}\Bigg\{B(\Delta, T) + \varphi(M) + \lim_{m\to 0}\Bigg[\frac{\Delta^4}{20}\mathrm{Si}\,(2m) + \frac{2m\Delta^2\omega_c^2 - \omega_c^2\Delta^2\sin 2m}{4m^2} +$$

$$+\frac{(\Delta^4 m^4 - \Delta^4 + 2\Delta^4 m^2)\cos 2m + \left(\frac{\Delta^4 m^3}{2} - 2\Delta^4 m\right)\sin 2m + \Delta^4}{40m^5}\Bigg]\Bigg\}$$

20. $\lim\limits_{m\to 0} \operatorname{Si}(2m) = 0;$

21. $$\lim_{m\to 0}\frac{2m\Delta^2\omega_c^2 - \omega_c^2\Delta^2\sin 2m}{4m^2} = \lim_{m\to 0}\frac{2\Delta^2\omega_c^2 - 2\omega_c^2\Delta^2\cos 2m}{8m} =$$

$$= \lim_{m\to 0}\frac{4\omega_c^2\Delta^2\sin 2m}{8} = 0.$$

Similarly, by successive application of the L'Hopital rule, we find

22. $$\lim_{m\to 0}\frac{(\Delta^4 m^4 - \Delta^4 + 2\Delta^4 m^2)\cos 2m + \left(\frac{\Delta^4 m^3}{2} - 2\Delta^4 m\right)\sin 2m + \Delta^4}{40m^5} =$$

$$= \lim_{m\to 0}\frac{\begin{gathered}(4\Delta^4 m^3 + 4\Delta^4 m)\cos 2m - (2\Delta^4 m^4 - 2\Delta^4 + 4\Delta^4 m^2)\sin 2m + \\ + \left(\frac{3}{2}\Delta^4 m^2 - 2\Delta^4\right)\sin 2m + (\Delta^4 m^3 - 4\Delta^4 m)\cos 2m\end{gathered}}{200m^4} =$$

$$= \lim_{m\to 0}\frac{5\Delta^4 m\cos 2m + \left(\frac{3}{2}\Delta^4 - 4\Delta^4 - 2\Delta^4 m^2\right)\sin 2m}{200m^2} =$$

$$= \lim_{m\to 0}\left[\frac{5\Delta^4\cos 2m - 10\Delta^4 m\sin 2m - 4\Delta^4 m\sin 2m}{400m} + \frac{(3\Delta^4 - 8\Delta^4 - 4\Delta^4 m^2)\cos 2m}{400m}\right] =$$

$$= \lim_{m\to 0}\frac{-14\Delta^4 m\sin 2m - 4\Delta^4 m^2\cos 2m}{400m} = 0.$$

Consequently [see (A.2.9)],

23. $$\frac{2}{\Delta}\int_0^M r_{y_{10}}^2(x)\,dx = \frac{2}{\Delta}\int_0^{\frac{\Delta T}{2}} r_{y_{10}}^2(x)\,dx = \frac{2}{\Delta}\left\{\left(\omega_c^4 + \frac{\Delta^4}{16} + \frac{\omega_c^2\Delta^2}{2}\right)\times\right.$$

$$\times\left[\mathrm{Si}(\Delta T)-\frac{2\sin^2\frac{\Delta T}{2}}{\Delta T}\right]+\frac{4\Delta^4\cos\Delta T}{5(\Delta T)^5}+\frac{4\Delta^4\sin\Delta T}{5(\Delta T)^4}-\frac{2\Delta^4\cos\Delta T}{5(\Delta T)^3}+$$

$$+\left(\omega_c^2\Delta^2-\frac{\Delta^4}{20}\right)\frac{\sin\Delta T}{(\Delta T)^2}-\frac{\Delta^4}{20\Delta T}\cos\Delta T-\frac{\Delta^4}{20}\mathrm{Si}(\Delta T)-\frac{4\Delta^4}{5(\Delta T)^5}-\frac{\Delta^2\omega_c^2}{\Delta T}\Big\}. \qquad \text{(A.2.10)}$$ /217

Since

$$24.\ \frac{2}{T}\int_0^T r^2_{y_{10}}(\tau)\,d\tau=\frac{2}{T}\left\{\frac{2}{\Delta}\int_0^M r^2_{y_{10}}(x)\,dx\right\},\qquad M=\frac{\Delta T}{2},$$

assuming $M \to \infty$ and noting that $\mathrm{Si}(\infty) = \pi/2$, we produce [see (A.2.5)]

$$25.\ \frac{4}{\Delta T}\int_0^\infty r^2_{y_{10}}(x)\,dx=\frac{2\pi}{\Delta T}\left(\omega_c^4+\frac{\omega_c^2\Delta^2}{2}+\frac{\Delta^4}{80}\right),\qquad \Delta=\Delta\omega_y \qquad \text{(A.2.11)}$$

If the inequality $\Delta\omega_y^2 \ll \omega_c^2$ is fulfilled, then (A.2.11) can be represented in the form

$$26.\ \frac{4}{\Delta T\omega_y}\int_0^\infty r^2_{y_{10}}(x)\,dx=\frac{2\pi\omega_c^4}{\Delta\omega_y T}. \qquad \text{(A.2.12)}$$

On the basis of (A.2.4), (A.2.7) and (A.2.8), we have [see also (4.3.83")]

$$27.\ \int_a^A r^2_{y_{10}y_0}(\tau)\,d\tau=\frac{2}{\Delta}\int_m^M r^2_{y_{10}y_0}(x)\,dx=\frac{2}{\Delta}\int_m^M\{[R'_{y_0}(x)]^2+[\omega_c R_{y_0}(x)]^2\}\,dx$$

$$\frac{\Delta\tau}{2}=x,\quad \frac{\Delta A}{2}=M,\quad \frac{\Delta a}{2}=m;$$

where

$$28.\ \frac{2}{\Delta}\int_{m}^{M} r^2_{y_1 y_0}(x)\,dx = \frac{2}{\Delta}\left\{\left\{\frac{\Delta^2}{4}\left\{\frac{1}{2}\left[\frac{1}{x}\right]_M^m + \left[-\frac{\cos v}{v} - \mathrm{Si}\,(v)\right]_{2m}^{2M}\right\} - \right.\right.$$

$$\left. - \frac{\Delta^2}{4}\left\{4\left[-\frac{\sin v}{2v^2} - \frac{\cos v}{2v} - \frac{\mathrm{Si}\,(v)}{2}\right]_{2m}^{2M}\right\} + \frac{\Delta^2}{4}\left\{\frac{1}{6}\left[\frac{1}{x^3}\right]_M^m - \right.$$

$$\left.\left. - 4\left[-\frac{\cos v}{3v^3} + \frac{\sin v}{6v^2} + \frac{\cos v}{6v} + \frac{\mathrm{Si}\,(v)}{6}\right]_{2m}^{2M}\right\} + \omega_c^2\left[C_1(\Delta, T)\right]\right\}\right\};$$

$$29.\ C_1(\Delta, T) = \int_m^M \frac{\sin^2 v}{v^2}\,dv.$$

Performing contractions and grouping terms, we produce

$$30.\ \frac{2}{\Delta}\int_m^M r^2_{y_{10} y_0}(x)\,dx = \frac{2}{\Delta}\left\{\frac{\Delta^2}{8}\left[\frac{1}{x}\right]_M^m + \frac{\Delta^2}{24}\left[\frac{1}{x^3}\right]_M^m + \left[\frac{\Delta^2}{3}\,\frac{\cos v}{v^3} + \right.\right.$$

$$\left.\left. + \frac{2\Delta^2 \sin v}{6v^2} + \frac{\Delta^2\cos v}{12v} + \frac{\Delta^2\,\mathrm{Si}\,(v)}{12}\right]_{2m}^{2M} + \omega_c^2 C_1(\Delta, T)\right\};$$

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$$31.\ \frac{2}{\Delta}\int_0^\infty r^2_{y_{10} y_0}(x)\,dx = \frac{2}{\Delta}\lim_{m\to 0}\frac{3m^2\Delta^2 + \Delta^2 - (\Delta^2 + \Delta^2 m^2)\cos 2m - 2\Delta^2\sin 2m}{24m^3} +$$

$$+ \frac{2}{\Delta}\left(\frac{\pi\Delta^2}{24} + \frac{\pi\omega_c^2}{2}\right);$$

$$32.\ \lim_{m\to 0}\frac{3m^2\Delta^2 + \Delta^2 - (\Delta^2 + \Delta^2 m^2)\cos 2m - 2\Delta^2 m \sin 2m}{24m^3} =$$

$$= \lim_{m\to 0}\frac{6m\Delta^2 + 2(\Delta^2 + \Delta^2 m^2)\sin 2m - 2\Delta^2 m\cos 2m - 2\Delta^2\sin 2m - 4\Delta^2 m\cos 2m}{72m^2} =$$

$$= \lim_{m\to 0}\left\{\left[\frac{1-\cos 2m}{2m}\right]\frac{6\Delta^2}{36} + \frac{2\Delta^2}{72}\sin 2m\right\} = 0.$$

Consequently,

$$33.\ \frac{2}{\Delta}\int_0^\infty r^2_{y_{10} y_0}(x)\,dx = \frac{\pi\Delta^2}{\Delta}\left(\frac{1}{12} + \frac{\omega_c^2}{\Delta^2}\right) = \pi\Delta\left(\frac{\omega_c^2}{\Delta^2} + \frac{1}{12}\right),\qquad \Delta = \Delta\omega_y. \tag{A.2.13}$$

In the case of $\omega_c^2 \gg \Delta^2$, equation (A.2.13) is represented in the form

$$34.\quad \frac{2}{\Delta}\int_0^\infty r^2_{y_0 y_0}(x)\,dx = \frac{\pi\omega_c^2}{\Delta\omega_y}. \tag{A.2.14}$$

Let us now calculate the second integral in the expression (A.2.5):

$$35.\quad \int_m^M x r^2_{y_0}(x)\,dx = \int_m^M \frac{\Delta^4}{4}\frac{\sin^2 x}{x^5}\,dx - \int_m^M \frac{\Delta^4}{4}\frac{\sin^2 x}{x^3}\,dx - \int_m^M \frac{\Delta^4}{4}\frac{\sin 2x}{x^4}\,dx -$$
$$- \int_m^M \frac{\Delta^2(4\omega_c^2-\Delta^2)}{8}\frac{\sin 2x}{x^2}\,dx + \int_m^M \frac{\Delta^4}{4}\frac{\cos^2 x}{x^3}\,dx + \int_m^M \left(\frac{\omega_c^4}{x}+\frac{\Delta^4}{16x}\right)\sin^2 x\,dx +$$
$$+ \int_m^M \frac{\Delta^2\omega_c^2(\sin^2 x + 2\cos^2 x)}{2x}\,dx, \quad M = \frac{\Delta T}{2};$$

$$36.\quad \int_0^{\frac{\Delta T}{2}} \left(\frac{\omega_c^4}{x}+\frac{\Delta^4}{16x}\right)\sin^2 x\,dx = \frac{1}{2}\int_0^{\frac{\Delta T}{2}} \left(\frac{\omega_c^4}{x}+\frac{\Delta^4}{16x}\right)(1-\cos 2x)\,dx =$$
$$= \frac{1}{2}\left(\omega_c^4+\frac{\Delta^4}{16}\right)\int_0^{\Delta T}\frac{1-\cos v}{v}\,dv = \frac{1}{2}\left[\omega_c^4+\frac{\Delta^4}{16}\right]\int_0^{\Delta T}\frac{dv}{v} - \frac{1}{2}\left[\omega_c^4+\frac{\Delta^4}{16}\right] \times$$
$$\times \int_0^{\Delta T}\frac{\cos v}{v}\,dv = \frac{1}{2}\left[\omega_c^4+\frac{\Delta^4}{16}\right]L(\Delta T) - \frac{1}{2}\left[\omega_c^4+\frac{\Delta^4}{16}\right]K(\Delta T), \quad \text{(II.2.16)} \tag{A.2.15}$$

where /219

$$L(\Delta T) = \int_0^{\Delta T}\frac{dv}{v}; \quad K(\Delta T) = \int_0^{\Delta T}\frac{\cos v}{v}\,dv;$$

$$37.\quad \int_0^{\frac{\Delta T}{2}} \frac{\Delta^2\omega_c^2}{2x}(\sin^2 x + 2\cos^2 x)\,dx = \int_0^{\frac{\Delta T}{2}} \frac{\Delta^2\omega_c^2}{2x}\left(\frac{3+\cos 2x}{2}\right)dx =$$
$$= \frac{1}{2}\int_0^{\Delta T}\frac{\Delta^2\omega_c^2}{v}\left(\frac{3+\cos v}{2}\right)dv = \frac{3\Delta^2\omega_c^2}{4}L(\Delta T) + \frac{\Delta^2\omega_c^2}{4}K(\Delta T). \tag{A.2.17}$$

Let us integrate the remaining terms of the sum (A.2.15):

$$38.\ \int_m^M Q(x)\,dx = \int_m^M \frac{\Delta^4}{4}\frac{\sin^2 x}{x^5}\,dx - \int_m^M \frac{\Delta^4}{4}\frac{\sin^2 x}{x^3}\,dx - \int_m^M \frac{\Delta^4}{4}\frac{\sin 2x}{x^4}\,dx -$$

$$-\int_m^M \frac{\Delta^2(4\omega_c^2-\Delta^2)}{8}\frac{\sin 2x}{x^2}\,dx + \int_m^M \frac{\Delta^4}{4}\frac{\cos^2 x}{x^3}\,dx = \frac{\Delta^4}{4}\left\{\frac{1}{8}\left[\frac{1}{x^4}\right]_M^m + \right.$$

$$+\left[\frac{2\cos v}{v^4} - \frac{2\sin v}{3v^3} - \frac{\cos v}{3v^2} + \frac{\sin v}{3v} - \frac{K(v)}{3}\right]_{2m}^{2M}\Bigg\} - \frac{\Delta^4}{4}\left\{\frac{1}{4}\left[\frac{1}{x^2}\right]_M^m + \right.$$

$$+\left[\frac{\cos v}{v^2} - \frac{\sin v}{v} + K(v)\right]_{2m}^{2M}\Bigg\} - \left\{\frac{\Delta^4}{4}\left[-\frac{8\sin v}{3v^3} - \frac{4\cos v}{3v^2} + \right.\right.$$

$$+\frac{4\sin v}{3v} - \frac{4K(v)}{3}\Bigg]_{2m}^{2M}\Bigg\} - \left\{\frac{\Delta^2(4\omega_c^2-\Delta^2)}{8}\left[-\frac{2\sin v}{v} + 2K(v)\right]_{2m}^{2M}\right\} +$$

$$+\frac{\Delta^4}{4}\left\{\frac{1}{4}\left[\frac{1}{x^2}\right]_M^m + \left[-\frac{\cos v}{v^2} + \frac{\sin v}{v} - K(v)\right]_{2m}^{2M}\right\}.$$

Grouping terms and performing contractions, we produce

$$39.\ \int_m^M Q(x)\,dx = \frac{\Delta^4}{32}\left[\frac{1}{x^4}\right]_M^m + \left[\frac{\Delta^4\cos v}{2v^4} + \frac{\Delta^4\sin v}{2v^3} - \frac{\Delta^4\cos v}{4v^2} + \Delta^2\omega_c^2\frac{\sin v}{v} - \right.$$

$$\left. - \Delta^2\omega_c^2 K(v)\right]_{2m}^{2M};$$

$$40.\ \int_0^{\frac{\Delta T}{2}} Q(x)\,dx = \frac{\Delta^4\cos\Delta T}{2(\Delta T)^4} + \frac{\Delta^4\sin\Delta T}{2(\Delta T)^3} - \frac{\Delta^4\cos\Delta T}{4(\Delta T)^2} + \frac{\Delta^2\omega_c^2\sin\Delta T}{\Delta T} -$$

$$-\frac{\Delta^4}{2(\Delta T)^4} - \Delta^2\omega_c^2 K(\Delta T) + \lim_{m\to 0}\left[\frac{\Delta^4}{32m^4} - \frac{\Delta^4\cos 2m}{32m^4} - \frac{\Delta^4\sin 2m}{16m^3} + \right.$$

$$\left. + \frac{\Delta^4\cos 2m}{16m^2}\right] - \Delta^2\omega_c^2.$$

The limit is calculated by using the L'Hopital rule /220

$$41.\ \lim_{m\to 0}\left[\frac{\Delta^4}{32m^4} - \frac{\Delta^4\cos 2m}{32m^4} - \frac{\Delta^4\sin 2m}{16m^3} + \frac{\Delta^4\cos 2m}{16m^2}\right] =$$

$$= \lim_{m\to 0} \frac{\Delta^4 - \Delta^4\cos 2m - 2\Delta^4 m \sin 2m + 2\Delta^4 m^2 \cos 2m}{32 m^4} =$$

$$= \lim_{m\to 0} \frac{2\Delta^4 \sin 2m - 2\Delta^4 \sin 2m - 4\Delta^4 m \cos 2m + 4\Delta^4 m \cos 2m - 4\Delta^4 m^2 \sin 2m}{128 m^3} = -\frac{\Delta^4}{16};$$

$$42. \quad \int_0^{\frac{\Delta T}{2}} Q(x)\,dx = \frac{\Delta^4 \cos \Delta T}{2(\Delta T)^4} + \frac{\Delta^4 \sin \Delta T}{2(\Delta T)^3} - \frac{\Delta^4 \cos \Delta T}{4(\Delta T)^2} + \frac{\Delta^2\omega_c^2 \sin \Delta T}{\Delta T} -$$

$$- \frac{\Delta^4}{2(\Delta T)^4} - \Delta^2\omega_c^2 - \frac{\Delta^4}{16} - K(\Delta T)\,\Delta^2\omega_c^2 . \qquad \text{(A.2.18)}$$

Adding (A.2.16), (A.2.17) and (A.2.18), we find the resulting expression for (A.2.15) (M = ΔT/2; m = 0):

$$43. \quad \int_0^{\frac{\Delta T}{2}} x r^2_{y_{10}}(x)\,dx = \frac{\Delta^4 \cos \Delta T}{2(\Delta T)^4} + \frac{\Delta^4 \sin \Delta T}{2(\Delta T)^3} - \frac{\Delta^4 \cos \Delta T}{4(\Delta T)^2} + \frac{\Delta^2\omega_c^2 \sin \Delta T}{\Delta T} -$$

$$- \frac{\Delta^4}{2(\Delta T)^4} - \Delta^2\omega_c^2 - \frac{\Delta^4}{16} + \left[\frac{1}{2}\left(\omega_c^4 + \frac{\Delta^4}{16}\right) + \frac{3\Delta^2\omega_c^2}{4}\right][\ln \Delta T - \mathrm{Ci}(\Delta T) + C_0],$$

(A.2.19)

where [see (A.1.4)] $\int_0^{\Delta T} \frac{1-\cos v}{v}\,dv = C_0 + \ln \Delta T - \mathrm{Ci}(\Delta T).$

On the basis of (A.2.10) and (A.2.19), integral (A.2.5) can be represented in the form

$$44. \quad \frac{2}{T^2}\int_0^T (T-\tau)\, r^2_{y_{10}}(\tau)\,d\tau = \frac{2}{T}\left\{\frac{2}{\Delta}\int_0^{\frac{\Delta T}{2}} r^2_{y_{10}}(x)\,dx\right\} -$$

$$- \frac{2}{T^2}\left\{\frac{4}{\Delta^2}\int_0^{\frac{\Delta T}{2}} x r^2_{y_{10}}(x)\,dx\right\} = \frac{4}{\Delta T}\left\{\left(\omega_c^4 + \frac{\Delta^4}{16} + \frac{\omega_c^2\Delta^2}{2}\right) \times\right.$$

$$\times \left[\mathrm{Si}(\Delta T) - \frac{2\sin^2\frac{\Delta T}{2}}{\Delta T}\right] + \frac{4\Delta^4 \cos \Delta T}{5(\Delta T)^5} + \frac{4\Delta^4 \sin \Delta T}{5(\Delta T)^4} - \frac{2\Delta^4 \cos \Delta T}{5(\Delta T)^3} +$$

$$\left. + \left(\omega_c^2\Delta^2 - \frac{\Delta^4}{20}\right)\frac{\sin \Delta T}{(\Delta T)^2} - \frac{\Delta^4 \cos \Delta T}{20\Delta T} - \frac{\Delta^4\,\mathrm{Si}(\Delta T)}{20} - \frac{4\Delta^4}{5(\Delta T)^5} - \frac{\Delta^2\omega_c^2}{\Delta T}\right\} -$$

$$-\frac{8}{(\Delta T)^2}\left\{\frac{\Delta^4\cos\Delta T}{2(\Delta T)^4}+\frac{\Delta^4\sin\Delta T}{2(\Delta T)^3}-\frac{\Delta^4\cos\Delta T}{4(\Delta T)^2}+\frac{\Delta^2\omega_c^2\sin\Delta T}{\Delta T}-\frac{\Delta^4}{2(\Delta T)^4}-\right.$$

$$\left.-\Delta^2\omega_c^2-\frac{\Delta^4}{16}+\left[\frac{1}{2}\left(\omega_c^4+\frac{\Delta^4}{16}\right)+\frac{3\Delta^2\omega_c^2}{4}\right][\ln\Delta T-\mathrm{Ci}(\Delta T)+C_0]\right\}. \qquad \text{(A.2.20)}$$ /221

With narrow band processes $\omega_c \gg \Delta$, expression (A.2.20) is somewhat simplified. Noting that

$$\frac{4}{\Delta T}\left(\omega_c^4+\frac{\Delta^4}{16}+\frac{\omega_c^2\Delta^2}{2}\right)\left[\mathrm{Si}(\Delta T)-\frac{2\sin^2\frac{\Delta T}{2}}{\Delta T}\right]=$$

$$=\frac{4}{\Delta T}\left(\omega_c^4+\frac{\Delta^4}{16}+\frac{\omega_c^2\Delta^2}{2}\right)\mathrm{Si}(\Delta T)-\frac{4}{(\Delta T)^2}\left(\omega_c^4+\frac{\Delta^4}{16}+\frac{\omega_c^2\Delta^2}{2}\right)+$$

$$+\frac{4}{(\Delta T)^2}\left(\omega_c^4+\frac{\Delta^4}{16}+\frac{\omega_c^2\Delta^2}{2}\right)\cos\Delta T,$$

we group coefficients with cos ΔT, sin ΔT, Si(ΔT):

$$45.\quad \cos\Delta T\left\{\frac{4\Delta^4}{5(\Delta T)^5}\cdot\frac{4}{\Delta T}-\frac{2\Delta^4}{5(\Delta T)^3}\cdot\frac{4}{\Delta T}-\frac{\Delta^4}{20\Delta T}\cdot\frac{4}{\Delta T}-\right.$$

$$-\frac{\Delta^4}{2(\Delta T)^4}\cdot\frac{8}{(\Delta T)^2}+\frac{\Delta^4}{4(\Delta T)^2}\cdot\frac{8}{(\Delta T)^2}+\frac{4}{(\Delta T)^2}\left(\omega_c^4+\frac{\Delta^4}{16}+\frac{\omega_c^2\Delta^2}{2}\right)=$$

$$=\frac{64\Delta^4-32\Delta^4(\Delta T)^2-4\Delta^4(\Delta T)^4-80\Delta^4+40\Delta^4(\Delta T)^2+80\left(\omega_c^4+\frac{\Delta^4}{16}+\frac{\Delta^2\omega_c^2}{2}\right)(\Delta T)^4}{20(\Delta T)^6}=$$

$$=\frac{-4\Delta^4+2\Delta^4(\Delta T)^2-(\Delta T)^4\Delta^2\omega_c^2\left(\frac{\Delta^2}{\omega_c^2}-20\frac{\omega_c^2}{\Delta^2}-\frac{20\Delta^2}{\omega_c^2}-10\right)}{5(\Delta T)^6}=$$

$$=\left.\frac{2\Delta^4(\Delta T)^2-4\Delta^4+20(\Delta T)^4\omega_c^4}{5(\Delta T)^6}\right\}; \qquad \text{(A.2.21)}$$

$$46.\quad \sin\Delta T\left\{\frac{4\Delta^4}{5(\Delta T)^4}\cdot\frac{4}{\Delta T}+\frac{\omega_c^2\Delta^2-\frac{\Delta^4}{20}}{(\Delta T)^2}\cdot\frac{4}{\Delta T}-\right.$$

$$-\frac{\Delta^4}{2(\Delta T)^3}\cdot\frac{8}{(\Delta T)^2}-\frac{\Delta^2\omega_c^2}{\Delta T}\cdot\frac{8}{(\Delta T)^2}=$$

$$=\frac{32\Delta^4+40(\Delta T)^2\omega_c^2\Delta^2-2(\Delta T)^2\Delta^4-40\Delta^4-80\Delta^2\omega_c^2(\Delta T)^2}{10(\Delta T)^5}= \qquad \text{(A.2.22)}$$

$$=\frac{-\Delta^2\omega_c^2\left[4\frac{\Lambda^2}{\omega_c^2}+(\Delta T)^2\left(20+\frac{\Delta^2}{\omega_c^2}\right)\right]}{5(\Delta T)^5}=\left.-\frac{4\Delta^2\omega_c^2\left[\frac{\Delta^2}{\omega_c^2}+5(\Delta T)^2\right]}{5(\Delta T)^5}\right\}$$

$$47.\quad \mathrm{Si}\,(\Delta T)\left\{\frac{4}{\Delta T}\left(\omega_c^4+\frac{\Delta^4}{16}+\frac{\omega_c^2\Delta^2}{2}\right)-\frac{4}{\Delta T}\cdot\frac{\Delta^4}{20}-\frac{4}{\Delta T}\,\omega_c^4\right\}. \tag{A.2.23}$$

The free term /222

$$48.\quad -\frac{4\Delta^4}{5\,(\Delta T)^5}\cdot\frac{4}{\Delta T}-\frac{\Delta^2\omega_c^2}{\Delta T}\cdot\frac{4}{\Delta T}+\frac{\Delta^4}{2\,(\Delta T)^4}\cdot\frac{8}{(\Delta T)^2}+\Delta^2\omega_c^2\,\frac{8}{(\Delta T)^2}+$$
$$+\frac{\Delta^{4}8}{16\,(\Delta T)^2}-\frac{4}{(\Delta T)^2}\left(\omega_c^4+\frac{\Delta^4}{16}+\frac{\Delta^2\omega_c^2}{2}\right)=$$
$$=\frac{8\Delta^4+\Delta^2\omega_c^2\,(\Delta T)^4\left[40+5\,\frac{\Delta^2}{\omega_c^2}-40\left(\frac{\omega_c^2}{\Delta^2}+\frac{\Delta^2}{16\omega_c^2}+\frac{1}{2}\right)\right]}{10\,(\Delta T)^6}=\frac{4\Delta^4-20\,(\Delta T)^4\,\omega_c^4}{5\,(\Delta T)^6} \tag{A.2.24}$$

and the last term of sum (A.2.20)

$$49.\quad -\frac{8}{(\Delta T)^2}\left[\frac{1}{2}\left(\omega_c^4+\frac{\Delta^4}{16}\right)+\frac{3\Delta^2\omega_c^2}{4}\right]\left[\ln\Delta T-\mathrm{Ci}\,(\Delta T)+C_0\right]=$$
$$=-\frac{4\omega_c^4}{(\Delta T)^2}\left[\ln\Delta T-\mathrm{Ci}\,(\Delta T)+C_0\right]. \tag{A.2.25}$$

Summing (A.2.21)-(A.2.25), we write the integral (A.2.25) for narrow band processes in the form

$$50.\quad \frac{2}{T^2}\int_0^T (T-\tau)\,r^2_{u_0}(\tau)\,d\tau=\frac{2\Delta^4(\Delta T)^2-4\Delta^4+20\,(\Delta T)^4\,\omega_c^4}{5\,(\Delta T)^6}\cos\Delta T-$$
$$-\frac{4\Delta^2\omega_c^2\left[\frac{\Delta^2}{\omega_c^2}+5\,(\Delta T)^2\right]}{5\,(\Delta T)^5}\sin\Delta T+\frac{4\omega_c^4}{\Delta T}\,\mathrm{Si}\,(\Delta T)-$$
$$-\frac{4\omega_c^4}{(\Delta T)^2}\left[\ln(\Delta T)-\mathrm{Ci}\,(\Delta T)+C_0\right]+\frac{4\Delta^4-20\,(\Delta T)^4\,\omega_c^4}{5\,(\Delta T)^6}. \tag{A.2.26}$$

Where $\Delta T \gg 1$, expression (A.2.26) leads to (A.2.12) (see Yanke, Emde, 1949):

$$\int_0^{\Delta T} \frac{1-\cos v}{v}\, dv \approx \ln \Delta T + C_0 - \frac{\sin(\Delta T)}{\Delta T}; \quad \mathrm{Si}(\Delta T) \approx \frac{\pi}{2} - \frac{\cos \Delta T}{\Delta T}.$$

Ignoring all terms in (A.2.26) ($\Delta T \gg 1$), except for the integral sine, we find

$$51.\ \frac{4\omega_c^4}{\Delta T}\, \mathrm{Si}(\Delta T) \approx \frac{4\omega_c^4 \pi}{2\Delta T} = \frac{2\omega_c^4 \pi}{\Delta T}.$$

Appendix 3

An estimate of the relative errors in calculation of the mathematical expectation $\delta[n_T(0)]$ and dispersion $\delta(\sigma_n^2)$ in expansion of function $n_T(0)$ into a Taylor series with retention of the first three terms (accuracy of linearization method). On the basis of (4.3.98), (4.3.100), (4.3.101), (4.3.104), (4.3.105), we have /223

$$1.\ n_T(0) = \frac{1}{\pi}\left(\frac{u_1}{u}\right)^{\frac{1}{2}};$$

$$2.\ \delta[\overline{n_T(0)}] = \left| \frac{\frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} \frac{\partial^2 [n_T(0)]}{\partial u_i \partial u_j} B_{ij}}{\overline{n_T(0)} + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} \frac{\partial^2 [n_T(0)]}{\partial u_i \partial u_j} B_{ij}} \right|; \tag{A.3.1}$$

(A.3.2)

$$3.\ \delta(\sigma_n^2) = \left| \frac{\frac{1}{4}\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} \frac{\partial^2 [n_T(0)]}{\partial u_i \partial u_j} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_k \partial u_l} (B_{ij}B_{kl} + B_{ik}B_{jl} + B_{il}B_{jk})}{\sigma_n^2 + \frac{1}{4}\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} \frac{\partial^2 [n_T(0)]}{\partial u_i \partial u_j} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_k \partial u_l} (B_{ij}B_{kl} + B_{ik}B_{jl} + B_{il}B_{jk})} \right|$$

$$4.\ \frac{\partial^2 [n_T(0)]}{\partial u_i \partial u_j} = \frac{\partial^2 [n_T(0)]}{\partial u \partial u_1} = \frac{u_1}{4\pi u^3}\left(\frac{u_1}{u}\right)^{-\frac{3}{2}} - \frac{1}{4\pi u^2}\left(\frac{u_1}{u}\right)^{-\frac{1}{2}} =$$

$$= \left(\frac{u_1}{u}\right)^{\frac{1}{2}} \left[\frac{u_1}{4\pi u^3}\left(\frac{u_1}{u}\right)^{-2} - \frac{1}{2\pi u^2}\left(\frac{u_1}{u}\right)^{-1}\right] = -\frac{1}{4\pi u_1 u}\left(\frac{u_1}{u}\right)^{\frac{1}{2}};$$

$$5.\ \frac{\partial^2[n_T(0)]}{\partial u^2} = \frac{u_1}{\pi u^3}\left(\frac{u_1}{u}\right)^{-\frac{1}{2}} - \frac{u_1^2}{4\pi u^4}\left(\frac{u_1}{u}\right)^{-\frac{3}{2}} =$$

$$= \left(\frac{u_1}{u}\right)^{\frac{1}{2}}\left[\frac{u_1}{\pi u^3}\left(\frac{u_1}{u}\right)^{-1} - \frac{u_1^2}{4\pi u^4}\left(\frac{u_1}{u}\right)^{-2}\right] = \frac{3}{4\pi u^2}\left(\frac{u_1}{u}\right)^{\frac{1}{2}};$$

$$6.\ \frac{\partial^2[n_T(0)]}{\partial u_1^2} = -\frac{1}{4\pi u^2}\left(\frac{u_1}{u}\right)^{-\frac{3}{2}} = -\frac{1}{4\pi u_1^2}\left(\frac{u_1}{u}\right)^{\frac{1}{2}};$$

$$7.\ \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\partial^2[n_T(0)]}{\partial u_i \partial u_j}B_{ij} = \frac{1}{2}\left\{\frac{\partial^2[n_T(0)]}{\partial u_1 \partial u_1}B_{11} + \frac{\partial^2[n_T(0)]}{\partial u_1 \partial u_2}B_{12} + \right.$$

$$\left. + \frac{\partial^2[n_T(0)]}{\partial u_2 \partial u_1}B_{21} + \frac{\partial^2[n_T(0)]}{\partial u_2 \partial u_2}B_{22}\right\} = \frac{1}{2}\left\{\frac{\partial^2[n_T(0)]}{\partial u^2}B_{uu} + \right.$$

$$\left. + 2\frac{\partial^2[n_T(0)]}{\partial u \partial u_1}B_{uu_1} + \frac{\partial^2[n_T(0)]}{\partial u_1^2}B_{u_1u_1}\right\}, \qquad (A.3.3)$$

where /224

$$u_1 = u; \quad u_2 = u_1; \quad B_{uu} = \sigma_u^2; \quad B_{u_1u_1} = \sigma_{u_1}^2.$$

Let us calculate the component sums of (A.3.3). Substituting into the coefficients with $B_{u_iu_j}$ the mathematical expectations of variables u, u_i and noting that

$$\frac{1}{\pi}\left(\frac{u_1}{u}\right)^{\frac{1}{2}} = \overline{n_T(0)},$$

we produce [see (4.3.83), (4.3.89), (4.3.93), (4.3.94), (4.3.97)]

$$8.\ \frac{\partial^2[n_T(0)]}{\partial u^2}\sigma_u^2 = \frac{3\sigma_u^2}{4\pi \bar{u}^2}\left(\frac{\bar{u}_1}{\bar{u}}\right)^{\frac{1}{2}} = \overline{n_T(0)}\,\frac{3}{4\sigma_y^4}\cdot\frac{2\pi\sigma_y^4}{\Delta\omega_y T} = \frac{3\pi}{2\Delta\omega_y T}\overline{n_T(0)};$$

$$9.\ \frac{\partial^2[n_T(0)]}{\partial u_1^2}\sigma_{u_1}^2 = -\frac{1}{4\pi \bar{u}_1^2}\left(\frac{\bar{u}_1}{\bar{u}}\right)^{\frac{1}{2}}\sigma_{u_1}^2 =$$

$$= -\overline{n_T(0)}\,\frac{2\pi\sigma_y^4\omega_c^4\left[\left(1 + \frac{\Delta\omega_y^2}{12\omega_c^2}\right)^2 + \frac{\Delta\omega_y^2}{3\omega_c^2} + \frac{\Delta\omega_y^4}{180\omega_c^4}\right]}{4\sigma_y^4\omega_c^4\left(1 + \frac{\Delta\omega_y^2}{12\omega_c^2}\right)^2\Delta\omega_y T}.$$

Ignoring the terms $\Delta\omega_4^y/180\ \omega_c^4$ for narrow band processes and assuming

$$\left(1+\frac{\Delta\omega_y^2}{12\omega_c^2}\right)^2 \approx 1+\frac{\Delta\omega_y^2}{6\omega_c^2},$$

we rewrite the last equation in the form

$$10.\quad \frac{\partial^2[n_T(0)]}{\partial u_1^2}\sigma_{u_1}^2 = -\frac{\overline{n_T(0)}\,\pi}{2\Delta\omega_y T}\left(1+\frac{2\Delta\omega_y^2}{6\omega_c^2+\Delta\omega_y^2}\right) = -\frac{\pi\overline{n_T(0)}}{2\Delta\omega_y T}(1+A),$$

where

$$A=\frac{2\Delta\omega_y^2}{6\omega_c^2+\Delta\omega_y^2};$$

$$11.\quad \frac{\partial^2[n_T(0)]}{\partial u_1\partial u}B_{uu_1} = -\frac{1}{4\pi u_1 u}\left(\frac{u_1}{u}\right)^{\frac{1}{2}}B_{uu_1} =$$

$$= -\frac{\overline{n_T(0)}\,2\pi\sigma_y^4\left(1+\frac{\Delta\omega_y^2}{12\omega_c^2}\right)\omega_c^2}{4\sigma_y^4\omega_c^2\left(1+\frac{\Delta\omega_y^2}{12\omega_c^2}\right)\Delta\omega_y T} = -\frac{\pi\overline{n_T(0)}}{2\Delta\omega_y T};$$

$$12.\quad \delta[\overline{n_T(0)}] = \left|\frac{\frac{1}{2}\overline{n_T(0)}\left[\frac{3\pi}{2\Delta\omega_y T}-\frac{2\pi}{2\Delta\omega_y T}-\frac{\pi}{2\Delta\omega_y T}(1+A)\right]}{\overline{n_T(0)}+\frac{1}{2}\overline{n_T(0)}\left[\frac{3\pi}{2\Delta\omega_y T}-\frac{2\pi}{2\Delta\omega_y T}-\frac{\pi}{2\Delta\omega_y T}(1+A)\right]}\right| =$$

$$= \left|\frac{-\frac{\pi A}{4\Delta\omega_y T}}{1-\frac{\pi A}{4\Delta\omega_y T}}\right| = \left|-\frac{\pi}{\frac{4\Delta\omega_y T}{A}-\pi}\right| = \left|-\frac{\pi}{\frac{4\Delta\omega_y T}{2\Delta\omega_y^2}(6\omega_c^2+\Delta\omega_y^2)-\pi}\right| =$$

$$= \left|-\frac{\pi}{2\Delta\omega_y T\left(6\frac{\omega_c^2}{\Delta\omega_y^2}+1\right)-\pi}\right| \approx \left|-\frac{\pi}{12\Delta\omega_y T\frac{\omega_c^2}{\Delta\omega_y^2}-\pi}\right|,\quad \frac{\Delta\omega_y^2}{\omega_c^2}\ll 1.$$

(A.3.4) /225

The possible combinations of indices in the quadruple sum (A.3.2) can be easily determined from the table of Figure 79, where the verticals and horizontals carry the combinations of indices of the double sum [see (A.3.3)]. On the basis of the relationship

$$\frac{\partial^2 [n_T(0)]}{\partial u_2 \partial u_1} = \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_2}; \quad B_{12} = B_{21}$$

and analysis of the table, we see that in our case the terms with numbers (2, 3, 5, 9)-(4, 13)-(6, 7, 10, 11)-(8.12, 14, 15) are equal. The calculation is performed by rows, left to right. Consequently, representing the quadruple sum in (A.3.2) as B(u, u_1), we produce six groups of terms, including terms with numbers 1 and 16:

$$\begin{aligned}
13.\ \frac{1}{4} B(u, u_1) = \frac{1}{4} \Big\{ & 4 \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_1} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_2} (B_{11}B_{12} + B_{11}B_{12} + B_{12}B_{11}) + \\
& + 2 \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_1} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_2} (B_{11}B_{22} + B_{12}B_{12} + B_{12}B_{12}) + \\
& + 4 \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_2} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_2} (B_{12}B_{12} + B_{11}B_{22} + B_{12}B_{21}) + \\
& + 4 \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_2} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_2 \partial u_2} (B_{12}B_{22} + B_{12}B_{22} + B_{12}B_{22}) + \\
& + \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_1} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_1 \partial u_1} (B_{11}B_{11} + B_{11}B_{11} + B_{11}B_{11}) + \\
& + \frac{\partial^2 [n_T(0)]}{\partial u_2 \partial u_2} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_2 \partial u_2} (B_{22}B_{22} + B_{22}B_{22} + B_{22}B_{22}) \Big\} = \\
= \frac{1}{4} \Big\{ & 12 \frac{\partial^2 [n_T(0)]}{\partial u^2} \cdot \frac{\partial^2 [n_T(0)]}{\partial u \partial u_1} B_{uu} B_{uu_1} + \\
& + 2 \frac{\partial^2 [n_T(0)]}{\partial u^2} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_1^2} (B_{uu}B_{u_1u_1} + 2B_{uu_1}B_{uu_1}) + \\
& + 4 \frac{\partial^2 [n_T(0)]}{\partial u \partial u_1} \cdot \frac{\partial^2 [n_T(0)]}{\partial u \partial u_1} (B_{uu}B_{u_1u_1} + 2B_{uu_1}B_{uu_1}) + \\
& + 12 \frac{\partial^2 [n_T(0)]}{\partial u \partial u_1} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_1^2} B_{uu_1}B_{u_1u_1} + 3 \frac{\partial^2 [n_T(0)]}{\partial u^2} \cdot \frac{\partial^2 [n_T(0)]}{\partial u^2} B_{uu}B_{uu} + \\
& + 3 \frac{\partial^2 [n_T(0)]}{\partial u_1^2} \cdot \frac{\partial^2 [n_T(0)]}{\partial u_1^2} B_{u_1u_1}B_{u_1u_1} \Big\}.
\end{aligned} \tag{A.3.5}$$

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	1	2	3	4		N-3	N-2	N-1	N
1	0	-1	-2	-3		-(N-4)	-(N-3)	-(N-2)	-(N-1)
2	1	0	-1	-2		-(N-5)	-(N-4)	-(N-3)	-(N-2)
3	2	1	0	-1		-(N-6)	-(N-5)	-(N-4)	-(N-3)
4	3	2	1	0		-(N-7)	-(N-6)	-(N-5)	-(N-4)
N-3	N-4	N-5	N-6	N-7		0	-1	-2	-3
N-2	N-3	N-4	N-5	N-6		1	0	-1	-2
N-1	N-2	N-3	N-4	N-5		2	1	0	-1
N	N-1	N-2	N-3	N-4		3	2	1	0

b (across columns), a (down rows)

A)

Figure 79

Let us calculate the elements of sum (A.3.5). We introduce the symbol

$$E = 1 + \frac{\Delta\omega_y^2}{12\omega_c^2}; \quad L = \frac{\Delta\omega_y^2}{3\omega_c^2} + \frac{\Delta\omega_y^4}{180\omega_c^4}$$

therefore

$$\bar{u}_1 = \sigma_y^2\left(1 + \frac{\Delta\omega_y^2}{12\omega_c^2}\right)\omega_c^2 = E\sigma_y^2\omega_c^2;$$

$$B_{uu_1} = \frac{2\pi\omega_c^2\sigma_y^4}{\Delta\omega_y T}\left(1 + \frac{\Delta\omega_y^2}{12\omega_c^2}\right) = \frac{2\pi\omega_c^2\sigma_y^4}{\Delta\omega_y T}E;$$

$$\sigma_{u_1}^2 = B_{u_1u_1} = \frac{2\pi\sigma_y^4\omega_c^4}{\Delta\omega_y T}\left(1 + \frac{\Delta\omega_y^2}{2\omega_c^2} + \frac{\Delta\omega_y^4}{80\omega_c^4}\right) = \frac{2\pi\sigma_y^4\omega_c^4}{\Delta\omega_y T}\left[\left(1 + \frac{\Delta\omega_y^2}{12\omega_c^2}\right)^2 + \frac{\Delta\omega_y^2}{3\omega_c^2} + \frac{\Delta\omega_y^4}{180\omega_c^4}\right] = \frac{2\pi\sigma_y^4\omega_c^4}{\Delta\omega_y T}(E^2 + L).$$

The partial derivatives of function $n_T(0)$ were calculated earlier. Substituting the mathematical expectations of variables u, u_1 into the final /227 results, we produce [see (A.3.5)]

$$14.\ 12\,\frac{\partial^2[n_T(0)]}{\partial u^2}\cdot\frac{\partial^2[n_T(0)]}{\partial u\,\partial u_1}\,B_{uu}B_{uu_1}=$$

$$=-\frac{12}{4\pi u_1 u}\left(\frac{u_1}{u}\right)^{1/2}\frac{3}{4\pi u^2}\left(\frac{\bar u_1}{u}\right)^{1/2}B_{uu}B_{uu_1}=$$

$$=-\frac{9\sigma_u^2 B_{uu_1}}{4\pi^2 u^4}=-\frac{9\cdot 2\pi\sigma_y^1\cdot 2\pi\omega_c^2\sigma_y^1 E}{4\pi^2\sigma_y^8\,\Delta\omega_y T\,\Delta\omega_y T}=-E\cdot\frac{9\omega_c^2}{(\Delta\omega_y T)^2};$$

$$15.\ 2\,\frac{\partial^2[n_T(0)]}{\partial u^2}\cdot\frac{\partial^2[n_T(0)]}{\partial u^2}\,(B_{uu}B_{u_1u_1}+2B_{uu_1}B_{uu_1})=$$

$$=\frac{2}{4\pi u_1^2}\left(\frac{u_1}{u}\right)^{\frac{1}{2}}\frac{3}{4\pi u^2}\,(\sigma_u^2\sigma_{u_1}^2+2B_{uu_1}^2)=\frac{3}{8\pi^2 u_1 u^3}\,(\sigma_u^2\sigma_{u_1}^2+2B_{uu_1}^2)=$$

$$=\frac{3}{8\pi^2 E\sigma_y^2\omega_c^2\sigma_y^6}\left[\frac{2\pi\sigma_y^1}{\Delta\omega_y T}\cdot\frac{2\pi\sigma_y^1\omega_c^1}{\Delta\omega_y T}\,(E^2+L)+\right.$$

$$\left.+2\left(\frac{2\pi\omega_c^2\sigma_y^1}{\Delta\omega_y T}\right)^2E^2\right]=\frac{3\omega_c^2}{2\,(\Delta\omega_y T)^2}\left(3E+\frac{L}{E}\right);$$

$$16.\ 4\,\frac{\partial^2[n_T(0)]}{\partial u\,\partial u_1}\cdot\frac{\partial^2[n_T(0)]}{\partial u\,\partial u_1}\,(B_{uu}B_{u_1u_1}+2B_{uu_1}B_{uu_1})=$$

$$=4\left[-\frac{1}{4\pi u_1 u}\left(\frac{u_1}{u}\right)^{\frac{1}{2}}\right]^2(\sigma_u^2\sigma_{u_1}^2+2B_{uu_1}^2)=\frac{4}{16\pi^2 u_1 u^3}\,(\sigma_u^2\sigma_{u_1}^2+2B_{uu_1}^2)=$$

$$=\frac{1}{4\pi^2 E\sigma_y^2\omega_c^2\sigma_y^6}\left[\frac{2\pi\sigma_y^1}{\Delta\omega_y T}\,\frac{2\pi\sigma_y^1\omega_c^1}{\Delta\omega_y T}\,(E^2+L)+2\left(\frac{2\pi\omega_c^1\sigma_y^1}{\Delta\omega_y T}\right)^2E^2\right]=$$

$$=\frac{\omega_c^2}{(\Delta\omega_y T)^2}\left(3E+\frac{L}{E}\right);$$

$$17.\ 12\,\frac{\partial^2[n_T(0)]}{\partial u_1\,\partial u}\cdot\frac{\partial^2[n_T(0)]}{\partial u_1^2}\,B_{uu_1}B_{u_1u_1}=$$

$$=12\,\frac{1}{4\pi u_1 u}\left(\frac{\bar u_1}{u}\right)^{\frac{1}{2}}\frac{1}{4\pi u_1^2}\left(\frac{\bar u_1}{u}\right)^{\frac{1}{2}}B_{uu_1}\sigma_{u_1}^2=\frac{3}{4\pi^2 u_1^2 u_2}\,B_{uu_1}\sigma_{u_1}^2=$$

$$=\frac{3}{4\pi^2\omega_y^1\omega_c^1E^2\sigma_y^1}\cdot\frac{2\pi\sigma_y^1\omega_c^2}{\Delta\omega_y T}\,E\,\frac{2\pi\omega_y^1\omega_c^1}{\Delta\omega_y T}\,(E^2+L)=\frac{3\omega_c^2}{(\Delta\omega_y T)^2}\left(E+\frac{L}{E}\right)$$

$$18.\quad 3\frac{\partial^2[n_T(0)]}{\partial u^2}\cdot\frac{\partial^2[n_T(0)]}{\partial u^2}B_{uu}B_{uu} = 3\left[\frac{3}{4\pi\bar{u}^2}\left(\frac{\bar{u}_1}{\bar{u}}\right)^{\frac{1}{2}}\right]^2\sigma_u^4 =$$

$$= 3\,\frac{9\bar{u}_1}{16\pi^2\bar{u}^5}\,\sigma_u^4 = \frac{27\sigma_y^2 E\,\omega_c^2 4\pi^2\sigma_y^8}{16\pi^2\sigma_y^{10}(\Delta\omega_y T)^2} = \frac{27\omega_c^2}{4(\Delta\omega_y T)^2}\,E;$$

$$19.\quad 3\frac{\partial^2[n_T(0)]}{\partial u_1^2}\cdot\frac{\partial^2[n_T(0)]}{\partial u_1^2}B_{u_1u_1}B_{u_1u_1} =$$

$$= 3\left[-\frac{1}{4\pi\bar{u}_1^2}\left(\frac{\bar{u}_1}{u}\right)^{\frac{1}{2}}\right]\sigma_{u_1}^4 = \frac{3}{16\pi^2 u_1^3\bar{u}}\,\sigma_{u_1}^4 =$$

$$= \frac{3\cdot 4\pi^2\sigma_y^8\omega_c^8(E^2+L)^2}{16\pi^2\sigma_y^6E^2\omega_c^6\sigma_y^2(\Delta\omega_y T)^2} = \frac{3\omega_c^2}{4(\Delta\omega_y T)^2}\left(E+2\frac{L}{E}+\frac{L^2}{E^3}\right);$$

Consequently,

$$20.\quad \frac{1}{4}B(u_1u_1) = \frac{1}{4}\left[-\frac{9\omega_c^2E}{(\Delta\omega_yT)^2} - \frac{3\omega_c^2}{2(\Delta\omega_yT)^2}\left(3E+\frac{L}{E}\right)+\right.$$

$$+\frac{\omega_c^2}{(\Delta\omega_yT)^2}\left(3E+\frac{L}{E}\right)+\frac{3\omega_c^2}{(\Delta\omega_yT)^2}\left(E+\frac{L}{E}\right)+$$

$$\left.+\frac{27\omega_c^2}{4(\Delta\omega_yT)^2}E+\frac{3\omega_c^2}{4(\Delta\omega_yT)^2}\left(E+2\frac{L}{E}+\frac{L^2}{E^3}\right)\right] =$$

$$= \frac{3\omega_c^2}{16(\Delta\omega_yT)^2}\left(\frac{2L}{E}+\frac{L^2}{E^3}\right).$$

From equation (4.3.108), we find

$$21.\quad \sigma_n^2 = \frac{\omega_c^2\left(\frac{\Delta\omega_y^2}{3\omega_c^2}+\frac{\Delta\omega_y^4}{180\omega_c^4}\right)}{2\pi\Delta\omega_yT\left(1+\frac{\Delta\omega_y^2}{12\omega_c^2}\right)} - \frac{\omega_c E}{2\pi\Delta\omega_yT},$$

from which follows

$$22.\ \frac{1}{4}B(uu_1) = \sigma_n^2 \frac{3\pi}{8\Delta\omega_y T}\left(2+\frac{L}{E^2}\right);$$

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$$23.\ \delta(\sigma_n^2) = \left| \frac{\frac{1}{4}B(uu_1)}{\sigma_n^2 + \frac{1}{4}B(uu_1)} \right| = \left| \frac{\sigma_n^2 \frac{3\pi}{8\Delta\omega_y T}\left(2+\frac{L}{E^2}\right)}{\sigma_n^2 + \sigma_n^2 \frac{3\pi}{8\Delta\omega_y T}\left(2+\frac{L}{E^2}\right)} \right| =$$

$$= \left| \frac{3\pi\left(2+\frac{L}{E^2}\right)}{8\Delta\omega_y T + 3\pi\left(2+\frac{L}{E^2}\right)} \right|.$$

Assuming for the narrow band processes

$$L \approx \frac{\Delta\omega_y^2}{3\omega_c^2}; \quad E^2 \approx 1 + \frac{\Delta\omega_y^2}{6\omega_c^2},$$

we can rewrite the expression for determination of the relative error in calculation of the dispersion $n_T(0)$ in the form

$$24.\ \delta(\sigma_n^2) = \left| \frac{\pi}{\frac{8\Delta\omega_y T}{(2+L/E^2)^3} + \pi} \right| = \left| \frac{\pi}{\frac{8\Delta\omega_y T E^2}{3(2E^2+L)} + \pi} \right| =$$

$$= \left| \frac{\pi}{\frac{8\Delta\omega_y T\left(1+\frac{\Delta\omega_y^2}{6\omega_c^2}\right)}{3\left(2+\frac{\Delta\omega_y^2}{3\omega_c^2}+\frac{\Delta\omega_y^2}{3\omega_c^2}\right)} + \pi} \right| = \left| \frac{\pi}{\frac{8\Delta\omega_y T(6\omega_c^2+\Delta\omega_y^2)}{3(12\omega_c^2+4\Delta\omega_y^2)} + \pi} \right| =$$

$$= \left| \frac{\pi}{\frac{2\Delta\omega_y T\left(6+\frac{\Delta\omega_y^2}{\omega_c^2}\right)}{3\left(3+\frac{\Delta\omega_y^2}{\omega_c^2}\right)} + \pi} \right|. \qquad (A.3.6)$$

If we consider further

$$6 + \frac{\Delta\omega_y^2}{\omega_c^2} \approx 6; \quad 3 + \frac{\Delta\omega_y^2}{\omega_c^2} \approx 3,$$

equation (A.3.6) can be represented as follows:

$$25.\ \delta(\sigma_n^2) \approx \left| \frac{\pi}{\frac{4}{3}\Delta\omega_y T + \pi} \right|. \qquad \text{(A.3.7)}$$

Appendix 4

Calculation of the sum of terms for various combinations of indices in the expression /230

$$1.\ B_{s^2}(\tau) = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sum_{a=1}^{N} \sum_{b=1}^{N} \sum_{c=1}^{N} \sum_{d=1}^{N} A_a A_b A_c A_d \cos(a\omega_0 t - \varphi_a) \times$$
$$\times \cos(b\omega_0 t - \varphi_b) \cos[(t-\tau)\omega_0 c - \varphi_c] \cos[(t-\tau)\omega_0 d - \varphi_d]\, dt. \quad \text{(II.4.1)} \qquad \text{(A.4.1)}$$

Let us analyze the products of the cosines of the general term H of the integrand quadruple sum [see (A.4.1)]

$$2.\ H = \cos(a\omega_0 t - \varphi_a)\cos(b\omega_0 t - \varphi_b)\cos[(t-\tau)\omega_0 c - \varphi_c] \times$$
$$\times \cos[(t-\tau)\omega_0 d - \varphi_d] = \frac{1}{4}\{\cos[(a-b)\omega_0 t - (\varphi_a - \varphi_b)] \times$$
$$\times \cos[(c-d)\omega_0 t - (c-d)\omega_0\tau - (\varphi_c - \varphi_d) +$$
$$+ \cos[(a-b)\omega_0 t - (\varphi_a - \varphi_b)]\cos[(c+d)\omega_0 t - (c+d)\omega_0\tau - (\varphi_c + \omega_d)] +$$
$$+ \cos[(a+b)\omega_0 t - (\varphi_a + \varphi_b)]\cos[(c-d)\omega_0 t - (c-d)\omega_0\tau - (\varphi_c - \varphi_d)] +$$
$$+ \cos[(a+b)\omega_0 t - (\varphi_a + \varphi_b)]\cos[(c+d)\omega_0 t - (c+d)\omega_0\tau - (\varphi_c + \varphi_d)]\}.$$

The sum of products of cosines produced consists of four components. We can represent them in order H_1, H_2, H_3 and H_4 and, determining the product of the cosines from known trigonometric formulas, represent it in the form

$$3.\ H_1 = \frac{1}{8}\{\cos[(a-b-c+d)\omega_0 t - (\varphi_a - \varphi_b - \varphi_c + \varphi_d) + (c-d)\omega_0\tau] +$$
$$+ \cos[(a-b+c-d)\omega_0 t - (\varphi_a - \varphi_b + \varphi_c - \varphi_d) - (c-d)\omega_0\tau]\}; \qquad \text{(A.4.2a)}$$

$$4.\ H_2 = \frac{1}{8}\{\cos[(a-b-c-d)\,\omega_0 t - (\varphi_a - \varphi_b - \varphi_c - \varphi_d) + (c+d)\,\omega_0\tau] +$$
$$+ \cos[(a-b+c+d)\,\omega_0 t - (\varphi_a - \varphi_b + \varphi_c + \varphi_d) - (c+d)\,\omega_0\tau]\}; \tag{A.4.2b}$$

$$5.\ H_3 = \frac{1}{8}\{\cos[(a+b-c+d)\,\omega_0 t - (\varphi_a + \varphi_b - \varphi_c + \varphi_d) + (c-d)\,\omega_0\tau] +$$
$$+ \cos[(a+b+c-d)\,\omega_0 t - (\varphi_a + \varphi_b + \varphi_c - \varphi_d) - (c-d)\,\omega_0\tau]\}; \tag{A.4.2c}$$

$$6.\ H_4 = \frac{1}{8}\{\cos[(a+b-c-d)\,\omega_0 t - (\varphi_a + \varphi_b - \varphi_c - \varphi_d) + (c+d)\,\omega_0\tau] +$$
$$+ \cos[(a+b+c+d)\,\omega_0 t - (\varphi_a + \varphi_b + \varphi_c + \varphi_d) - (c+d)\,\omega_0\tau]\}. \tag{A.4.2d}$$

Let us turn to expression (A.4.1). The integral of the form

$$7.\ \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(n\omega_0 t - \psi)\cos[m\omega_0(t-\tau) - \varphi]\,dt \tag{A.4.3}$$

differs from zero only in the case $|m| = |n|$, where n and m are integers, positive or negative. Consequently, considering the inequalities

$$\begin{array}{ll} 1 \leqslant a \leqslant N & 1 \leqslant c \leqslant N \\ 1 \leqslant b \leqslant N & 1 \leqslant d \leqslant N \end{array}$$

we will be interested in the following combinations of indices a, b, c, d in terms H_1, H_2, H_3, H_4:

$$\begin{array}{ll} \overbrace{\begin{array}{l} 1)\ a-b = c-d \\ 2)\ -(a-b) = -(c-d) \\ 3)\ -(a-b) = c-d \\ 4)\ a-b = -(c-d) \end{array}}^{H_1} & \overbrace{\begin{array}{l} 1)\ a-b = c+d \\ 2)\ -(a-b) = c+d \end{array}}^{H_2} \\ \overbrace{\begin{array}{l} 1)\ a+b = c-d \\ 2)\ a+b = -(c-d) \end{array}}^{H_3} & \overbrace{\begin{array}{l} 1)\ a+b = c+d \end{array}}^{H_4} \end{array} \tag{A.4.4}$$

As a result of averaging H_1, H_2, H_3, H_4 over period T_0 [see (A.4.3)], various combinations of indices produce various components K, L, M, P, R, S, V.

Term H_1:

a) combination of indices

$$-(a-b)=-(c-d);\ (a-b)=c-d \tag{A.4.5a}$$

$$8.\quad K=\frac{1}{16}\cos\left[(c-d)\,\omega_0\tau-(\varphi_a-\varphi_b-\varphi_c+\varphi_d)\right];$$

b) combination of indices

$$-(a-b)=(c-d);\ a-b=-(c-d)$$

$$9.\quad L=\frac{1}{16}\cos\left[(c-d)\,\omega_0\tau+(\varphi_a-\varphi_b+\varphi_c-\varphi_d)\right].$$

Term H_2:

a) combination of indices (a - b) = (c + d)

$$10.\quad M=\frac{1}{16}\cos\left[(c+d)\,\omega_0\tau-(\varphi_a-\varphi_b-\varphi_c-\varphi_d)\right]; \tag{A.4.5b}$$

b) combination of indices: -(a - b) = (c + d)

$$11.\quad P=\frac{1}{16}\cos\left[(c+d)\,\omega_0\tau+(\varphi_a-\varphi_b+\varphi_c+\varphi_d)\right].$$

Term H_3:

a) combination of indices (a + b) = (c - d)

$$12.\quad R=\frac{1}{16}\cos\left[(c-d)\,\omega_0\tau-(\varphi_a+\varphi_b-\varphi_c+\varphi_d)\right]; \tag{A.4.5c}$$

b) combination of indices (a + b) = -(c - d);

$$13.\quad S=\frac{1}{16}\cos\left[(c-d)\,\omega_0\tau+(\varphi_a+\varphi_b+\varphi_c-\varphi_d)\right].$$

Term H_4:

combination of indices: (a + b) = (c + d)

$$14.\quad V=\frac{1}{16}\cos\left[(c+d)\,\omega_0\tau-(\varphi_a+\varphi_b-\varphi_c-\varphi_d)\right]. \tag{A.4.5.d}$$

Let us now find the total number Y of terms of type K and L. Figure 80 A shows the quadratic matrix $\|\delta_{ab}\|$, the elements of which are the possible varieties of indices a and b: /232

15. $\delta_{ab} = a - b$.

It is not difficult to see that elements of this matrix are located symmetrically in relation to the main diagonal ($\delta_{ab} = 0$; $a = b$), equal in magnitude and opposite in sign:

$$16.\ \delta_{ab} = -\delta_{ba};$$
$$17.\ 0 \leqslant |\delta_{ab}| \leqslant N - 1. \qquad (A.4.6)$$

Any two diagonals, symmetrical to the main diagonal, consist of terms identical in absolute value, and are characterized by the equation

$$18.\ |\delta_{ab}| = |a - b| = \text{const},$$

while the total number of elements W_{ab} in each is determined by the relationship

$$19.\ W_{ab} = (N - |\delta_{ab}|).$$

Analyzing the quadruple sum [see (A.4.1)] for a certain fixed value of the difference of indices

$$20.\ \delta_{ab} = \delta_{cd} = \text{const},$$

we note that it is necessary to make a selection of each element of the diagonal row of the matrix $\|\delta_{ab}\|$ (or $\|\delta_{cd}\|$) with all elements of the corresponding diagonal of the matrix $\|\delta_{cd}\|$ (or $\|\delta_{ab}\|$); consequently, in this case the number Y_k of terms of type k in expression (A.4.1) is equal to

$$21.\ Y_k = (N - |\delta_{ab}|)^2,\ |\delta_{ab}| = \text{const}.$$

Determining the sign of the modulus and considering the equation $\delta_{ab} = -\delta_{ba}$, we produce

$$22.\ Y_k = 2(N - \delta_{ab})^2,\quad \delta_{ab} > 0. \qquad (A.4.7)$$

In the case $\delta_{ab} = \delta_{cd} = 0$ and $a - b = c - d = 0$, the resulting number of terms Y_{ab} is determined by the formula

$$23.\ Y_{ab} = N^2, \quad \delta_{ab} = 0, \tag{A.4.8}$$

where for the situation $\delta_{ab} = \delta_{cd} = 0$ and $a = b = c = d$, the following equation is correct:

$$24.\ Y_{ab} = Y'_{ab} = N. \tag{A.4.9}$$

It is obvious that the sum Y_L of components of type L resulting from combinations $\delta_{cd} = -\delta_{ab}$ and $\delta_{ab} = -\delta_{cd}$ with a certain fixed value of the difference of indices i will be equal to

$$25.\ Y_L = Y_k = 2(N - \delta_{ab})^2, \quad \delta_{ab} > 0 \tag{A.4.10}$$

on the strength of symmetry of matrices $\|\delta_{ab}\|$; $\|\delta_{cd}\|$ relative to the main diagonals.

Considering the above, we write with the j-th value of the difference of indices δ_{ab} the formula for the number of terms Y_j of expression (A.4.1), which are determined by four groups of combinations of indices (A.4.4), in the form

$$26.\ Y_j = Y_{ab} + Y_{Lj} + Y_{kj}, \quad j = \delta_{ab} \geqslant 0. \tag{A.4.11}$$

Substituting formulas (A.4.7), (A.4.8), (A.4.10) into (A.4.11), and keeping in /233
mind (A.4.6), we produce for possible selections of values δ_{ab} an equation determining the total sum Y of components of type K and L:

$$27.\ Y = N^2 + 4\sum_{j=1}^{N-1} (N - j)^2.$$

Performing addition and using the relationships below (Bronshteyn, Semendyayev, 1945)

$$28.\quad \sum_{j=1}^{N-1} j = \frac{(N-1)N}{2}; \qquad \sum_{j=1}^{N-1} j^2 = \frac{(N-1)(N-2)N}{6}, \tag{A.4.12}$$

we finally write

$$29.\quad Y = N^2 + 2\frac{N(N-1)(2N-1)}{3} \tag{A.4.13}$$

a \ b	1	2	3	4		N-3	N-2	N-1	N
1	2	3	4	5		N-2	N-1	N	N+1
2	3	4	5	6		N-1	N	N+1	N+2
3	4	5	6	7		N	N+1	N+2	N+3
4	5	6	7	8		N+1	N+2	N+3	N+4
N-3	N-2	N-1	N	N+1		2N-6	2N-5	2N-4	2N-3
N-2	N-1	N	N+1	N+2		2N-5	2N-4	2N-3	2N-2
N-1	N	N+1	N+2	N+3		2N-4	2N-3	2N-2	2N-1
N	N+1	N+2	N+3	N+4		2N-3	2N-2	2N-1	2N

B)

Figure 80

Similar discussions can be presented for calculation of the total number x of terms of type V [see (A.4.5)]. Figure 80,B shows a quadratic matrix $\|\gamma_{ab}\|$, the elements of which are the possible sums of indices a and b:

$$30.\quad \gamma_{ab} = a + b,$$

where /234

$$31.\quad 2 \leqslant \gamma_{ab} \leqslant 2N.$$

All diagonals of the matrix parallel to diagonals defined by the equation

$$32.\ \gamma_{ab} = a + b = N + 1, \tag{A.4.14}$$

contain equal elements γ_{ab} = const, while the total number of terms W'_{ab} in each of these is equal to

$$33.\ W'_{ab} = (N - |N + 1 - \gamma_{ab}|). \tag{A.4.15}$$

It follows from the last expression that the sum X_{ab} of components of type V [see (A.4.1), (A.4.5)] for a certain value of combinations of indices γ_{ab} = const (a + b = c + d) can be determined usng the formula

$$34.\ X_{ab} = (N - |N + 1 - \gamma_{ab}|)^2. \tag{A.4.16}$$

Determining in (A.4.16) the sign of the modulus

$$35.\ X_{ab} = (2N - \gamma_{ab} + 1)^2, \qquad N + 1 < \gamma_{ab} \leqslant 2N;$$
$$36.\ X_{ab} = (\gamma_{ab} - 1)^2, \qquad 2 \leqslant \gamma_{ab} \leqslant N + 1$$

and considering

$$37.\ X = N^2, \qquad \gamma_{ab} = N + 1,$$

we produce, keeping in mind the symmetry of the number of diagonals and the number of elements in them relative to diagonal γ_{ab} = N + 1, an equation for the total number X of terms of type V, corresponding to possible combinations of the indices a + b = c + d in the expression (A.4.1) [see (A.4.12)]

$$38.\ X = N^2 + 2\sum_{j=2}^{N}(j - 1)^2 = N^2 + \frac{N(N-1)(2N-1)}{3}. \tag{A.4.17}$$

We note that in the case a + b = c + d and a = b = c = d, the resulting number of terms X' for various combinations of indices is equal to N:

$$39.\ X' = N. \qquad (A.4.18)$$

The sum Z of components of type M and P [see (A.4.5b)] with combinations of indices

$$40. \qquad a - b = c + d; \; -(a - b) = c + d \qquad (A.4.19)$$

is determined on the basis of combined analysis of matrices $\|\delta_{ab}\|$ and $\|\gamma_{cd}\|$ where

$$\begin{aligned} &41.\ \delta_{ab} = a - b; \quad \gamma_{cd} = c + d; \\ &42.\ 0 \leqslant |\delta_{ab}| \leqslant N - 1; \qquad 2 \leqslant \gamma_{cd} \leqslant 2N. \end{aligned} \qquad (A.4.20)$$

From the latter inequalities, when condition (A.4.19) is fulfilled, it follows that

$$43.\ 2 \leqslant \gamma_{cd} = |\delta_{ab}| \leqslant N - 1.$$

The sum Z_{ab}, consisting of the sum Z_M of terms of type M and the sum Z_P of terms of type P,

$$44.\ Z_{ab} = Z_M + Z_P = 2Z_M = 2Z_P,$$

corresponding to a certain fixed combination of indices /235

$$45.\ \gamma_{cd} = |\delta_{ab}| = \mathrm{const}, \qquad (A.4.21)$$

on the strength of symmetry of matrices $\|\delta_{ab}\|$ considering the sign of the modulus in (A.4.21), is determined by the equation

$$46.\ Z_{ab} = 2(\delta_{ab} - 1)(N - \delta_{ab}), \quad \delta_{ab} > 0. \qquad (A.4.22)$$

The total number Z of components of type M and P in expression (A.4.1), corresponding to possible values of the combinations of indices

$$47. \quad a - b = c + d; \; -(a - b) = c + d,$$

can be found from the relationship

$$48. \quad Z = 2 \sum_{j=2}^{N-1} (j-1)(N-j),$$

which, considering (A.4.12), gives us

$$49. \quad Z = \frac{N(N-1)(N-2)}{3}. \tag{A.4.23}$$

The sum of terms of type R and S with various combinations of indices [see (A.4.5c)] is determined similarly.

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